Appendix A: Mathematical description and derivations

The following appendix contains the mathematical description of the experimental market and the derivations used in the analysis in the thesis. The appendix is fairly voluminous, in large part because it also serves as a record of the derivations. The first section describes the market structure and derives the competitive and collusive equilibria. Following this are three sections, one for each price regime, in which the optimal policies and the rational-expectations stochastic equilibria are derived. In some cases one cannot find exact analytic expression but must instead rely on linear- or quadratric approximations and numerical analysis. Section 5 shows the variance and spectral density of output and prices in rational-expectations equilibrium, for each of the six regimes. The last three sections contain subsidiary notes on optimal control of linear systems with first-order terms, comparison of the simple averages X, P to the non-linear aggregates $\tilde{X}$, $\tilde{P}$, and on the expectation $E\{|n|\}$, respectively.

Notation

To facilitate reading of equations, the following conventions are observed in the following:

- Upper-case symbols (X,Y,N, etc.) denote market averages or aggregates while lower-case symbols (x, y, n, etc.) denote the corresponding individual firm variables. Moreover, for non-linear aggregations, a superscript squiggle (~) will be used. Otherwise, the aggregates are all arithmetic averages.

- Boldface symbols (**z**, **Z**, **u**, etc.) denote vectors while matrices are denoted by boldface with a double underline ($\underline{\underline{\mathbf{P}}}$, $\underline{\underline{\mathbf{Q}}}$, $\underline{\underline{\mathbf{R}}}$, etc.).

- Parameters are denoted by greek letters while variables and derived constants are denoted by roman letters.

- At its **first introduction**, each symbol is printed in **boldface**.

- Likewise, **assumptions** will be highlighted whenever made.

Appendix A

1. Market system

Firms

The market consists of **K firms**, indexed by **i = 1, ..., K. Time** is divided into discrete periods, indexed by **t = 0, 1, ...** .

At the beginning of each period, t, each firm, i, must decide how much **production to initiate, $y_{i,t}$,** to initiate and what **price, $p_{i,t}$,** to charge for its product this period. Production starts are constrained to be non-negative. The firm must make these decisions *ex ante,* i.e. without knowing the demand it faces this period.

Each firm maintains a goods **inventory, $n_{i,t}$,** to accommodate fluctuations in demand. Inventories can be negative, corresponding to an order backlog. The inventory is decreased by **sales, $x_{i,t}$** , and increased by production. There may be a **lag of δ periods** between the initiation of production, and the time it arrives in inventory. Thus, the movement of inventories/backlogs is described by

(1.1) $$n_{i,t+1} = n_{i,t} + y_{i,t-\delta} \text{ for all i,t.}$$

Profits, $v_{i,t}$, each period is the difference between revenue and costs. Costs consist of production cost, proportional to sales, and inventory holding costs, proportional to the absolute value of the inventory at the beginning of the period. Production costs and revenues are assumed to be incurred at the time the goods are ordered by buyers, regardless of when the good is actually produced and/or delivered. Given **unit production costs** ω and **unit inventory/backlog holding costs** γ, firm i's profit in period t is

(1.2) $$v_{i,t} = p_{i,t}x_{i,t} - \omega x_{i,t} - \gamma | n_{i,t} | \text{ for all i,t.}$$

The inventory backlog cost is conceived as a combination of holding and processing costs and, in the case of a backlog, a "discount" offered to consumers to compensate for lost utility due to late delivery.

Firms have "full aggregate information" in the sense that they can observe past values of all their "own" variables, such as price, inventory, and production, and they can observe past values of the market average values of these variables.

Appendix A

Demand side

The demand side of the market is assumed to consist of an arbitrary but large number of buyers who purchase (or order) goods from individual firms so as to maximize their utility. It is assumed that buyer utility is a Constant-Elasticity-of-Substitution (CES) function of goods bought from individual firms, with an **elasticity of substitution** $\varepsilon > 1$, and that buyer utility is "obtained" at the time of purchase or ordering, regardless of when the good is in fact delivered. (The loss of utility from late delivery is assumed to be fully compensated by firms, in the form of the backlog cost, γ.) Thus, buyer utility is an instantaneous function of the "**aggregate good**", $\tilde{X}_t$, determined by

$$(1.3) \qquad \tilde{X}_t = \left(\frac{1}{K}\left(x_{1,t}{}^{(\varepsilon-1)/\varepsilon} + \dots + x_{K,t}{}^{(\varepsilon-1)/\varepsilon}\right)\right)^{\varepsilon/(\varepsilon-1)}, \text{ for all t.}$$

Given the **average amount of consumer spending per firm, I** on all the goods in the market during a particular period, consumers thus solve the problem of maximizing $\tilde{X}$ over the individual goods x_i, subject to the budget constraint

$$(1.4) \qquad p_{1,t}x_{1,t} + \dots + p_{K,t}x_{K,t} = K\,I, \text{ for all t.}$$

The first-order conditions for this problem yield

$$(1.5) \qquad \frac{x_{i,t}{}^{(\varepsilon-1)/\varepsilon}}{x_{j,t}{}^{(\varepsilon-1)/\varepsilon}} = \frac{p_{i,t}{}^{1-\varepsilon}}{p_{j,t}{}^{1-\varepsilon}}, \text{ for all i, j, t.}$$

Summing this expression over all j and inserting into (1.3) yields

$$(1.6) \qquad x_{i,t} = \tilde{X}_t(p_{i,t}/\tilde{P}_t)^{-\varepsilon}\ \forall\ i, t,$$

where the "**aggregate" price, $\tilde{P}_t$,** is defined by

$$(1.7) \qquad \tilde{P}_t = \left(\frac{1}{K}\left(p_{1,t}{}^{1-\varepsilon} + \dots + p_{K,t}{}^{1-\varepsilon}\right)\right)^{1/(1-\varepsilon)}, \forall\ t.$$

Multiplying (1.7) by $p_{i,t}$, summing over i, and inserting in the budget constrant (1.4) further yields

$$(1.8) \qquad I_t = \tilde{X}_t\tilde{P}_t, \forall\ t.$$

Thus, $\tilde{P}_t$ can be interpreted as an aggregate price index, or the "price" of the aggregate good $\tilde{X}_t$.

Buyers are further assumed to embed their total purchases from this market in the context of a larger optimization problem involving other markets, savings, leasure, etc., so that the aggregate market demand (or demand for the aggregate good) in period t, $\tilde{X}_t$, is a function of the the aggregate price, $\tilde{P}_t$.

Moreover, the aggregate market demand is assumed to depend on the overall level of activity in that market, i.e. there is a "multiplier" effect from production to demand. Economically, the effect can be interpreted either as a Keynesian consumption multiplier, where wage income is related to the level of activity, or as an input-output mutliplier, where firms use part of each others products as inputs to the production process.

Specifically, $\tilde{X}_t$ is determined by

(1.9) $$\tilde{X}_t = X_t^* f(\tilde{P}_t);$$

(1.10) $$X_t^* = (1-\alpha)G_t + \alpha\frac{1}{\delta+1}(Y_t+S_t);\ 0\leq \alpha \leq 1;$$

(1.11) $$Y_t = (y_{1,t} + \dots + y_{K,t})/K;$$

(1.12) $$S_t = (Y_{t-\delta} + \dots + Y_{t-1}),\ \forall\ t.$$

X_t^* is the **"reference demand,"** which is multiplied by the function f(.) of the aggregate price to get actual demand. The "reference" demand consists of **autonomous demand, G_t**, and a proportion attributed to the multiplier effect. The parameter α is the indicates the **strength of the multiplier effect**, i.e. the increase in reference demand for each unit increase in the average production level, i.e., the average production over all firms and all δ production stages. In equations (1.10) to (1.12) a distinction is made for notational purposes between the **average production starts, Y_t** and the **average "supply line", S_t**, of previously started but not yet completed production.

The **price-dependence of industry demand, f(.)**, is assumed to have a **constant elasticity, μ**, around the **perfect competition equilibrium price, p^***.

(p^* is a function only of the unit production cost, ω, and the elasticity of substitution, ε, as shown below but can be considered to be a derived parameter of the system.) The elasticity μ is assumed to be less than unity, reflecting the idea that, while the goods offered by different firms in the market are fairly close substitutes, the overall industry demand is inelastic. As the aggregate price moves further away from the competitive-equilibrium value, however, aggregate demand becomes a linear function of price. (If aggregate price-elasticity of demand was constant and less than unity for all prices, colluding firms could earn arbitrarily large profits by charging an arbitrarily large price.) Figure 1.1 shows a plot of the function f(.).

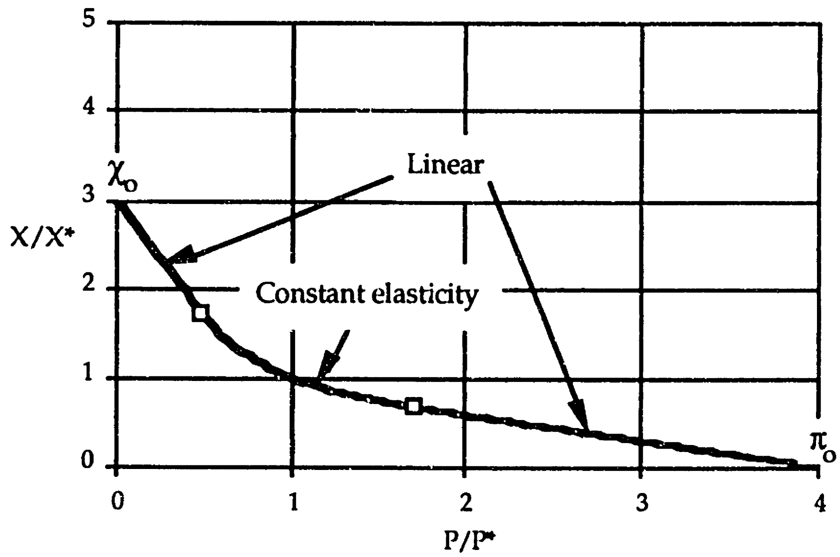

Figure 1.1
Plot of aggregate demand relative to "reference" aggregate demand, as a function of aggregate price relative to "reference" price.

Specifically, the function f(.) is formulated in terms of the ratio $\tilde{P}_t/p^*$, according to

$$(1.16) \qquad f(\tilde{P}_t) = \begin{cases} \chi_0 - b_1\tilde{P}_t/p^*, & 0 \le \tilde{P}_t/p^* < c_1; \\ (\tilde{P}_t/p^*)^{-\mu}, & c_1 \le \tilde{P}_t/p^* \le c_2; \\ a_2 - b_2\tilde{P}_t/p^*, & c_2 < \tilde{P}_t/p^* \le \pi_0; \\ 0, & \pi_0 < \tilde{P}_t/p^*, \forall\, t. \end{cases}$$

The parameter χ_0 is the ratio of demand to reference demand at zero price, and π_0 is the ratio of price to competitive equilibrium price at which

demand is zero. Given these cut-off points and the requirement that f(.) be differentiable, the **derived parameters** b_1, c_1, a_2, b_2, **and** c_2 fulfill

$$(1.17) \qquad b_1 = \frac{\mu\chi_o}{c_1(1+\mu)};\ c_1 = \left(\frac{\chi_o}{1+\mu}\right)^{-1/\mu};\ a_2 = b_2\pi_o;\ b_2 = \mu c_2^{-1-\mu};\ c_2 = \frac{\mu\pi_o}{1+\mu}.$$

Steady-state competitive equilibrium

If the market is in steady-state equilibrium with a constant autonomous demand component, G_t=G, inventories will be zero. If **furthermore all firms are assumed to act competitively,** they face the profit maximization problem

$$(1.18) \qquad \text{Max } v_i(p_i) = x_i(p_i - \omega),$$

where x_i is determined by the structural equations given above. The **steady state,** requires that (assuming firms are producing approximately the same outout)

$$X^* = G(1-\alpha) + \alpha\,\tilde{X} = G(1-\alpha) + \alpha\,X^*\,f(\tilde{P}) \Rightarrow$$

$$(1.19) \qquad \tilde{X}_{ss} = G\frac{(1-\alpha)\,f(\tilde{P})}{1-\alpha\,f(\tilde{P})}, \text{ where "ss" stands for steady state.}$$

The first-order condition, assuming other firms hold their prices (not their quantities) constant, yields

$$\frac{dv_i}{dp_i} = 0, \Rightarrow x_i + (p_i-\omega)\frac{dx_i}{dp_i} = 0, \Rightarrow$$

$$x_i + (p_i-\omega)\left(-\varepsilon x_i/p_i + (\varepsilon x_i/\tilde{P} + \frac{x_i}{\tilde{X}_{ss}}\frac{d\tilde{X}_{ss}}{d\tilde{P}})\frac{d\tilde{P}}{dp_i}\right) = 0, \Rightarrow$$

$$x_i + x_i(p_i-\omega)\left(-\varepsilon/p_i + (\varepsilon/\tilde{P} + \frac{f'(\tilde{P})}{f(\tilde{P})[1-\alpha f(\tilde{P})]})\frac{1}{K}(p_i/\tilde{P})^{-\varepsilon}\right) = 0, \Rightarrow$$

$$p_i + (p_i - \omega)\left(-\varepsilon + (\varepsilon + \frac{\tilde{P}\, f'(\tilde{P})}{f(\tilde{P})} \frac{1}{1-\alpha f(\tilde{P})}) \frac{1}{K} (p_i/\tilde{P})^{1-\varepsilon} \right) = 0. \tag{1.20}$$

