Capacity Planning in a General Supply Chain with Multiple Contract Types

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***Abstract*— The successful commercialization of any new product depends to a degree on the ability of a firm to match its supply to market demand. In an emerging industry where products have little similarity with the products in existing industries, it is very hard to predict demand patterns. In this paper, we will develop a general mathematical model for providing decision support for the design of supply chains for emerging industries. In particular, we will focus on how capacity investments in a general supply chain can be made in the presence of demand uncertainty and different types of contracts. We will develop an efficient and practical algorithm for finding the optimal capacity planning strategy in a multi-product and multi-stage supply chain model and study the properties of the optimal strategies.**

***Index Terms*— Capacity planning, new product, supply chain design, contract option.**

I. Introduction

One of the key challenges of the commercialization process of products in an emerging industry is to design an effective supply chain that can meet market demand with high quality products in a timely fashion at competitive prices. Since there is little data on the commercial uptake of the products in these industries, it is difficult to predict the demand patterns of the products. One example that motivates our research is the micro-fluidic devices industry. Researchers have demonstrated that micro-fluidic devices will benefit many industries and research processes. These devices can be used by cancer research laboratories and drug development companies to perform specific biological analysis tests. Many companies believe that these devices have a bright future and are starting to commercialize these products. However, there is limited information on the commercial uptake of these devices by the pharmaceutical industry in their research process; thus, the micro-fluidic manufacturing firms must plan their capacities at a time when the demand patterns of these devices are currently unknown.

Besides uncertainty of demands, the manufacturers also face the difficulties of planning resources for multiple products at the same time. Due to the wide range of applications, the manufacturers need to produce a variety of generic or custom-made micro-fluidic devices to meet the requirements of their customers. Such variety in products adds complexity to the manufacturers' supply chain and requires them to plan their resources in a general setting. Manufacturers, however, are looking for efficient and practical algorithms for solving capacity planning problem in a general setting.

Since micro-fluidic device manufacturers are still in the early stages of designing their supply chains, they have the privilege to incorporate different types of capacity contracts without high administrative cost. Traditionally, a manufacturer establishes a fixed-cost capacity contract with its suppliers to buy a fixed amount of capacity. They need to pay the price whether they use the capacity or not. In practice, the cost of capacity might have two components: a fixed cost and a variable cost. In an option contract, the manufacturers buy rights to use a fixed amount of capacity with an upfront fixed payment. If they decide to execute their rights and use these capacities, they need to pay an exercise price for each unit of capacity that they actually use.

In this paper, we develop a mathematical model to study capacity planning in a multi-product and multi-stage supply chain with different types of capacity contracts. We study the properties of the optimal capacity planning strategies. We also develop an efficient and practical algorithm to find the optimal capacity planning strategies. The rest of the paper is organized as follows. Section II states the formulation of a multi-product multi-stage capacity planning problem. Section III outlines and compares different algorithms for solving the capacity planning problem. Section IV studies the properties of the optimal capacity planning strategy. Section V discusses how to extend the single-period model to a multi-period setting. Finally, Section VI concludes the paper.

Related Literature. There is a large amount of

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literature studying capacity planning under uncertainty and fixed-cost contracts. Fine and Freund (1990) consider capacity investment strategies in flexible resources, Barahona et al. (2005) examine capacity acquisition schedule in the context of semiconductor tool planning, Huang and Ahmed (2006) study the problem where the capacity decision can be revised as more demand information is revealed, and Zhang, et al. (2004) look at the capacity expansion problem with special demand structure. These works either have more restrictive assumptions on supply chain structure or demand distribution or focus on some particular industries such as semiconductor. Van Meighem and Rudi (2002) propose a newsvendor network which is closely related to the model that we use. All of these papers only consider a fixed-cost contract for determining the capacity level.

The consideration of option contracts in supply chains is a more recent research topic. Martinez-de-Albéniz and Simchi-Levi (2002) analyze the optimal option contract for a case of single product and single supplier. Yazlali and Erhun (2006) consider option contracts in a single product dual supply problem. Both of these works take lead time into consideration. Even though we do not consider lead time, our model allows a more general setting.

Another stream of literature that is related to our work is that for algorithms for stochastic linear programming. We refer readers to Kall and Mayer (2006) for a review. Our model for capacity planning problem in supply chain system can be viewed as a stochastic linear program.