It is possible to show (numerically, at least) that (1.20) has only one positive solution for p_i for any given aggregate price level, $\tilde{P}$. This in turn implies that the symmetric equilibrium where all firms charge the same price and produce the same output is the only possible Nash equilibrium. If, moreover, the function f(.) is in the constant-elasticity region, the **competitive-equilibirum price for K firms, $p_K{}^*$,** is given by

$$p_K{}^* = \frac{\omega\, \varepsilon_K}{\varepsilon_K - 1}, \text{ where } \varepsilon_K = \frac{K-1}{K} \varepsilon + \frac{1}{K} \mu\, [1 - \alpha\, f(p_K{}^*)]^{-1}. \tag{1.21}$$

The equation (1.21) cannot be solved analytically, but are easy to solve numerically. Table 1.1 shows the values of $p_K{}^*$ and teh **competitive-equilibrium output for K firms, $x_K{}^*$**,respectively, for various values of K. The assumption underlying these derivation was that firms optimize assuming other firms hold their <u>prices</u> constant (the so-called Bertrand game equilibrium). If, instead, firms assume that other firms hold their <u>sales</u> constant, one can go through a similar set of derivations, to find the **Cournot-equilibrium price, $p_K{}^{**}$, and output, $x_K{}^{**}$**, respectively. These are also listed in Table 1.1. If the number of firms goes to infinity, the solution converges to the **"perfect"-competitive-equilibrium price,**

$$p^* = \omega\varepsilon/(\varepsilon - 1) \tag{1.22}$$

and the **"perfect"-competitive-equilibrium output, G.**

K	Bertrand equilibrium				Cournot equilibrium			
	p_K^*/p^*		x_K^*/G		p_K^{**}/p^*		x_K^{**}/G	
	α=0	α=.5	α=0	α=.5	α=0	α=.5	α=0	α=.5
2	1.56	1.27	0.56	0.72		1.66		0.52
3	1.25	1.13	0.73	0.84	2.08	1.26	0.58	0.72
4	1.16	1.09	0.80	0.88	1.64	1.16	0.69	0.81
5	1.12	1.07	0.85	0.91	1.45	1.12	0.76	0.85
6	1.10	1.05	0.87	0.93	1.35	1.09	0.80	0.88
7	1.08	1.04	0.89	0.94	1.29	1.08	0.83	0.90
8	1.07	1.04	0.91	0.95	1.24	1.07	0.85	0.91
9	1.06	1.03	0.92	0.95	1.21	1.06	0.87	0.92
10	1.05	1.03	0.93	0.96	1.18	1.05	0.88	0.93
∞	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 1.1
Non-cooperative steady-state price and output level as a function of the number of firms.

Collusive equilibrium

If, instead, firms collude perfectly, they must solve the joint profit-maximization problem

(1.23) $\max V = (\tilde{P} - \omega)\tilde{X}$, where $\tilde{X}$ must satisfy (1.19).

It is clear that, since the elasticity of aggregate demand is $\mu<1$ around the constant-elasticity region of the function f(.), the collusive equilibrium must lie in the high-price, linear region.[1] By differentiating (1.23) and

1 If $\alpha > (1-1/\varepsilon)^{\mu}$ (which it isn't with the parameters chosen for the experiment), colluding firms could theoretically earn arbitrarily large profits by charging a price which would render the denominator in (1.19) zero: in that case the system has no equilibrium point, and demand can grow forever, fueled by the multiplier process.

inserting (1.19), one can find the profit-maximizing collusive price and output. The process is straightforward but tedious, and only the final results are shown here. The **collusive steady-state output, x^M**, is

$$(1.24) \qquad x^M = \begin{cases} qG/2, & \alpha = 0; \\ (1-\alpha)G\dfrac{1-q}{\alpha q}, & 0<\alpha<1, \end{cases}$$

and the **steady-state collusive price, p^M**, is

$$(1.25) \qquad p^M = \begin{cases} (\omega + \pi_0)/2, & \alpha = 0 \\ p^*(\alpha a_2 - 1 + q)/(\alpha b_2), & 0<\alpha<1 \end{cases}, \text{ where}$$

$$q = \sqrt{1 - \alpha(a_2 - b_2(1 - 1/\varepsilon))}.$$

Figure 1.2 shows steady-state profits and output as a function of price, all normalized by the competitive-equilibrium values. Thus, colluding firms could earn about 50% more in the complex condition (α=0.5) and about double in the simple condition (α=0).

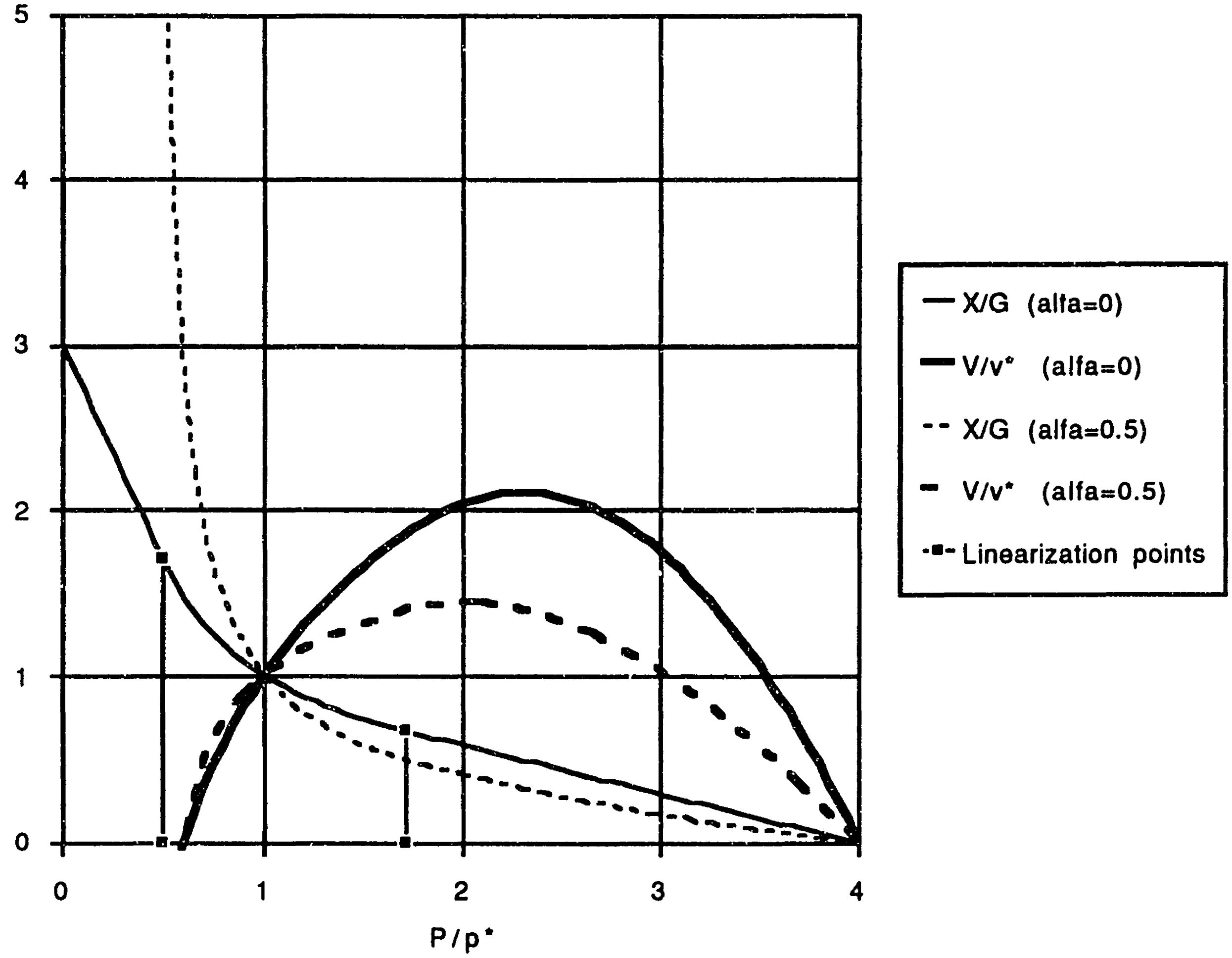

Figure 1.2
Steady-state output and profits as a function of price.
The parameters in f(.), etc., are those used in the experiment. Note that, for $\alpha>0$, the system does not have a steady-state solution below a certain price level.

2. Fixed prices

Optimal policy

In the following, firms are assumed to know the complete structure of the system and its parameters. Moreover, in solving for rational expectations, firms are assumed to follow the optimal policy subject only to a random, serially uncorrelated error. Thus, this section does not consider optimal (Bayesian) learning.

Appendix A

Since prices are held fixed at the perfectly-competitive level, p^*, the task of each firm is to meet incoming demand and minimize inventory costs. The demand for each firm is the same, equal to the reference demand, X^*, i.e.,

$$(2.1) \qquad x_{i,t} = (1-\alpha)G + \alpha\frac{1}{\delta+1}(Y_t+S_t).$$

Assuming first that the number of firms, K, is large or that firms believe the effect of their own actions on the aggregate to be negligible, and further assuming that the non-negativity constraint on production is not binding, the dynamic cost minimization problem can be decomposed into separate problems for each time period. One period's production decision affects only the inventory level $\delta+1$ periods hence; demand is assumed to be unaffected by the decision, and the inventory in later periods can be fully regulated by subsequent production decisions. Then, since costs are proportional to the absolute value of inventory, the problem becomes to choose output, $y_{i,t}$ to minimize the expected inventory costs, i.e., to minimize the expected absolute value of inventory. Using the results in Section 8, one finds the first-order condition

$$(2.2) \qquad 0 = \frac{\partial}{\partial y_{i,t}} E_t\{|n_{i,t+\delta+1}|\}$$

$$= [1 - 2F(n_{i,t+\delta+1})]\frac{\partial n_{i,t+\delta+1}}{\partial y_{i,t}}$$

$$= [1 - 2F(n_{i,t+\delta+1})],$$

where E_t is the expectation operator and F(.) is the cumulative distribution function of $n_{i,t+\delta+1}$, conditional on $y_{i,t}$. The first-order condition thus requires the <u>median</u> of $n_{i,t+\delta+1}$ to be zero. Further, since, by successive substitution in equation (1.1),

$$(2.3) \qquad n_{i,t+\delta+1} = n_{i,t} + y_{i,t-\delta} + ... + y_{i,t-1} + y_{i,t} - x_{i,t} - ... - x_{i,t+\delta},$$

the optimal policy, $y_{i,t}^*$, is

$$(2.4) \qquad y_{i,t}^* = M_t\{x_{i,t} + ... + x_{i,t+\delta}\} - (y_{i,t-\delta} + ... + y_{i,t-1} + n_{i,t}),$$

where M_t is the median operator. If the distribution of $x_{i,t} + ... + x_{i,t+\delta}$ is approximately symmetric, as will be assumed in the following, the median and the mean approximately coincide. One can now write the optimal production policy as

(2.5) $$y_{i,t}^* = E_t\{x_{i,t} + \dots + x_{i,t+\delta}\} - (y_{i,t-\delta} + \dots + y_{i,t-1} + n_{i,t}).$$

Defining the **individual "supply line", $s_{i,t}$**,

(2.6) $$s_{i,t} = y_{i,t-\delta} + \dots + y_{i,t-1}, \forall\, i,t,$$

and the **desired "supply line", $s_{i,t}^*$**, as

(2.7) $$s_{i,t}^* = E_t\{x_{i,t} + \dots + x_{i,t+\delta-1}\}, \forall\, i,t,$$

one can rewrite (2.5) as

(2.8) $$y_{i,t}^* = E_t\{x_{i,t+\delta}\} + (n^* - n_{i,t})/\tau^* + \beta^*(s_{i,t}^* - s_{i,t})/\tau^*, \text{ where}$$

$$\tau^* = \beta^* = 1,\ n^* = 0.$$

Expressed this way, the optimal rule thus has the same general form as the suggested behavioral stock adjustment rule.

Rational expectations, perfect competition

In the following, firms are assumed to be following the optimal policy (2.8) up to a random, serially uncorrelated error, $u_{i,t}$ and to have rational expectations. The number of firms is still assumed to be very large. Thus,

(2.9) $$y_{i,t} = y_{i,t}^* + u_{i,t}, \forall\, i,\ t.$$

Since all firms receive identical demand and have the same aggregate information, the desired supply line and expected demand is the same for each and (2.9) is readily aggregated into

(2.10) $Y_t = Y_t^* + U_t,$

$= E_t\{X_{t+\delta} - N_{t+\delta}\} + U_t,$

$= E_t\{X_t + \dots + X_{t+\delta}\} - (Y_{t-\delta} + \dots + Y_{t-1} + N_t) + U_t,$

$= E_t\{X_t + \dots + X_{t+\delta}\} - (S_t + N_t) + U_t, \forall\, t,$[2] where

(2.11) $U_t = \frac{1}{K}(u_{1,t} + \dots + u_{K,t})$ is the aggregate random decision error.

Now consider

(2.12) $E_t\{Y_{t+s}\} = E_t\{\, E_{t+s}\{X_{t+\delta+s} - N_{t+\delta+s}\} + U_{t+s}\,\},$

$= E_t\{X_{t+\delta+s}\} - E_t\{N_{t+\delta+s}\} + E_t\{U_{t+s}\},$

$= E_t\{X_{t+\delta+s}\}, \forall t\ \forall s>0,$

since the expected error is zero and expected future inventories are zero. Further inserting the equation for demand (2.1) yields

$$E_t\{Y_{t+s}\} = (1-\alpha)G + \frac{\alpha}{\delta+1}(\,E_t\{Y_{t+s}\} + \dots + E_t\{Y_{t+\delta+s}\}\,), \Rightarrow$$

(2.13) $E_t\{Y_{t+s}\} - G = \frac{\alpha}{1-\alpha+\delta}\,\frac{1}{\delta+1}\,(\,E_t\{Y_{t+1+s}\}-G + \dots + E_t\{Y_{t+\delta+s}\}-G), \forall t\ \forall s>0.$

Since

$$\frac{\alpha}{1-\alpha+\delta} < 1,$$

the process (2.13) is stable (Y converges to G) going <u>backwards</u> in time. Conversely, the process is unstable going <u>forwards</u> in time. Thus, the only non-exploding solution to (2.13) is

(2.14) $E_t\{Y_{t+s}\} = G; E_t\{X_{t+\delta+s}\} = G, \forall t\ \forall s>0.$

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2 Since all firm demands are equal, the non-linear aggregate demand, $\tilde{X}$ is identical to the simple arithmetic average, X, which is used in the following.

Appendix A

In effect, the rational-expectations solution requires that all observed imbalances are eliminated "immediately" by adjusting production, so that future production is on average in equilibrium. In the absense of future errors, production would exhibit a one-time spike or dip and then jump to equilibrium.

To find Y_t, first consider demand in the current and future δ periods.

$$(2.15)\qquad E_t\{X_t\} = (1-\alpha)G + \frac{\alpha}{\delta+1}(Y_{t-\delta} + ... + Y_{t-1} + E_t\{Y_t\}),$$

$$E_t\{X_{t+1}\} = (1-\alpha)G + \frac{\alpha}{\delta+1}(Y_{t-\delta+1} + ... + Y_{t-1} + E_t\{Y_t\} + E_t\{Y_{t+1}\}),$$

...,

$$E_t\{X_{t+\delta}\} = (1-\alpha)G + \frac{\alpha}{\delta+1}(E_t\{Y_t\} + ... + E_t\{Y_{t+\delta}\}).$$

Summing the δ+1 equations (2.15), and using (2.14), one gets

$$(2.16)\qquad E_t\{X_t + ... + X_{t+\delta}\} = (\delta+1)(1-\alpha)G +$$

$$\frac{\alpha}{\delta+1}\Big[Y_{t-\delta} + 2Y_{t-\delta+1} + ... + \delta Y_{t-1} + (\delta+1)E_t\{Y_t\} + G\delta(\delta+1)/2\Big],$$

$$= (\delta+1)G + \alpha(E_t\{Y_t\} - G) + Z_t, \forall\, t,$$

where the **auxillary variable, Z_t**, is defined by

$$(2.17)\qquad Z_t = \frac{\alpha}{\delta+1}\Big[Y_{t-\delta}-G + 2(Y_{t-\delta+1}-G) + ... + \delta(Y_{t-1}- G)\Big], \forall\, t.$$

Inserting this in (2.10) allows one to solve for $E_t\{Y_t\}$.

$$E_t\{Y_t\} = E_t\{X_t + ... + X_{t+\delta}\} - (S_t + N_t)$$

$$= (\delta+1)G + \alpha(E_t\{Y_t\} - G) + Z_t - (S_t + N_t), \forall\, t, \Rightarrow$$

$$(2.18)\qquad E_t\{Y_t\} = G + \frac{1}{1-\alpha}(Z_t - (S_t - \delta G + N_t)), \forall\, t.$$

One can now find $E_t\{X_{t+\delta}\}$ by inserting (2.17) and (2.18) in (2.15), to get

$$(2.19)\qquad E_t\{X_{t+\delta}\} = (1-\alpha)G + \frac{\alpha}{\delta+1}(E_t\{Y_t\} + \delta G)$$

$$= G + \frac{\alpha}{\delta+1}(E_t\{Y_t\} - G)$$

$$= G + \frac{1}{\delta+1}\frac{\alpha}{1-\alpha}(Z_t - (S_t - \delta G + N_t)), \forall\ t.$$

In similar fashion, one finds S_t^* to be

(2.20) $$S_t^* = E_t\{X_t + ... + X_{t+\delta}\} - E_t\{X_{t+\delta}\} =$$

$$= (\delta+1)G + \alpha(E_t\{Y_t\} - G) + Z_t - G - \frac{\alpha}{\delta+1}(E_t\{Y_t\} - G)$$

$$= \delta G + \frac{\delta}{\delta+1}\frac{\alpha}{1-\alpha}(Z_t - (S_t - \delta G + N_t)) + Z_t, \forall\ t.$$

To summarize, under rational expectations with many firms, and in the absence of binding non-negativity constraints on production, the production policy is

(2.21) $$y_{i,t}^* = E_t\{x_{i,t+\delta}\} + (n^* - n_{i,t})/\tau^* + \beta^*(s_{i,t}^* - s_{i,t})/\tau^*,$$

$$E_t\{x_{i,t+\delta}\} = G + \frac{1}{\delta+1}\frac{\alpha}{1-\alpha}(Z_t - (S_t - \delta G + N_t)),$$

$$s_{i,t}^* = \delta G + Z_t + \frac{\delta}{\delta+1}\frac{\alpha}{1-\alpha}(Z_t - (S_t - \delta G + N_t)), \forall\ t, \text{ where}$$

$$Z_t = \frac{\alpha}{\delta+1}\Big[Y_{t-\delta} - G + 2(Y_{t-\delta+1} - G) + ... + \delta(Y_{t-1} - G)\Big], \forall\ t, \text{ and}$$

$$n^* = 0;\ \tau^* = \beta^* = 1.$$

Note that if $\delta = 0$, $Z_t = s_{i,t}^* = s_{i,t} = 0$, and (2.21) reduces to

(2.22) $$y_{i,t}^* = E_t\{x_{i,t+\delta}\} + (n^* - n_{i,t})/\tau^*,$$

$$E_t\{x_{i,t+\delta}\} = G - \frac{\alpha}{1-\alpha}N_t, \forall\ t.$$

Likewise, if $\alpha = 0$, (2.21) reduces to

(2.23) $$y_{i,t}^* = E_t\{x_{i,t+\delta}\} + (n^* - n_{i,t})/\tau^* + \beta^*(s_{i,t}^* - s_{i,t})/\tau^*,$$

$$E_t\{x_{i,t+\delta}\} = G,$$

$$s_{i,t}^{*} = \delta G, \forall\ t.$$

Performance in stochastic equilibrium

The the aggregate production can be written as

$$\text{(2.24)}\qquad Y_t = E_t\{Y_t\} + U_t$$

$$= G + \frac{1}{1-\alpha}(Z_t - (S_t - \delta G + N_t)) + U_t.$$

Taking first differences on both sides yield

$$\text{(2.25)}\qquad (1-L)Y_t = \frac{1}{1-\alpha}((1-L)Z_t - (1-L)(S_t + N_t)) + (1-L)U_t,$$

where L is the lag operator $L\ x_t = x_{t-1}$; $L\ E_t x_t = E_{t-1}\ x_{t-1}$. Taking first differences on both sides of (2.17) yields

$$\text{(2.26)}\qquad (1-L)Z_t = \frac{\alpha}{\delta+1}\Big[Y_{t-\delta} + 2Y_{t-\delta+1} + ... + \delta Y_{t-1} - Y_{t-\delta-1} - 2Y_{t-\delta} - ... - \delta Y_{t-1}\Big]$$

$$= \frac{\alpha}{\delta+1}\Big[\delta Y_{t-1} - Y_{t-\delta-1} - ... - Y_{t-2}\Big]$$

$$= \frac{\alpha}{\delta+1}(\delta Y_{t-1} - S_{t-1}).$$

Likewise, taking first differences on both sides of (1.12) yields

$$\text{(2.27)}\qquad (1-L)S_t = Y_{t-\delta} + ... + Y_{t-1} - Y_{t-\delta-1} - ... - Y_{t-2}$$

$$= Y_{t-1} - Y_{t-\delta-1}.$$

Finally, aggregating the equation (1.1) for inventories yields

$$\text{(2.28)}\qquad (1-L)N_t = Y_{t-\delta-1} - X_{t-1}.$$

By inserting this results and the equation for demand (2.1) into (2.24) one gets

$$(1-L)Y_t = \frac{1}{1-\alpha}\left(\frac{\alpha}{\delta+1}(\delta Y_{t-1} - S_{t-1}) - (Y_{t-1} - X_{t-1})\right) + (1-L)U_t,$$

$$= \frac{1}{1-\alpha}\left(\frac{\alpha}{\delta+1}(\delta Y_{t-1} - S_{t-1}) - Y_{t-1} + (1-\alpha)G + \frac{\alpha}{\delta+1}(Y_{t-1} + S_{t-1})\right) + (1-L)U_t,$$

$$= G - Y_{t-1} + (1-L)U_t, \Rightarrow$$

(2.29) $Y_t - G = U_t - U_{t-1}$.

Further, inserting (2.29) in (2.1) yields

(2.30) $X_t - G = \frac{\alpha}{\delta+1}(Y_t - G + ... + Y_{t-\delta} - G)$

$= \frac{\alpha}{\delta+1}(U_t - U_{t-1} + ... + U_{t-\delta} - U_{t-\delta-1})$

$= \frac{\alpha}{\delta+1}(U_t - U_{t-\delta-1})$.

Finally, inserting in the equation for inventory yields

$(1-L)N_t = L^{\delta+1}Y_t - LX_t$

$= L^{\delta+1}(1-L)U_t - \frac{\alpha}{\delta+1}L(1 - L^{\delta+1})U_t, \Rightarrow$

(2.31) $N_t = \left(L^{\delta+1} - \frac{\alpha}{\delta+1}L\frac{1 - L^{\delta+1}}{1-L}\right)U_t$

$= -\frac{\alpha}{\delta+1}(U_{t-1} + ... + U_{t-\delta}) + (1 - \frac{\alpha}{\delta+1})U_{t-\delta-1}$.

Turning now to the production and inventory of individual firms, one first observes from (2.9) and (2.21) that

(2.32) $y_{i,t} - G = Y_t - G - U_t + S_t + N_t - s_{i,t} - n_{i,t} + u_{i,t}$.