II. MODEL

We consider a multi-product, multistage supply chain consisting of M products, J processes, and K resources. The production of each product requires a certain amount (possibly zero) of each type of process. For instance, we might have two process types – assembly process and testing process. A resource provides capacity for one or more processes. For instance, a resource might be an assembly line with the capability to assemble a single product type. A flexible resource might be an assembly line capable of assembling several different product types. We might also imagine a resource with capability to provide more than one type of process; for instance, a resource might do both assembly and test for a single product type. Without loss of generality, we assume that to produce one unit of product, it requires one unit of each of its required processes; we also assume that to get one unit of a process, we need one unit of capacity from one of its resources.

There are multiple options for procuring or reserving capacity for each resource. A firm can reserve capacity on a resource with a fixed-cost capacity contract; alternatively a firm can reserve capacity on a resource with an option contract where there is smaller upfront fixed cost and then a variable cost for the use of this capacity. For instance, under a fixed-cost capacity contract, the price for one unit of capacity is 1 dollar. Under option contract, the firm might pay a fixed cost 30 cents upfront for one unit of the capacity. If the firm decides to use the capacity that it has reserved, it needs to pay another 80 cents per unit. Given these alternatives, the firm wants to find the types of resources and contracts to use so that the resulting supply chain can maximize the firm's expected profit.

We denote

- D A vector of random variables, with probability density function $f(D)$, that represents the demand of products. (Vector of size M)
- d A realization of random demand D. (Vector of size M)
- z Amount of products that are produced. (Vector of size M)
- x_{jk} Amount of resource k provided under a fixed-cost capacity contract that is used to provide capacity to process j. (Scalar)
- x The vector of x_{jk}. (Vector of size JK)
- y_{jk} Amount of resource k provided under an option capacity contract that is used to provide capacity to process j. (Scalar)
- y The vector of y_{jk}. (Vector of size JK)
- A An $J \times M$ matrix such that
$$A(j,m) = \begin{cases} 1, & \text{if product } m \text{ requires process } j; \\ 0, & \text{otherwise.} \end{cases}$$
- B An $J \times JK$ matrix such that
$$B(j,(j,k)) = \begin{cases} 1, & \text{if resource } k \text{ can provide} \\ & \text{capacity to process } j; \\ 0, & \text{otherwise.} \end{cases}$$
- H A $K \times JK$ matrix such that
$$H(k,(j,k)) = \begin{cases} 1, & \text{if resource } k \text{ can provide} \\ & \text{capacity to process } j; \\ 0, & \text{otherwise.} \end{cases}$$
- C The amount of fixed-cost capacity that the firm has reserved. (Vector of size K)
- G The total amount of capacity, including fixed-cost and option capacity, that the firm has reserved. (Vector of size K)
- r Unit profit generated from filled products. (Vector of size M)
- p Unit price of resources under fixed-cost contract. (Vector of size K)
- q Unit upfront price of resources under option contract. (Vector of size K)
- e Unit exercise price of resources under option contract. (Vector of size K)

We assume that any demand that cannot be filled is lost. We also assume a two-stage sequential decision process. In the first stage, the firm determines the types and sizes of the contracts with its suppliers or contract manufacturers.

In the second stage, demand is realized and the firm allocates production capacity to meet demand.

We now formulate the second stage problem as a single period production planning problem with the objective to maximize the profit of the firm. We are given the demand realization d as well as the decisions on C, the amount of resource to reserve with fixed-cost contract and G, the total amount of resource to reserve. We have the following linear optimization problem:

$$\begin{aligned} P_2(C,G,d) = \max_{x,y,z} \quad & \pi(C,G,d) = r'z - e'Hy \\ s.t. \quad & z \le d \\ & Az \le B(x+y) \\ & Hx \le C \\ & Hy \le G - C \\ & x, y, z \ge 0. \end{aligned} \tag{1.1}$$

By solving this optimization problem, we can find the profit maximizing production level for a given demand realization and the capacity planning decisions. The firm ultimately wants to find the optimal capacity planning strategy under demand uncertainty:

$$\begin{aligned} P_1 = \max_{C,G} \quad & \prod(C,G) = E[P_2(C,G,D)] - p'C - q'(G-C) \\ s.t. \quad & C \le G \end{aligned} \tag{1.2}$$

Proposition 1: $\prod(C,G)$ is concave in both C and G.