Taking first differences on both sides, one gets

$$(1\text{-}L)y_{i,t} = (1\text{-}L)Y_t - (1\text{-}L)U_t + (1\text{-}L)(S_t + N_t) - (1\text{-}L)(s_{i,t} + n_{i,t}) + (1\text{-}L)u_{i,t}$$

$$= (1\text{-}L)Y_t - (Y_t - G) + (Y_{t-1} - X_{t-1}) - (y_{i,t-1} - x_{i,t-1}) + (1\text{-}L)u_{i,t}$$

$$= G - y_{i,t-1} - X_{t-1} + x_{i,t-1} + (1\text{-}L)u_{i,t}, \Rightarrow$$

$$y_{i,t} - G = (1\text{-}L)u_{i,t} = u_{i,t} - u_{i,t-1}. \tag{2.33}$$

In similar fashion, one finds for inventories that

$$(1\text{-}L)n_{i,t} = L^{\delta+1}y_{i,t} - Lx_{i,t}$$

$$= L^{\delta+1}(1\text{-}L)u_{i,t} - \frac{\alpha}{\delta+1}L(1 - L^{\delta+1})U_t, \Rightarrow$$

$$n_{i,t} = L^{\delta+1}u_{i,t} - \frac{\alpha}{\delta+1}L\frac{1 - L^{\delta+1}}{1\text{-}L}U_t = u_{i,t-\delta-1} - \frac{\alpha}{\delta+1}(U_{t-1} + \dots + U_{t-\delta-1}). \tag{2.34}$$

It is now possible to calculate the expected invenotry costs, if, for instance, one further **makes the assumption that the errors, u, are normally distributed.** Since n must then also be normally distributed, one finds from Section 8 one finds the expected absolute value of inventory is

$$E_t\{|n_{i,t}|\} = \sqrt{\frac{2}{\pi}}\,\sigma_n, \tag{2.35}$$

where σ_n^2 **is the variance of** $n_{i,t}$. Now, further **assume that individual errors are serially uncorrelated but correlated accross firms in any given period with correlation coefficent** $E\{u_iu_j\} = \rho$. Then, by squaring (2.34) and taking expectations on both sides, one finds that the **variance,** $V\{\,.\,\}$ of $n_{i,t}$ is

$$V\{n_{i,t}\} = \sigma_n^2 = V\{u_{i,t-\delta-1}\} \tag{2.36}$$

$$+ \left(\frac{\alpha}{\delta+1}\right)^2(V\{U_{t-1}\} + \dots + V\{U_{t-\delta-1}\}) - 2\frac{\alpha}{\delta+1}\,E\{U_{t-\delta-1}u_{i,t-\delta-1}\}$$

$$= \sigma_u^2 + \frac{\alpha^2}{\delta+1}V\{U\} - 2\frac{\alpha}{\delta+1}E\{u_iU\},$$

where σ_u **is the variance of** $u_{i,t}$. Further, using (2.11) and taking expectations readily yields

(2.37) $$V\{U\} = (\tfrac{1}{K})^2 E\{u_1{}^2+\ldots+u_K{}^2+2u_1u_2+\ldots+2u_1u_K+2u_2u_3+\ldots+2u_{K-1}u_K\}$$

$$= \frac{1}{K}\sigma_u{}^2 + \frac{K-1}{K}\rho\,\sigma_u{}^2 = (\frac{1}{K} + \frac{K-1}{K}\rho)\,\sigma_u{}^2, \text{ and}$$

(2.38) $$E\{u_i U\} = (\frac{1}{K} + \frac{K-1}{K}\rho)\,\sigma_u{}^2.$$

Inserting this in (2.36) one further finds

(2.39) $$V\{n_{i,t}\} = \sigma_u{}^2\left(1 - \frac{1-(1-\alpha)^2}{\delta+1}(\rho(1-1/K)+1/K)\right)$$

It can be seen from (2.39) that the variance of inventories is <u>less than or equal</u> to the variance in production, due to the fact that production affects demand. If, for instance, production is too high, demand will also increase somewhat, thus partly offsetting the full impact of the error on inventories. As expected, the effect is greater for higher correlation of errors accross firms, fewer firms, a higher multiplier effect, or a shorter time lag. Under the assumptions used, the average inventory cost will be

(2.40) $$\gamma E\{|n_{i,t}|\} = \gamma\,\sigma_u\sqrt{\frac{2}{\pi}\left(1 - \frac{1-(1-\alpha)^2}{\delta+1}(\rho(1-1/K)+1/K)\right)}$$

Minimum-variance criterion in stochastic equilibrium

The rational-expectations solution above was derived under the assumption that the number of firms was essentially infinite. When the number of firms is finite, the optimal policy cannot be derived analytically, but one can instead use optimal control theory to obtain a numerical solution. In this section, the optimal policy is found, using these methods, and both the policy and its performance is compared to the infinite-firm policy at the end of the section.

If the distribution of inventories are approximately normal with a zero mean, the average inventory costs are proportional to the standard deviation of inventories (cf. Section 8.) Thus, minimizing the variance in inventories will also minimize average costs. This means that standard methods for optimal control in linear systems can be used (as long as the non-negativity constraint on production is not binding.)

In such linear systems, the optimal policy is a linear feedback rule, i.e. the optimal production is a linear function of the system states. From the point of view of an individual firm the states of the system are its own inventory and supply line, and the aggregate inventory and supply line. Thus, the optimal policy of the task of an individual firm has the form

$$(2.41) \qquad y_{i,t}^{*} = -(g_{o1}n_{i,t}+g_{11}y_{i,t-1}+\ldots+g_{\delta 1}y_{i,t-\delta})$$

$$-(g_{o2}N_{t}+g_{12}Y_{t-1}+\ldots+g_{\delta 2}Y_{t-\delta})+u_{i,t}, \ \forall\ i,t,$$

or, in vector notation,

$$(2.42) \qquad y_{i,t}^{*} = -\,g\begin{bmatrix} z_{i,t} \\ Z_{t} \end{bmatrix} \equiv -\,g\,\tilde{z}_{i,t}, \text{ where}$$

$$g = [g_{o1}\ g_{11} \cdots g_{\delta 1}\ \ g_{o2}\ g_{12} \cdots g_{\delta 2}],$$

$$z_{i,t} = [n_{i,t}\ y_{i,t-1} \cdots y_{i,t-3}]' \text{, and}$$

$$Z_{t} = [N_{t}\ Y_{t-1} \cdots Y_{t-3})]'.$$

Moreover, knowing that all other firms follow the same policy, one can aggregate (2.42) to get the equations of motions for the entire system. The manipulations are straight-forward but laborious. The result, for δ=3, is

$$(2.43) \qquad \tilde{z}_{i,t+1} = \begin{bmatrix} z_{i,t+1} \\ Z_{t+1} \end{bmatrix} = \underline{\underline{P}}\,\tilde{z}_{i,t} + q\,y_{i,t} + \underline{\underline{R}}\,\tilde{u}_{i,t}, \text{ where}$$

$$\underline{\underline{P}} = \begin{bmatrix} \underline{\underline{P}}_{11} \underline{\underline{P}}_{12} \\ \underline{\underline{P}}_{21} \underline{\underline{P}}_{22} \end{bmatrix}, \quad \tilde{u}_{i,t} = \begin{bmatrix} u_{i,t} \\ U_{t} \end{bmatrix},$$

$$\underline{\underline{P}}_{11} = \begin{bmatrix} 1-\dfrac{\alpha\gamma_{o1}}{K(\delta+1)} & -\dfrac{\alpha\gamma_{11}}{K(\delta+1)} & -\dfrac{\alpha\gamma_{21}}{K(\delta+1)} & 1-\dfrac{\alpha\gamma_{31}}{K(\delta+1)} \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

Appendix A

$$\underline{\underline{P}}_{12} = \frac{\alpha}{(\delta+1)} \begin{bmatrix} \gamma_{o1}+\frac{(K-1)\gamma_{o2}}{K} & \gamma_{11}-1+\frac{(K-1)\gamma_{12}}{K} & \gamma_{21}-1+\frac{(K-1)\gamma_{22}}{K} & \gamma_{31}-1+\frac{(K-1)\gamma_{32}}{K} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\underline{\underline{P}}_{21} = \frac{1}{K} \begin{bmatrix} -\frac{\alpha\gamma_{o1}}{\delta+1} & -\frac{\alpha\gamma_{11}}{\delta+1} & -\frac{\alpha\gamma_{21}}{\delta+1} & -\frac{\alpha\gamma_{31}}{\delta+1} \\ \gamma_{o1} & \gamma_{11} & \gamma_{21} & \gamma_{31} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\underline{\underline{P}}_{22} =$$

$$\begin{bmatrix} 1+\frac{\alpha}{\delta+1}(\gamma_{o1}+\frac{(K-1)\gamma_{o2}}{K}), & \frac{\alpha}{\delta+1}(\gamma_{11}+\frac{(K-1)\gamma_{12}}{K}-1), & \frac{\alpha}{\delta+1}(\gamma_{21}+\frac{(K-1)\gamma_{22}}{K}-1), & 1+\frac{\alpha}{\delta+1}(\gamma_{31}+\frac{(K-1)\gamma_{32}}{K}-1) \\ -(\gamma_{o1}+\frac{(K-1)\gamma_{o2}}{K}) & -(\gamma_{11}+\frac{(K-1)\gamma_{12}}{K}) & -(\gamma_{21}+\frac{(K-1)\gamma_{22}}{K}) & -(\gamma_{31}+\frac{(K-1)\gamma_{32}}{K}) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$q = \begin{bmatrix} -\frac{\alpha}{K(\delta+1)} \\ 1 \\ 0 \\ 0 \\ -\frac{\alpha}{K(\delta+1)} \\ 1/K \\ 0 \\ 0 \end{bmatrix}, \underline{\underline{R}} = \begin{bmatrix} 0 & -\frac{\alpha}{\delta+1} \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -\frac{\alpha}{\delta+1} \\ 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

That individual errors, $u_{i,t}$, do not seem to make any difference in the system is due to the fact that the aggregate effects of this error are subsumed in the aggregate error, U_t. Thus, rather than distinguish between $u_{i,t}$ and all other $u_{j,t}$, we have chosen to consider U_t as the relevant variable. (Note that U_t and $u_{i,t}$ are correlated, even if $u_{i,t}$ is uncorrelated with $u_{j,t}$).

The individual firms seeks to minimize the variance of its inventory, i.e. it faces the optimization problem

(2.44) $\min J_i = E\{ \tilde{z}_{i,t} \underline{\underline{F}} \tilde{z}_{i,t}' \}$ for all t, where,

(2.45) $$\underline{\underline{F}} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \dots & & & \\ 0 & 0 & \dots & 0 \end{bmatrix}$$

and subject to the equations of motion (2.43). The optimal policy, i.e. the optimal "gain vector," **g**, is

(2.46) $$g = (q'\underline{\underline{H}}\, q)^{-1}\, q'\underline{\underline{H}}\,\underline{\underline{P}},$$

where $\underline{\underline{H}}$ is the solution to the matrix Riccatti equation

(2.47) $$\underline{\underline{H}} = \underline{\underline{P}}'\underline{\underline{H}}\,\underline{\underline{P}} - \underline{\underline{P}}'\underline{\underline{H}}\, q(q'\underline{\underline{H}}\, q)^{-1}\, q'\underline{\underline{H}}\,\underline{\underline{P}} + \underline{\underline{F}}.$$[3]

The equations (2.43), (2.45), (2.46), and (2.47) together identify the rational-expectations solution. They cannot be solved analytically but must instead be determined numerically. The Riccatti equation, taken as an iterative method for finding $\underline{\underline{H}}$, will always converge, as long as the system is controllable and observable and the covariance matrix of the errors is positive definite (see e.g. F.C. Schweppe, F.C. *Uncertain dynamic systems.* Englewood Cliffs, NJ: Prentice-Hall, 1973). The complication here, however, is that the feedback gain, **g**, also enters into the system matrix, $\underline{\underline{P}}$. In practice, the solutions were found by first iterating $\underline{\underline{H}}$ for a given **g**, then iterating **g** for a given $\underline{\underline{H}}$, etc.

Table 2.1 shows the optimal feedback gain for various values of K, the number of firms. As K goes to infinity, the optimal control law converges to the solution found analytically in the previous section.

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[3] This result can be found in any standard textbook on optimal control.

	Gain in individual feedback rules, $y_{i,t} = -g^*\tilde{z}_{i,t}$, $\tilde{z}_{i,t} = (n_{i,t}, y_{i,t-1}, \ldots, Y_{t-3})$.								Aggregate gain $Y_t = -G^*\tilde{Z}_t$, $\tilde{Z}_t = (N_t, \ldots, Y_{t-3})$.			
K	$n_{i,t}$	$y_{i,t-1}$	$y_{i,t-2}$	$y_{i,t-3}$	N_t	Y_{t-1}	Y_{t-2}	Y_{t-3}	N_t	Y_{t-1}	Y_{t-2}	Y_{t-3}
1									1.143	1.000	1.000	1.000
2	0.800	0.933	0.867	0.800	0.800	0.200	0.400	0.600	1.600	1.133	1.267	1.400
3	0.870	0.957	0.913	0.870	0.870	0.217	0.435	0.652	1.739	1.174	1.348	1.522
4	0.903	0.968	0.935	0.903	0.903	0.226	0.452	0.677	1.806	1.194	1.387	1.581
5	0.923	0.974	0.949	0.923	0.923	0.231	0.462	0.692	1.846	1.205	1.410	1.615
10	0.962	0.987	0.975	0.962	0.962	0.241	0.481	0.722	1.924	1.228	1.456	1.684
∞	1.000	1.000	1.000	1.000	1.000	0.250	0.500	0.750	2.000	1.250	1.500	1.750

Table 2.1

Optimal decision rules in fixed-price complex condition, using minimum variance criterion, as a function of the number of firms, K.

We note that, for four or more firms, the optimal policy is quite close to the infinite-firm policy: the coefficients are nearly the same. Thus, it is probably not too bad an approximation to use the infinite-firm policy with four or more firms.

To find the average inventory costs, first find the covariance matrix of the noise, i.e.,

$$\text{(2.48)} \qquad E\{\tilde{u}_{i,t}\,\tilde{u}_{i,t}'\} = \underline{\underline{W}} = \begin{bmatrix} E\{u_{i,t}^2\} & E\{u_{i,t}U_t\} \\ E\{u_{i,t}U_t\} & E\{U_t^2\} \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{K} + \rho\frac{K-1}{K} \\ \frac{1}{K} + \rho\frac{K-1}{K} & \frac{1}{K} + \rho\frac{K-1}{K} \end{bmatrix} \sigma^2.$$

Then, the steady-state covariance matrix of the system states, $E\{\tilde{z}_{i,t}\,\tilde{z}_{i,t}'\} = \underline{\underline{\Gamma}}$ satisfies the matrix equation

$$\text{(2.49)} \qquad \underline{\underline{\Gamma}} = (\underline{\underline{P}} - q\,g)\,\underline{\underline{\Gamma}}\,(\underline{\underline{P}} - q\,g)' + \underline{\underline{R}}\,\underline{\underline{W}}\,\underline{\underline{R}}'.$$

The equation is easy to solve numerically. The computed variances in output and inventories are shown in Table 2.2. Also shown is the performance resulting from following the infinite-firm optimal policy derived previously. The coefficients in Table 2.1 indicated that the finite-firm and infinite-firm policies are not very different. The 2.2 shows that the performances of the two rules are very close, i.e., that the infinite-firm policy does almost as well as the optimal, even with only a single firm! For instance, in the case of a single firm, the optimal policy results in an average reduction in inventory costs of only 3% compared to the infinite-firm policy. Moreover, if a random-error standard deviation of about 10% of the average production level, inventory costs in the infinite-firm policy amount only to a few percent of average profits. So the difference in performance is neglible.

K	ρ	$V\{n_{i,t}\}/\sigma^2$		$V\{y_{i,t}\}/\sigma^2$		$V\{N_t\}/\sigma^2$		$V\{Y_t\}/\sigma^2$	
1		0.766	*0.812*	1.750	*2.000*	0.766	*0.812*	1.750	*2.000*
2	0	0.898	*0.906*	1.875	*2.000*	0.391	*0.406*	0.937	*1.000*
	0.5	0.840	*0.859*	1.875	*2.000*	0.586	*0.609*	1.406	*1.500*
	1	0.781	*0.812*	1.875	*2.000*	0.781	*0.812*	1.875	*2.000*
3	0	0.934	*0.937*	1.917	*2.000*	0.264	*0.270*	0.639	*0.666*
	0.5	0.863	*0.875*	1.917	*2.000*	0.527	*0.541*	1.278	*1.333*
	1	0.791	*0.812*	1.917	*2.000*	0.791	*0.812*	1.917	*2.000*
4	0	0.951	*0.953*	1.937	*2.000*	0.199	*0.203*	0.484	*0.500*
	0.5	0.874	*0.882*	1.937	*2.000*	0.497	*0.507*	1.211	*1.250*
	1	0.796	*0.812*	1.937	*2.000*	0.796	*0.812*	1.937	*2.000*
5	0	0.961	*0.962*	1.950	*2.000*	0.160	*0.162*	0.390	*0.400*
	0.5	0.880	*0.887*	1.950	*2.000*	0.479	*0.487*	1.170	*1.200*
	1	0.799	*0.812*	1.950	*2.000*	0.799	*0.812*	1.950	*2.000*
10	0	0.981	*0.981*	1.975	*2.000*	0.081	*0.081*	0.197	*0.200*
	0.5	0.893	*0.896*	1.975	*2.000*	0.443	*0.446*	1.086	*1.100*
	1	0.806	*0.812*	1.975	*2.000*	0.806	*0.812*	1.975	*2.000*
∞	0	1.000	*1.000*	2.000	*2.000*	0.000	*0.000*	0.000	*0.000*
	0.5	0.906	*0.906*	2.000	*2.000*	0.406	*0.406*	1.000	*1.000*
	1	0.812	*0.812*	2.000	*2.000*	0.812	*0.812*	2.000	*2.000*

Table 2.2

Variance of production and inventories in stochastic equilibrium.

The figures shown are relative to the random-error variance, using the optimal minimum-variance decision rule (plain text) and the infinite-firm rule (*italic numbers*), respectively. The numbers are shown for three alternative values of the cross-firm correlation, ρ, of the random errors.

3. Clearing prices

In this condition, the computer determines the set of prices which will equate output (i.e. finished production) and demand each period. Thus, inventories are constantly equal to zero. **Assuming firms act competitively,** the optimization problem of the individual firm is

$$\text{(3.1)} \qquad \text{Max } E_{i,t}\{p_{i,t}x_{i,t} - \omega x_{i,t}\}, \text{ where } p_{i,t} = \tilde{P}_t(x_{i,t}/\tilde{X}_t)^{-1/\varepsilon}.$$

If the number of firms, K, is very large, the aggregate quantities are independent of the individual firm's actions, and the first-order conditions become

$$\text{(3.2)} \qquad E_{i,t}\{\tilde{P}_t\tilde{X}_t^{1/\varepsilon}\}\, x_{i,t}^{-1/\varepsilon}\,(\varepsilon-1)/\varepsilon - \omega = 0, \Rightarrow$$

$$x_{i,t}^* = E_{i,t}\{\tilde{P}_t\tilde{X}_t^{1/\varepsilon}\}^{-1/\varepsilon}(p^*)^{1/\varepsilon}.$$

Note that this means that

$$\text{(3.3)} \qquad E_{i,t}\{p_{i,t}\} = p^*.$$

If, futher, variances are small, one can use the approximation $E\{f(x)\} \approx f(E\{x\})$, so that the approximate optimal competitive policy for many firms becomes

$$\text{(3.4)} \qquad x_{i,t}^* = E_{i,t}\{\tilde{X}_t\}\,(E_{i,t}\{\tilde{P}_t\}/p^*)^{-1/\varepsilon}.$$

If the number of firms, K, is small, and one can no longer ignore the effect of the firm's actions on aggregate variables, one must turn to the derivations in Section 1. If the variances are small, one may use the equations in that section directly, substituting aggregates with their expected values, conditional on the choice of $x_{i,t}$. It would seem most natural to use the Cournot equilibrium (where other firms' output is assumed to be constant) instead of the Bertrand equilibrium (where other firms' prices are assumed to be constant.) As mentioned in Section 1, the equation (1.21), or, for that matter, the corresponding Cournot equation, has no analytical solution, except when all firms have equal output and there is no multiplier effect. **Under rational expectations with small random deviations,** however, the solution would approximately equal the constant output. Hence, the **optimal policy in rational-expectations competitive equilibrium** is

$$\text{(3.5)} \qquad x_{i,t}^* = x_K^* \text{ (Bertrand) or}$$

$$\text{(3.6)} \qquad x_{i,t}^* = x_K^{**} \text{ (Cournot).}$$

Note that this is so for both the simple and complex condition since, in the latter, the pipeline is completely cleared for past outputs by the time the current output decision becomes available for sale.

4. Posted prices

Simple condition

In the simple posted-price condition, the firm seeks to maximize its profits before inventory costs while minimizing the latter. It is obvious that, unless the non-negativity constraint on production is binding, the two problems can be separated: since production is costless, the optimal policy is always to choose the output that will minimize expected inventory costs. Conversely, in choosing its price, the firm can ignore inventory considerations. Furthermore, for **small error variances**, one can make the approximation $E\{ f(x) \} \approx f(E\{x\})$, and the derivations for the steady-state equilibria in Section 1 apply to the price decision. **Assuming also that firms act competitively**, the approximate optimal pricing policy is the price that solves

$$(4.1) \qquad p_i + (p_i - \omega)\left(-\varepsilon + \left(\varepsilon + \frac{E\{\tilde{P}\}\, f'(E\{\tilde{P}\})}{f(E\{\tilde{P}\})} \frac{1}{1-\alpha f(E\{\tilde{P}\})}\right) \frac{1}{K} (p_i / E\{\tilde{P}\})^{1-\varepsilon} \right) = 0,$$

where the expectation E{ } is conditional upon p_i (cf. Section 1), and the time subscripts have been dropped to simplify notation. Further, it is shown in Section 8 that the optimal production policy is the one that sets the median of next period's inventory to zero. Moreover, **if the distribution of demand is approximately symmetric**, the median and the expected value coincide. Finally, **assume that $\tilde{P}$ is close to the competitive-equilibrium value, p_K^* or that K is very large**. Then, the optimal policy is

$$(4.2) \qquad p_{i,t}^* = p_K^*;$$

$$(4.3) \qquad y_{i,t}^* = E_{i,t}\{x_{i,t}\} + (n^* - n_{i,t})/\tau^*;\; n^* = 0;\; \tau^* = 1.$$

Complex condition

The market system in the posted complex condition is too complicated to solve analytically for the optimal policy, except in the special case where there are no random errors. However, the system can be approximated around the equilibrium point with a Taylor expansion, yielding a linear-quadratic optimal control problem which can be analyzed using standard techniques. In the following, the simple posted condition, the deterministic posted complex condition, and the linear-quadratic approximate posted complex condition are treated in turn.

<u>Deterministic case:</u>

As in the simple condition, since there is no direct cost of initiating production, y_t, firms should choose y_t to minimize expected inventory cost. Moreover, **assuming that the number of firms, K, is very large,** firms can ignore the effect of their output on demand, and the optimal policy is therefore to choose output so that $n_{t+\delta+1} = 0$. Note that the firm cannot influence earlier inventories since it is assumed that the multiplier effect of production on demand is negligible. Thus, the optimal deterministic output policy (for $K=\infty$) is

(4.3) $$y_{i,t}^* = x_{i,t} + \ldots + x_{i,t+\delta} - n_{i,t} - y_{i,t-1} - \ldots - y_{i,t-\delta}.$$

In effect, the production policy "clears the system memory" so that, beyond time $t+\delta$, the system remains in equilibrium, with output and prices at their competitive-equilibrium values G and p^*, respectively. Moreover, since inventory in time $t+\delta+1$ is "taken care of" by the output decision, $p_{t+\delta}$ is equal to the unconstrainted optimal price, p^*. It now remains to find the choice of prices that maximize the expected profits in the intervening periods. The firm thus faces the problem of choosing prices $p_{i,t}, \ldots, p_{i,t+\delta-1}$ to

(4.4) $$\text{Max } V = v_{i,t} + \ldots + v_{i,t+\delta}$$

$$= x_{i,t}(p_{i,t}-\omega) + \ldots + x_{i,t+\delta-1}(p_{i,t+\delta-1}-\omega) - \gamma|n_{i,t+1}| - \ldots - \gamma|n_{i,t+\delta}|.$$

To solve this problem, it is easiest to use "dynamic programming", i.e., to work backwards from the period $t+\delta-1$. First, consider the **inventory-clearing price, p^c,** i.e., the price that will bring next period's inventory to zero. The existence of this price presumes that the current inventory plus the production due to arrive in inventory together are non-negative; otherwise, the p^c is best thought of as "infinite." In the following, the firm index, i, has been dropped for notational convenience. One has

(4.5) $$p^c_{t+\delta-1} = \tilde{P}_{t+\delta-1}\left(\frac{n_{t+\delta-1}+y_{t-1}}{\tilde{X}_{t+\delta-1}}\right)^{-1/\varepsilon},\ n_{t+\delta-1} + y_{t-1} > 0.$$

Note that in the deterministic case, V is not differentiable around this point. Instead of a stationarity condition, consider the effect separately of small changes in price above or below the clearing value. One has

(4.6) $$\frac{dV}{dp_{t+\delta-1}} = x_{t+\delta-1} + \frac{dx_{t+\delta-1}}{dp_{t+\delta-1}}(p_{t+\delta-1} - \omega + \gamma)$$

$$= x_{t+\delta-1}(1 - \varepsilon(p_{t+\delta-1} - \omega + \gamma)/p_{t+\delta-1}) \text{ for } p_{t+\delta-1} > p^c_{t+\delta-1}$$

$$\frac{dV}{dp_{t+\delta-1}} = x_{t+\delta-1}(1 - \varepsilon(p_{t+\delta-1} - \omega - \gamma)/p_{t+\delta-1}) \text{ for } p_{t+\delta-1} < p^c_{t+\delta-1}.$$

Consider now the conditions for the clearing price to be optimal. That would imply that the profit-function has a maximum at that point, i.e., that

$$\frac{dV}{dp_{t+\delta-1}} < 0 \text{ for } p_{t+\delta-1} > p^c_{t+\delta-1}$$

$$\frac{dV}{dp_{t+\delta-1}} > 0 \text{ for } p_{t+\delta-1} < p^c_{t+\delta-1},\ \Rightarrow$$

$$1 - \varepsilon(p_{t+\delta-1} - \omega + \gamma)/p_{t+\delta-1} < 0 \text{ for } p_{t+\delta-1} > p^c_{t+\delta-1}$$

$$1 - \varepsilon(p_{t+\delta-1} - \omega - \gamma)/p_{t+\delta-1} > 0 \text{ for } p_{t+\delta-1} < p^c_{t+\delta-1},\ \Rightarrow$$