Proposition 1 guarantees the existence of an optimal solution for problem (1.2) and also the convergence of algorithms given in the following section.

III. SOLVING THE CAPACITY PLANNING PROBLEM

In this section, we examine two alternative algorithms for solving the capacity planning problem (1.2).

A. Sub-gradient Method

Van Meighem and Rudi (2002) found the necessary and sufficient conditions for a different but similar capacity planning problem. In their model, the firm cannot reserve capacity through options. They propose an algorithm for their problem:

1. Given capacity $C^{(i)}$ [2], solve the LP (1.1) and find the associated dual variables $\lambda(C^{(i)}, d^{(j)})$ numerically for each sample demand vector $d^{(j)}$. Take the average of the $\lambda(C^{(i)}, d^{(j)})$ over all j as an unbiased estimate of $E[\lambda(C^{(i)}, d^{(j)})]$, and use it to compute an estimate of the sub-gradient $\nabla\prod(C^{(i)})$.
2. If $|\nabla\prod(C^{(i)}) - p|$ is smaller than some tolerance level, then stop. Otherwise, adjust capacity in the direction of the sub-gradient: $C^{(i+1)} = C^{(i)} + \xi(\nabla\prod(C^{(i)}) - p)$, where ξ is some step-size (or perform a line-search), and iterate.

The algorithm uses sub-gradient method. At each step, it will need to solve S LPs where S is the number of sample demand points that is used to estimate the sub-gradient. The computational requirements at each step can be very intensive depending upon the number of sample points. The algorithm can take a very long time converge, due to the following observations:

1. *The convergence rate is constrained by the bottleneck processes.* To produce a product, the firm needs to plan the capacity of all processes for the product at the same time. If one of the processes is short of capacity, the production is constrained by the bottleneck process, which dictates the sub-gradient. Consider the following example: The firm produces a single product that requires two types of processes a and b. Resource 1 can provide fixed capacity to process a at price 5 per unit and resource 2 can provide fixed capacity to process 2 at price 4 per unit. The demand for the product follows a uniform distribution between 100 and 120. The price for the product is 12 per unit. The optimal capacity strategy will be $100 \le C_1 = C_2 \le 120$ for some value of $C_1 = C_2$. Now, suppose we start with initial point $C_1 = 10$ and $C_2 = 11$. Since $C_1 < C_2 < 100$, $\nabla_{C_1}\prod = 12 - 5 = 7$ and $\nabla_{C_2}\prod = 0 - 4 = -4$. The sub-gradient algorithm will adjust the capacity as follows:
$$\begin{bmatrix} C_{1,new} \\ C_{2,new} \end{bmatrix} = \begin{bmatrix} C_{1,old} \\ C_{2,old} \end{bmatrix} + \xi \begin{bmatrix} 7 \\ -4 \end{bmatrix}$$
We also observe that when $C_2 < C_1 < 100$, the sign of the sub-gradient is reversed. Thus, depending upon how we set the step size, the sub-gradient algorithm can take a long time to converge as it will cycle back and forth between these two sub-gradients.
2. *The convergence rate is constrained by the non-uniqueness of the sub-gradient.* In a typical capacity planning problem, the number of processes is larger than the number of products and the number of resources is larger than the number of processes. Therefore, for some capacity planning strategies (C,G) and demand d, the solution of the dual problem of (1.1) is not unique. Therefore, the sub-gradient at some capacity strategies (C,G) is not unique. Following different sub-gradients will have very different convergence rates.
3. *The convergence rate depends heavily on the starting point.*
4. *The convergence rate depends heavily on the step size.*
5. *Lack of good termination criterion.* Due to sampling error, the termination criteria, $|\nabla\prod| < \varepsilon$, is hard to satisfy.