(4.7) $$p^* - \gamma(1-1/\varepsilon) < p^c_{t+\delta-1} < p^* + \gamma(1-1/\varepsilon).$$

Thus, as long as the inventory-clearing price is within the range in (4.7), it is optimal to charge that price. If the clearing price falls outside this range, V will be differentiable and one can instead use the stationarity condition

(4.8) $$\frac{dV}{dp_{t+\delta-1}} = 0,\ \Rightarrow$$

$$x_{t+\delta-1}(1 - \varepsilon(p_{t+\delta-1} - \omega + \gamma)/p_{t+\delta-1}) = 0 \text{ for } p^c_{t+\delta-1} < p^*(1 - \frac{\gamma}{\omega});$$

$$x_{t+\delta-1}(1 - \varepsilon(p_{t+\delta-1} - \omega - \gamma)/p_{t+\delta-1}) = 0 \text{ for } p^c_{t+\delta-1} > p^*(1 + \frac{\gamma}{\omega}), \Rightarrow$$

$$p_{t+\delta-1} = p^* - \gamma(1\text{-}1/\varepsilon) \text{ for } p^c_{t+\delta-1} < p^* - \gamma(1\text{-}1/\varepsilon);$$

$$p_{t+\delta-1} = p^* + \gamma(1\text{-}1/\varepsilon) \text{ for } p^c_{t+\delta-1} > p^* + \gamma(1\text{-}1/\varepsilon).$$

Thus, the optimal rule, $p_{i,t+\delta-1}{}^*$ is to charge the clearing price if possible but to limit it to remain within the range (4.7). It is straightforward to show that V is differentiable with respect to the inventory $n_{t+\delta-1}$ and

$$\text{(4.9)} \qquad \frac{dV}{dn_{t+\delta-1}} = (1\text{-}1/\varepsilon)p_{t+\delta-1}{}^* - \omega \equiv g_{t+\delta-1} \in [-\gamma, \gamma].$$

Now consider the previous period's price, $p_{t+\delta-2}$. Again, the point of departure is the price that will clear inventories, but this time letting the next period's price vary endogenously. Hence, the differentiation of V is done "forwards in time" in the sense that all variables in the current or later periods are allowed to vary endogenously. One gets

$$\text{(4.10)} \qquad \frac{dV}{dp_{t+\delta-2}} = x_{t+\delta-2} + \frac{dx_{t+\delta-2}}{dp_{t+\delta-2}}(p_{t+\delta-2} - \omega + \gamma - \frac{dV}{dn_{i,t+\delta-1}})$$

$$= x_{t+\delta-2}(1 - \varepsilon(p_{t+\delta-2} - \omega + \gamma - g_{t+\delta-1})/p_{t+\delta-2}) \text{ for } p_{t+\delta-2} > p^c_{t+\delta-2}$$

$$\frac{dV}{dp_{t+\delta-2}} = x_{t+\delta-2}(1 - \varepsilon(p_{t+\delta-2} - \omega - \gamma - g_{t+\delta-1})/p_{t+\delta-2}) \text{ for } p_{t+\delta-2} < p^c_{t+\delta-2}.$$

In completely analogous fashion as before, one finds that the optimal policy is to charge the inventory-clearing price $p^c_{t+\delta-2}$, truncated in the interval

$$\text{(4.11)} \qquad p_{t+\delta-1}{}^* - \gamma(1\text{-}1/\varepsilon) < p^c_{t+\delta-2} < p_{t+\delta-1}{}^* + \gamma(1\text{-}1/\varepsilon).$$

Of course, $p_{t+\delta-1}{}^*$ is a function of $p_{t+\delta-1}$. Nonetheless, one can first compute the inventory-clearing price, then compute the optimal next-period price and then check whether (4.11) holds. If so, the optimal policy is the clearing price. If not, the fact that $p_{t+\delta-1}{}^*$ as a function of $p_{t+\delta-2}$ has a negative derivative means that one can always find a price $p_{t+\delta-2}^*$ for which the lower or upper limit in (4.11)--whichever is applicable--holds with equality.

One may continue analogously backwards in time, so that the complete optimal pricing policy is characterized by the conditions

$$
\begin{aligned}
(4.12) \qquad p_t^* &= p_{t+1}^* - \gamma(1-1/\varepsilon) && \text{for } p_t^c \le p_{t+1}^* - \gamma(1-1/\varepsilon) \\
&= p_t^c && \text{for } p_{t+1}^* - \gamma(1-1/\varepsilon) \le p_t^c \le p_{t+1}^* + \gamma(1-1/\varepsilon) \\
&= p_{t+1}^* + \gamma(1-1/\varepsilon) && \text{for } p_t^c > p_{t+1}^* + \gamma(1-1/\varepsilon); \\
&\ldots \\
p_{t+\delta-1}^* &= p_{t+\delta}^* - \gamma(1-1/\varepsilon) && \text{for } p_{t+\delta-1}^c \le p_{t+\delta}^* - \gamma(1-1/\varepsilon) \\
&= p_{t+\delta-1}^c && \text{for } p_{t+\delta}^* - \gamma(1-1/\varepsilon) \le p_{t+\delta-1}^c \le p_{t+\delta}^* + \gamma(1-1/\varepsilon) \\
&= p_{t+\delta}^* + \gamma(1-1/\varepsilon) && \text{for } p_{t+\delta-1}^c > p_{t+\delta}^* + \gamma(1-1/\varepsilon); \\
p_{t+\delta}^* &= p^*.
\end{aligned}
$$

To actually calculate (4.12) requires a limited amount of trial and error but is doable in a finite number of steps. In practice, a wide variety of conditions lead to the inventory-clearing prices being optimal, including the initial conditions used in the experiment.

<u>Stochastic case</u>

An approximation to the optimal policy with less than infite firms and stochastic errors can be obtained by linearizing the equations of motion around the steady-state equilibrium and using a quadratic approximation to the profit function. In Section 6 it is shown that the first-order terms in the profit function can be eliminated by redefining the problem in terms of deviations from the steady state so that standard methods for linear-quadratic optimal control can be used.

The derivations are straightforward in principle but quite tedious. Therefore, only the final results will be presented here.[4] However, one part deserves special mention. The expected inventory cost are a function of the expected absolute inventory level. **If errors are normally distributed**, which will be assumed throughout, then it is shown in Section 8 that

$$(4.13) \qquad E\{|n|\} = \frac{2}{\sqrt{2\pi}}\sqrt{Var\{n\}}.$$

This is thus a deviation from all the other terms in the objective function which are proportional to the variance-covariance matrix E{z z'}. However, one can still achieve a quadratic approximation by considering that the variance in inventories consists of two components, one, call it s^2, that is related to the policy followed, and one, call it e^2, that comes from the random errors each period and is independent of the policy. (The two terms are analogous to the two terms of the right-hand side of equation (2.49) in the fixed-price case above.) Hence,

$$(4.14) \qquad Var\{n\} = s^2 + e^2, \Rightarrow$$

$$(4.15) \qquad E\{|n|\} = \frac{2}{\sqrt{2\pi}}\sqrt{s^2 + e^2}.$$

Now performing a first-order Taylor expansion of (4.15) with respect to s^2, one gets

$$(4.16) \qquad E\{|n|\} \approx \frac{2}{\sqrt{2\pi}}\left(\sqrt{e^2} + \frac{s^2}{2\sqrt{e^2+s^2}}\right) = \frac{2}{\sqrt{2\pi}}\left(\sqrt{e^2} + \frac{Var\{n\} - e^2}{2\sqrt{e^2+s^2}}\right)$$

Moreover, if $e^2 >> s^2$, a good approximation to the expected inventory cost is

$$(4.17) \qquad E\{\gamma|n|\} \approx \frac{\gamma e}{\sqrt{2\pi}} + \frac{\gamma}{e\sqrt{2\pi}} Var\{n\}.$$

Figure 8.1 shows a plot of the approximation compared with the true function. It is evident that even for s^2 approaching e^2, the approximation is excellent. Hence, as long as the systematic part of the variance is small relative to the unsystematic one, the profit function is well approximated by for purposes of the optimal-control derivations by

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[4] The derivations were done in *Mathematica* and are thus readily reproducible. The file is available upon request.

$$(4.18) \qquad v_{i,t} \approx x_{i,t}(p_{i,t}-\omega) + \frac{\gamma e}{\sqrt{2\pi}} - \frac{\gamma}{e\sqrt{2\pi}} n_{i,t}^{2}.$$

The results are shown in Tables 4.1 through 4.3. The gain vector, g, for the optimal production rule is shown in Table 4.1, for various values of both the number of firms and the standard deviations of the random errors in production and price, respectively. (These errors matter because the unsystematic error variance e^2 is part of the coefficients in the profit function, cf. equation 4.18). Table 4.2 shows the same results for the optimal price rule. It is quite clear that the rules mimic the inventory-clearing rule found in the deterministic case above: The coefficients for production are all quite low while the coefficients for prices are close to the linearized values from the clearing-price rule.

The performance results are shown in Table 4.3. The values shown are the reduction (i.e., a positive number) in profits as a percent of the steady-state profit. The reduction is partitioned into reduced profits before inventory costs, and inventory costs. It is quite clear that the reduction is very slight: by far the greatest portion comes from the unsystematic variance in inventories. This is further underlined by the last column of the table which shows the ratio of the systmatic and unsystematic error in inventory (s/e, cf. equation 4.14). That column also shows that s<<e, so that the quadratic approximation (4.17) is highly accurate (cf. also Figure 8.1).

K	Sy	Sp	Gain vector, g, in feedback rule, $y_{i,t} = -g\,\tilde{z}_{i,t}$, $\tilde{z}_{i,t} = (n_{i,t}, y_{i,t-1}, \ldots, Y_{t-3})$							
			n(t)	y(t-1)	y(t-2)	y(t-3)	N(t)	Y(t-1)	Y(t-2)	Y(t-3)
4	.01	.01	-.03	-.03	-.07	-.03	.02	.09	-.03	.02
4	.05	.01	-.03	-.02	-.07	-.03	.02	.09	-.03	.02
4	.01	.05	-.04	.06	-.05	-.04	.05	.27	.07	.05
4	.05	.05	-.04	.06	-.05	-.04	.05	.27	.07	.05
4	.00	.10	-.04	.14	-.02	-.04	.12	.35	.17	.11
4	.10	.00	-.02	-.04	-.07	-.02	.02	.02	-.05	.02
4	.10	.10	-.04	.14	-.02	-.04	.12	.35	.17	.11
∞	.01	.01	.00	.02	.00	.00	.00	.16	.01	.00
∞	.05	.01	.00	.02	.00	.00	.00	.16	.01	.00
∞	.01	.05	.00	.10	.01	.00	.04	.40	.11	.04
∞	.05	.05	.00	.10	.01	.00	.04	.40	.11	.04
∞	.00	.10	.01	.17	.03	.01	.13	.49	.22	.11
∞	.10	.00	.00	.00	.00	.00	.00	.00	.00	.00
∞	.10	.10	.01	.17	.03	.01	.13	.49	.22	.11

Table 4.1
Coefficients in optimal linear-quadratic production rule.

K	Sy	Sp	Gain vector, g, in feedback rule, $p_{i,t} = -g\,\tilde{z}_{i,t}$, $\tilde{z}_{i,t} = (n_{i,t}, y_{i,t-1}, \ldots, Y_{t-3})$							
			n(t)	y(t-1)	y(t-2)	y(t-3)	N(t)	Y(t-1)	Y(t-2)	Y(t-3)
4	.01	.01	.48	.00	.01	.48	1.05	-.18	-.12	.87
4	.05	.01	.48	.00	.01	.48	1.05	-.17	-.12	.87
4	.01	.05	.44	.01	.05	.44	.83	-.08	.06	.68
4	.05	.05	.44	.01	.05	.44	.83	-.08	.06	.68
4	.00	.10	.40	.01	.07	.40	.70	-.01	.14	.57
4	.10	.00	.49	.00	.00	.49	1.11	-.20	-.18	.93
4	.10	.10	.40	.01	.07	.40	.70	-.01	.14	.57
∞	.01	.01	.39	.00	.01	.39	.85	-.13	-.08	.69
∞	.05	.01	.39	.00	.01	.39	.85	-.13	-.08	.69
∞	.01	.05	.36	.00	.04	.36	.66	-.04	.07	.53
∞	.05	.05	.36	.00	.04	.36	.66	-.04	.07	.53
∞	.00	.10	.33	.01	.06	.33	.55	.02	.13	.44
∞	.10	.00	.40	.00	.00	.40	.93	-.17	-.17	.77
∞	.10	.10	.33	.01	.06	.33	.55	.02	.13	.44

Table 4.2
Coefficients in optimal linear-quadratic pricing rule.