B. Supporting Hyperplane Algorithm

Let's consider a new problem:

[2] Since the firm cannot reserve option capacity, $C = G$.

$$\min \quad f$$
$$s.t. \quad f + E[P_2(C,G,D)] - p'C - q'(G-C) \geq 0 \quad (2.1)$$
$$C \leq G$$

It can been shown that (C^*,G^*) solves problem (1.2) iff (C^*, G^*, f^*) solves problem (2.1) with $f^* + E[P_2(C^*,G^*,D)] - p'C^* - q'(G^* - C^*) = 0$. To solve problem (2.1), we can use the supporting hyperplane algorithm suggested by Veinott (1967). Let C_{upper} (C_{lower}) and G_{upper} (G_{lower}) be the upper (lower) bounds of the fixed and total capacities. Let f_{upper} (f_{lower}) be the upper (lower) bound of f. Let

$$T^0 = \{(C,G,f) : C \in [C_{lower}, C_{upper}], G \in [G_{lower}, G_{upper}], \\ f \in [f_{lower}, f_{upper}], C \leq G\}.$$

Let $s = 0$, the algorithm consists of the following steps:

1. Solve the linear program of minimizing f, subject to $(C,G,f) \in T^s$, and let (C^s, G^s, f^s) be the optimal solution. If
$$f^s + E[P_2(C^s,G^s,D)] - p'C^s - q'(G^s - C^s) \geq -\varepsilon$$
where ε is a small positive number chosen by the user, stop. Otherwise, go to step 2.
2. Use the simulation method given in the sub-gradient algorithm to calculate the sub-gradient $\Delta_C \prod$ and $\Delta_G \prod$. Add linear constraint
$$f + \prod(C^s,G^s) + \\ [(C,G) - (C^s,G^s)]^T (\nabla \prod_{C^s}, \nabla \prod_{G^s}) \geq 0 \quad (2.3)$$
to the set T^s. Let the new set be T^{s+1}. Set $s = s+1$ and go to step 1.

Geometrically, the supporting hyperplane method approximates function $\prod(C,G)$ with hyperplanes. At each step, the algorithm uses all the sub-gradients that it has calculated so far. Therefore, it overcomes observations 1 and 2 of the sub-gradient algorithm. By the nature of supporting hyperplane algorithm, it does not require a starting point or a step size. Finally, at each step - f^s is an upper bound of $\prod(C^*,G^*,D)$. Therefore, ε is an upper bound for $|\prod(C^s,G^s,D) - \prod(C^*,G^*,D)|$. This termination criterion is a better indicator of whether the solution is close enough to the optimum or not.

The supporting hyperplane method also suffers from the high cost of calculating the sub-gradient. This, however, can be improved by using a stochastic update method suggested by Higle and Sen (1991). At each iteration s, the algorithm simulates one demand realization d^s. Let $V^s = \{d^1, ..., d^s\}$ denote the set of demand realizations that have been simulated so far. The supporting hyperplane algorithm with stochastic update is as follows:

1. Set $s = 1$, $V^0 = \varnothing$, and
$$T^0 = \{(C,G,f) : C \in [C_{lower}, C_{upper}], G \in [G_{lower}, G_{upper}], \\ f \in [f_{lower}, f_{upper}], C \leq G\}$$
2. Simulate a demand realization d^s and let $V^s = V^{s-1} \cup d^s$. Add linear cut
$$f + \prod(C^s,G^s,V^s) + \\ [(C,G) - (C^s,G^s)]^T (\nabla \prod_{C^s}, \nabla \prod_{G^s}) \geq 0 \quad (2.4)$$
For $k = 1..s-1$, update all previous cuts
$$f + \prod(C^k,G^k,V^k)\frac{s}{s+1} + P_2(C^k,G^k,d^s)\frac{1}{s+1} + \\ [(C,G) - (C^k,G^k)]^T \begin{bmatrix} (\nabla \prod_{C^k}, \nabla \prod_{G^k})\frac{s}{s+1} \\ +(\nabla P_{2C^k}, \nabla P_{2G^k})\frac{1}{s+1} \end{bmatrix} \geq 0 \quad (2.5)$$
Let the new set of constraint to be T^s.
3. Solve the linear program of minimizing f, subject to $(C,G,f) \in T^s$, and let (C^s, G^s, f^s) be the optimal solution. If
$$f^s + E[P_2(C^s,G^s,V^s)] - p'C^s - q'(G^s - C^s) \geq -\varepsilon \quad (2.6)$$
where ε is a small positive number chosen by the user, stop. Otherwise, set $s = s+1$ and go to step 2.

IV. Properties of Optimal Strategy

In this section we will study the properties of optimal strategies. We first look at the effects of changes of demands. Let I be a $J \times K$ matrix such that

$$I(j,k) = \begin{cases} 1, \text{ if } p_k = \min\{p_n \mid B(j,(j,n)) = 1\}; \\ 0, \text{ otherwise.} \end{cases}$$

If $I(j,k) = 1$, it means that using resource k is the cheapest way to provide capacity to process j. WLOG, we assume that there is an unique k for each j such that $I(j,k) = 1$.