K	Sy	Sp	Reduction in gross profits % of steady-state profits	Inventory cost % of steady-state profits	Total reduction % of steady-state profits	Systematic relative to unsystematic inventory variance s/e
4	.01	.01	0.0	1.0	1.0	.04
4	.05	.01	0.2	1.0	1.2	.15
4	.01	.05	0.3	5.0	5.3	.12
4	.05	.05	0.4	5.0	5.4	.16
4	.00	.10	0.9	10.1	11.0	.20
4	.10	.00	0.8	0.3	1.2	.32
4	.10	.10	1.2	10.2	11.4	.25
∞	.01	.01	0.0	1.5	1.5	.03
∞	.05	.01	0.1	1.5	1.6	.07
∞	.01	.05	0.4	7.5	7.9	.10
∞	.05	.05	0.4	7.5	8.0	.12
∞	.00	.10	1.3	15.2	16.5	.17
∞	.10	.00	0.3	0.0	0.3	.14
∞	.10	.10	1.5	15.2	16.7	.20

Table 4.3
Performance of optimal linear-quadratic rules.

5. Variation in rational-expectations stochastic equilibrium.

In the following, it is assumed that the market is in rational-expectations stochastic equilibrium, i.e., that all firms have rational expectations and that they follow the optimal infinite-firm policies (K=∞), except for an independent and identically distributed random eror. Errors are assumed to be both serially and cross-sectionally uncorrelated. Hence,

(5.1) $$y_{i,t} = y_{i,t}^* + u_{i,t};$$

$$p_{i,t} = p_{i,t}^* + w_{i,t};$$

$$E\{u_{i,t}\} = E\{u_{i,t}u_{j,t}\} = E\{u_{i,t}u_{i,s}\} = 0;\ E\{u_{i,t}^2\} = \sigma_u^2,\ \forall\ i,\ j{\neq}i,\ s{\neq}t.$$

$$E\{w_{i,t}\} = E\{w_{i,t}w_{j,t}\} = E\{w_{i,t}w_{i,s}\} = 0;\ E\{w_{i,t}^2\} = \sigma_w^2,\ \forall\ i,\ j{\neq}i,\ s,\ t.$$

$$E\{u_{i,s}w_{j,t}\} = 0,\ \forall\ i,\ j,\ s,\ t.$$

It is also helpful to define the aggregate average errors

(5.2) $$U_t = \frac{1}{K}(u_{1,t} + \dots + u_{K,t});$$

$$W_t = \frac{1}{K}(w_{1,t} + \dots + w_{K,t}).$$

The derivations below are based on a linear first-order approximation around the equilibrium point. The approximation is derived by a Taylor expansion around the equilibrium with respect to the random noise variables. Hence, the results below hold only for small deviations from equilibrium. The results are summarized in Table 5.1, which shows the individual and aggregate production and prices as a function of the individual and random errors.

Condition	Y_t-G	y_t-G	P_t-p^*	p_t-p^*
Fixed simple	U_t-U_{t-1}	u_t-u_{t-1}		
Fixed complex	U_t-U_{t-1}	u_t-u_{t-1}		
Clearing simple	U_t	u_t	$-U_t/\mu$	$(U_t-u_t)/\varepsilon - U_t/\mu$
Clearing complex	U_t	u_t	$\frac{\alpha}{4\mu}(U_t+U_{t-1}+U_{t-2})+$ $\frac{1}{\mu}(\frac{\alpha}{4}-1)U_{t-3}$	$\frac{\alpha}{4\mu}(U_t+U_{t-1}+U_{t-2})+$ $\frac{1}{\mu}(\frac{\alpha}{4}-1)U_{t-3}+$ $(U_{t-3}-u_{t-3})/\varepsilon$
Posted simple	$U_t-U_{t-1}-$ μV_{t-1}	u_t-u_{t-1} $-\varepsilon v_{t-1}$ $+(\varepsilon-\mu)W_{t-1}$	W_t	w_t
Posted complex	U_t	u_t	$W_t-W_{t-1}+$ $\frac{\alpha}{4\mu}(2U_{t-1}+U_{t-2})+$ $\frac{1}{\mu}(\frac{\alpha}{4}-1)U_{t-3}$	$w_t-w_{t-1}-u_{t-3}/\varepsilon$ $+(\frac{1}{\varepsilon}-\frac{1}{\mu})U_{t-3}+$ $\frac{\alpha}{4\mu}(2U_{t-1}+U_{t-2}+U_{t-3})$

Table 5.1

Random deviations in stochastic, rational-expectations equilibrium.

The table shows the deviations of average and individual prices, as functions of the average and individual random errors. The expressions have been linearized around the mean.

From the analytical expressions in Table 5.1, it is straightforward to calculate the variance in average production (Table 5.2) and average price (Table 5.3).

Condition	$Var\{Y_t\}$	Value (K=4)	$Var\{y_{i,t}\}$	Value (K=4)
Fixed simple	$\frac{1}{K}2\sigma_u^2$	$0.50\,\sigma_u^2$	$2\sigma_u^2$	$2.00\,\sigma_u^2$
Fixed complex	$\frac{1}{K}2\sigma_u^2$	$0.50\,\sigma_u^2$	$2\sigma_u^2$	$2.00\,\sigma_u^2$
Clearing simple	$\frac{1}{K}\sigma_u^2$	$0.25\,\sigma_u^2$	σ_u^2	$1.00\,\sigma_u^2$
Clearing complex	$\frac{1}{K}\sigma_u^2$	$0.25\,\sigma_u^2$	σ_u^2	$1.00\,\sigma_u^2$
Posted simple	$\frac{1}{K}2\sigma_u^2+\frac{1}{K}\mu^2\sigma_w^2$	$0.50\,\sigma_u^2+$ $0.14\,\sigma_w^2$	$2\sigma_u^2+$ $[\frac{1}{K}\mu^2+\frac{K-1}{K}\varepsilon^2]\sigma_w^2$	$2.00\,\sigma_u^2+$ $4.83\,\sigma_w^2$
Posted complex	$\frac{1}{K}\sigma_u^2$	$0.25\,\sigma_u^2$	σ_u^2	$1.00\,\sigma_u^2$

Table 5.2
Variance of production in stochastic rational-expectations equilibrium

Condition	$Var\{P_t\}$	Value (K=4)	$Var\{p_{i,t}\}$	Value (K=4)
Clearing simple	$\frac{1}{K}\mu^{-2}\sigma_u^2$	$0.44\,\sigma_u^2$	$[\frac{K-1}{K}\varepsilon^{-2}+\frac{1}{K}\mu^{-2}]\sigma_u^2$	$0.56\,\sigma_u^2$
Clearing complex	$\frac{1}{K}\mu^{-2}(1+\alpha^2/4 -$ $\alpha/2)\sigma_u^2$	$0.36\,\sigma_u^2$	$[\frac{K-1}{K}\varepsilon^{-2}+\frac{1}{K}\mu^{-2}(1+\frac{1}{4}\alpha^2-$ $\frac{1}{2}\alpha)]\sigma_u^2$	$0.48\,\sigma_u^2$
Posted simple	$\frac{1}{K}\sigma_w^2$	$0.25\,\sigma_w^2$	σ_w^2	$1.00\,\sigma_w^2$
Posted complex	$\frac{1}{K}2\sigma_w^2+\frac{1}{K}\mu^-$ $2)(1+\frac{3}{8}\alpha^2-\frac{1}{2}\alpha)\sigma_u^2$	$0.50\,\sigma_w^2+$ $0.38\,\sigma_u^2$	$2\sigma_w^2+[\frac{K-1}{K}\varepsilon^{-2}+\frac{1}{K}\mu^-$ $2)(1+\frac{3}{8}\alpha^2-\frac{1}{2}\alpha)]\sigma_u^2$	$2.00\,\sigma_w^2+$ $0.50\,\sigma_u^2$

Table 5.3
Variance of prices in stochastic rational-expectations equilibrium

In order to calculate the spectral density of the the variance, recall that, for a q'th-order moving-average process

(5.3) $z_t = \theta_o e_t + \theta_1 e_{t-1} + ... + \theta_q e_{t-q} = \theta(L)e_t;$
$E\{e_t\} = 0; E\{e_t e_{t-i}\} = 0$ for $i \neq 0$ and σ^2 for $i=0$,

the spectrum, i.e., the distribution of the variance accross frequencies, ω, is

(5.4) $f(\omega) = \frac{\sigma^2}{2\pi}\theta(e^{i\omega})\theta(e^{-i\omega}), -\pi \leq \omega \leq \pi.$

For a third-order process, this amounts to

$$(5.5)\qquad f(\omega)\frac{2\pi}{\sigma^2} = \frac{\sigma^2}{2\pi}\Big(\theta_o{}^2+\theta_1{}^2+\theta_2{}^2+\theta_3{}^2 + [\theta_o\theta_1+\theta_1\theta_2+\theta_2\theta_3]\cos(\omega) + [\theta_o\theta_2+\theta_1\theta_3]\cos(2\omega) + \theta_o\theta_3\cos(3\omega)\Big),\ \pi\leq\omega\leq\pi.$$

Note also that, since the Fourier transformation is linear, the spectrum of a sum of two such processes is simply the sum of the spectra.

The spectral density functions, $f(\omega)$ have been calculated in Table 5.4. The functions are plotted in Figure 5.1 under the assumption that, in the posted-price conditions, the error variances in prices (σ_w) and output (σ_u) are equal. Figures 5.2 through 5.5 plots $f(\omega)$ in the posted-price conditions for various assumptions about the relative sizes of σ_w and σ_u.

Condition	$f[\omega]\ Y_t\text{-}\pi\le\omega\le\pi$	$f[\omega]\ P_t\text{-}\pi\le\omega\le\pi$
Fixed simple	$\frac{1}{K}\frac{\sigma_u^2}{\pi}(1-\cos(\omega))$	
Fixed complex	$\frac{1}{K}\frac{\sigma_u^2}{\pi}(1-\cos(\omega))$	
Clearing simple	$\frac{1}{K}\frac{\sigma_u^2}{2\pi}$	$\frac{1}{K}\frac{\sigma_u^2}{2\pi\,\mu^2}$
Clearing complex	$\frac{1}{K}\frac{\sigma_u^2}{2\pi}$	$\frac{1}{K}\frac{\sigma_u^2}{2\pi\,\mu^2}\left[(1+\frac{\alpha^2}{4}-\frac{\alpha}{2}) + (\frac{3\alpha^2}{8}-\frac{\alpha}{2})\cos(\omega) + (\frac{\alpha^2}{4}-\frac{\alpha}{2})\cos(3\omega) + (\frac{\alpha^2}{8}-\frac{\alpha}{2})\cos(3\omega)\right]$
Posted simple	$\frac{1}{K}\frac{1}{2\pi}(2\sigma_u^2+\mu^2\sigma_w^2 - 2\cos(\omega))$	$\frac{1}{K}\frac{\sigma_w^2}{2\pi}$
Posted complex	$\frac{1}{K}\frac{\sigma_u^2}{2\pi}$	$\frac{1}{K}\frac{1}{\pi}\sigma_w^2+\frac{1}{K}\frac{1}{2\pi}(\frac{\sigma_u}{\mu})^2\left[1+\frac{3}{8}\alpha^2-\frac{1}{2}\alpha + (\frac{3}{8}\alpha^2-\frac{1}{2}\alpha)\cos(\omega) + (\frac{1}{4}\alpha^2-\frac{1}{2}\alpha)\cos(2\omega)\right]$

<u>Table 5.4</u>

<u>Spectral densities of average price and production in stochastic, rational-expectations equilibrium.</u>

The table shows the spectral density as a function of the frequency, ω. The expressions are based on the linearized expressions in Table 5.1.