Proposition 2: *Let* (C^*,G^*) *be the optimal solution of capacity planning problem* (D,A,B,H,r,p,q,e) *. Let* $\hat{D}$ *be another set of random demand that is different to D only in its first moments. Let* $\Delta = E\left[\hat{D}\right] - E\left[D\right]$. *Let* $\left(\hat{C}^*, \hat{G}^*\right)$ *be the optimal solution of capacity planning problem* $\left(\hat{D},A,B,H,r,p,q,e\right)$. *Then* $\hat{C}^* = C^* + I'A\Delta$ *and* $\hat{G}^* = G^* + I'A\Delta$.

This proposition suggests that if the first moment of the demand vector changes, the firm doesn't need to recalculate the optimal capacity planning strategy. The new optimal strategy can be obtained by using the method suggested in the proposition.

The effects of unit profits and unit prices on optimal capacities are more complicated and less intuitive. For example,

- If unit profits for some products increase, the optimal total capacities for some resources might decrease. When unit profits increase, one would

expect that the firm will reserve as least as much capacity as before. This, however, might not always be true.

- Let (C^*, G^*) be the optimal capacity planning strategy for problem (D, A, B, H, r, p, q, e). Let's assume that $G^* > C^*$; as the unit profits r for some products increase, the optimal fixed capacity for some resources might also increase. If $G^* > C^*$, C^* indicates the optimal trade-off threshold between fixed capacity and option capacity. One might expect that this threshold only depends on the price ratio between fixed capacity and option capacity as in the single product case. However, for the case of multi-products, it also depends on the unit profits of the other products.

To illustrate the effects of unit profits, we consider the following example which contains 5 products, 9 processes, and 9 resources. The structure of the supply chain is given in Figure 1. The demand for each product follows a normal distribution $N(120, 10)$. We set $r = [30, 50, 46, 41, 25]$, $p_k = 10\ \forall k$, $q_k = 8\ \forall k$, and $e_k = 3\ \forall k$. We plot the change of optimal strategy for one of the resources as unit profits increase in Figure 2. We can see that both the optimal total capacity and the fixed capacity increase as unit profits increase. Also, the ratio between the option capacity and the fixed capacity increases as the unit prices increase. This means that as unit prices increase the firm will increase the amount of option capacity in the optimal strategy. Moreover, both curves have a concave structure. This is because the utilization of an additional unit of capacity deceases as the total capacity and fixed capacity increase.

Finally, we look at the effects of unit prices. We set $q + e$ to be a constant and increase q. When q is small, the firm pays less up-front cost to reserve capacity and a higher exercise price. When q is large, the firm will pay more to reserve and less to use the capacity. If $q + e$ is a constant, the firm prefers to pay less up-front cost since the penalty of over reservation is less. This intuition is confirmed by the plot given in Figure 3. When q is small, the firm reserves more capacity in total and less fixed capacity and when q is large the firm reserves less capacity in total and more fixed capacity.

V. Multi-period capacity planning and integer constraint

In this section, we discuss how to extend the single period model to a multi-period setting and how to solve the problem if capacity only can be reserved in indivisible units.

Depending on the time length of the contracts, there are different ways to formulate a multi-period capacity planning problem. If the contracts require a long term commitment, after the firm signs the contract to acquire capacity, the same amount of capacity might need to be bought or reserved in each period until the end of the planning horizon. On the other hand, if the contracts are short term, the firm can reserve different amounts of capacity for different periods. Huang, et al. (2006), Roundy et al. (2004), Barahona et al. (2005), and Martinez-de-Albéniz and Simchi-Levi (2002) consider long term contracts while Yazlali and Erhun (2006) use short term contract. For both formulations, we can show that if demands from different periods are independent, we can decompose the multiple-period problem into a series of single period problems. However, if demands from different periods are not independent, a simple decomposition algorithm might not be applicable. We will address this problem in our future research.

In practice, the capacity might only be procured or reserved in bulk units. This requires that the decision variables, C and G, to be integer multiples of some base unit. Having integer decision variables will increase the difficulty of solving the problem. Barahona et al. (2005) and Ahmed and Garcia (2003) have proposed some approximation algorithms that can be used to cope with these difficulties. Their algorithms need to solve an LP-relaxation of the integer programming problem. Similar types of technique might be used with the algorithm given in this paper to solve integer capacity planning problem.

VI. Conclusion

In this paper, we propose a model to study capacity planning in a multi-product and multi-stage supply chain with multiple types of contracts. The model is very general so that manufacturers can use it to plan their resources and also design their supply chain structure. We also give a practical algorithm for solving the capacity planning problem. We believe that our work opens the door to many future research topics.

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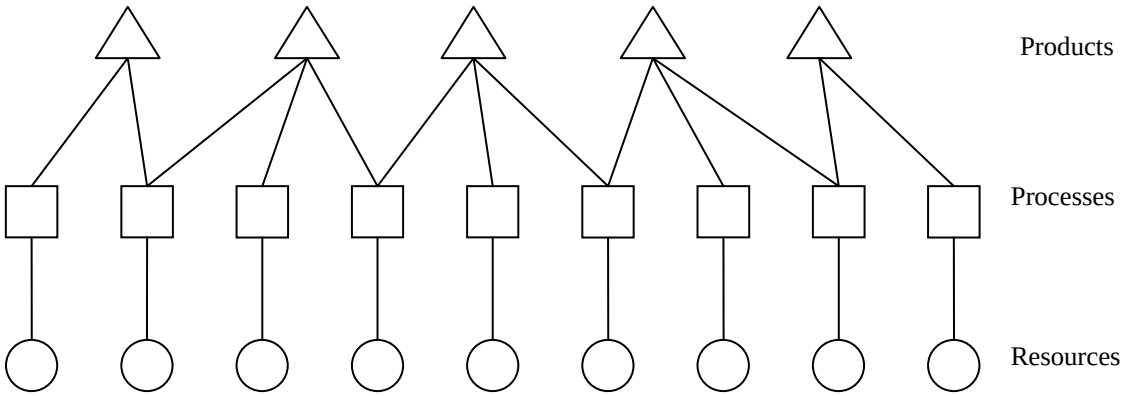

Figure 1: A supply chain with 5 products, 9 processes, and 9 resources.

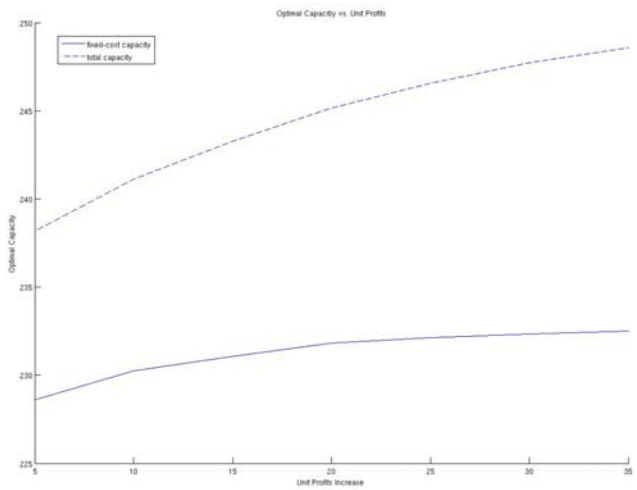

Figure 2: Optimal Capacity vs. Unit Profits Increment

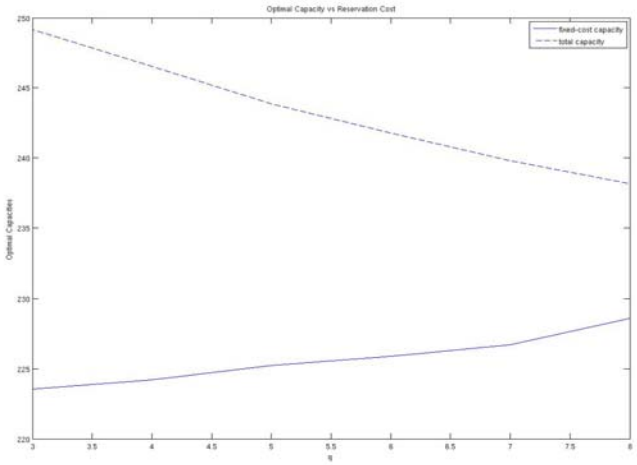

Figure 3: Optimal Capacity vs. Up-front Reservation Price