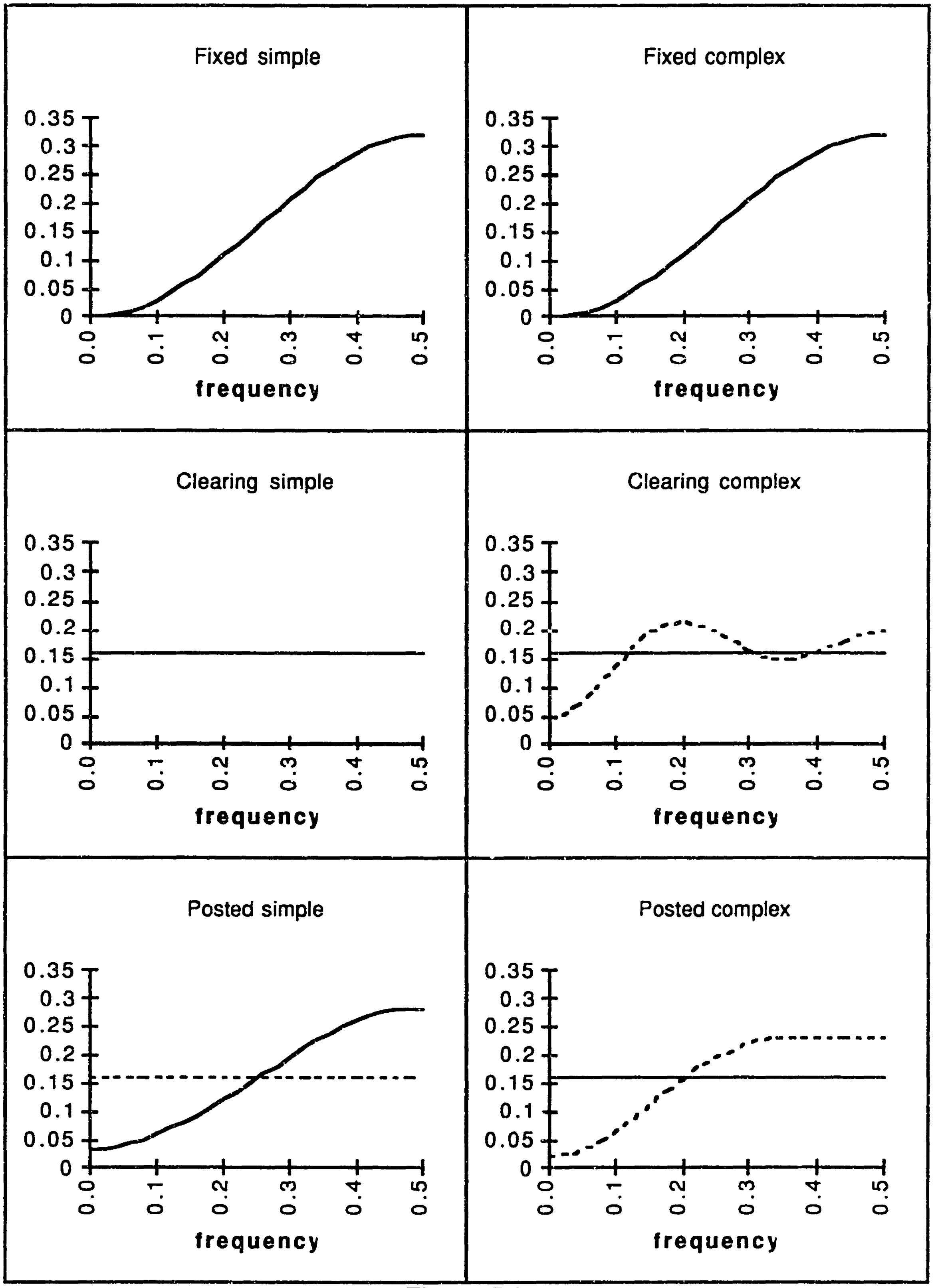

Figure 5.1
Spectral densities for $\sigma_u = \sigma_w$. Dashed line is prices, solid line is output.

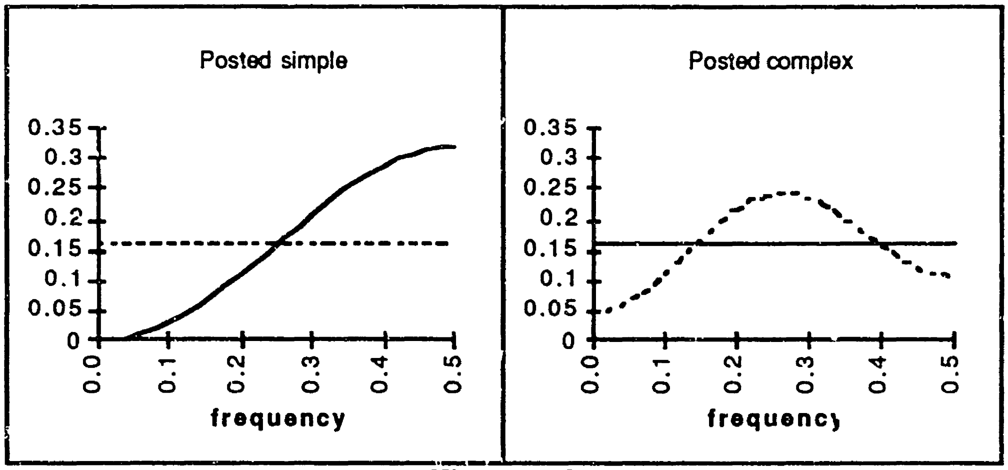

Figure 5.2

Spectral densities for posted prices, for $\sigma_u >> \sigma_w$

Dashed line is prices, solid line is output.

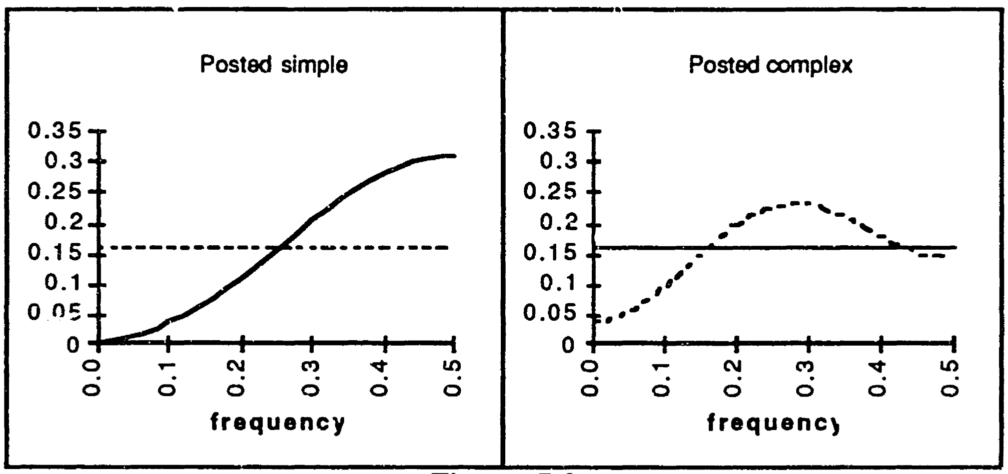

Figure 5.3

Spectral densities for posted prices, for $\sigma_u = 2.5\ \sigma_w$

Dashed line is prices, solid line is output.

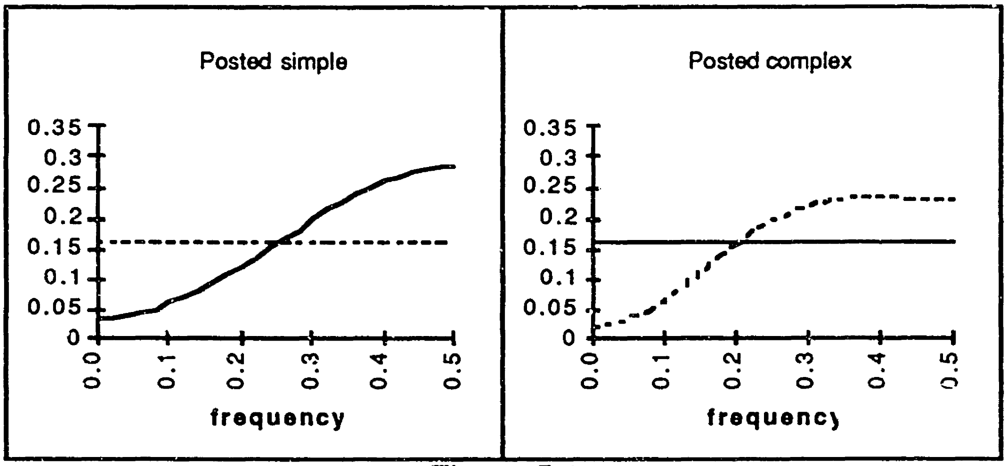

Figure 5.4

Spectral densities for posted prices, for $\sigma_u = \sigma_w$

Dashed line is prices, solid line is output.

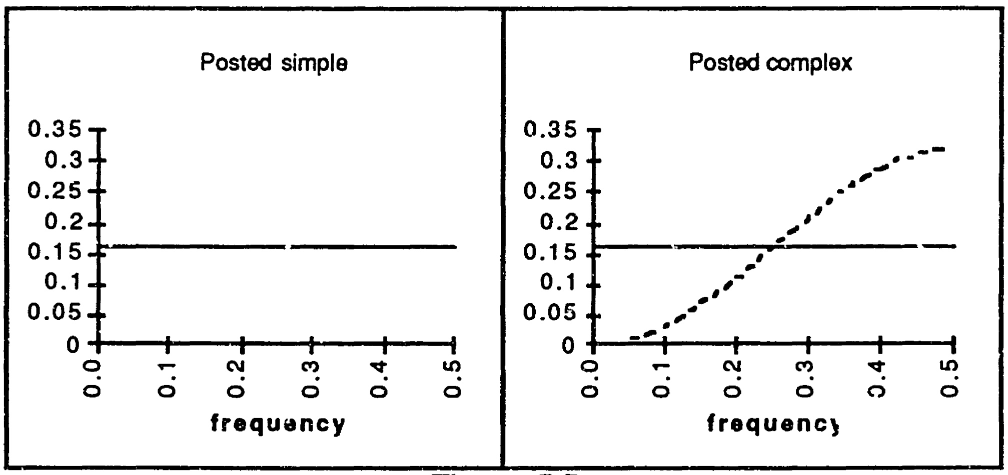

Figure 5.5

Spectral densities for posted prices, for $\sigma_u << \sigma_w$

Dashed line is prices, solid line is output.

6. Note regarding optimal control of linear (time invariant) systems with first-order terms

Normally, linear-quadratic optimal control problems do not involve first-order terms in the objective function. However, as is shown below, it is straightforward to transform a system with first-order terms into one with only quadratic terms in the objective function. Consider the control problem

(6.1) $\min J =$

$$\lim\{T\text{->}\infty\}\ \frac{1}{T}\sum_{t=0}^{T-1}\left(\mathbf{x}_t'\,\tfrac{1}{2}\underline{\underline{F}}_{xx}\,\mathbf{x}_t + \mathbf{u}_t'\,\tfrac{1}{2}\underline{\underline{F}}_{uu}\,\mathbf{u}_t + \mathbf{x}_t'\,\underline{\underline{F}}_{xu}\mathbf{u}_t + \mathbf{h}_x'\mathbf{x}_t + \mathbf{h}_u\mathbf{u}_t\right);$$

s.t.

(6.2) $$\mathbf{x}_{t+1} = \underline{\underline{P}}\,\mathbf{x}_t + \underline{\underline{Q}}\,\mathbf{u}_t + \mathbf{a}, \quad t\geq 0$$

(6.3) $$\mathbf{x}_o = \mathbf{x}^o \text{ (initial conditions).}$$

$\mathbf{x}$ is a n-dimensional vector of state variables, $\mathbf{u}$ is a p-dimensional vector of control variables, $\underline{\underline{F}}_{xx}, \underline{\underline{F}}_{uu}, \underline{\underline{F}}_{xu}$ are positive semidefinite matrices of dimensions n x n, p x p, and n x p, respectively, and $\mathbf{h}_x$ and $\mathbf{h}_u$ are n- and p-dimensional vectors.

The first-order (necessary) conditions are (in addition to (6.2) and (6.3))

(6.4) $$\frac{\partial L}{\partial \mathbf{x}_t} = \underline{\underline{F}}_{xx}\mathbf{x}_t + \underline{\underline{F}}_{xu}\mathbf{u}_t + \mathbf{h}_x - \psi_t + \underline{\underline{P}}'\psi_{t+1} = 0, \quad t\geq 0$$

(6.5) $$\frac{\partial L}{\partial \mathbf{u}_t} = \underline{\underline{F}}_{xu}'\mathbf{x}_t + \underline{\underline{F}}_{uu}\mathbf{u}_t + \mathbf{h}_u + \underline{\underline{Q}}'\psi_{t+1} = 0, \quad t\geq 0,$$

where ψ is the n-dimensional co-state vector (i.e. the Lagrangian multiplier).

Now assume that there is some optimal operating point, $\mathbf{x}^*$, such that, if the system is at that point, it is optimal to let it remain there. More formally, assume that the linear system of equations

(6.6) $$\mathbf{x}^* = \underline{\underline{P}}\,\mathbf{x}^* + \underline{\underline{Q}}\,\mathbf{u}^* + \mathbf{a} \text{ (steady state);}$$

(6.7) $\underline{\underline{F}}_{xx}x^* + \underline{\underline{F}}_{xu}u^* + h_x - \psi^* + \underline{\underline{P}}'\psi^* = 0;$

(6.8) $\underline{\underline{F}}_{xu}'x^* + \underline{\underline{F}}_{uu}u^* + h_u + \underline{\underline{Q}}'\psi^* = 0$

has a solution (x^*,u^*,ψ^*), and define a new system in terms of the deviations from this operating point, as

(6.9) $z_t = x_t - x^*, \qquad t\geq 0;$

(6.10) $v_t = u_t - u^*, \qquad t\geq 0;$

(6.11) $\phi_t = \psi_t - \psi^*, \qquad t\geq 0.$

Now the first-order conditions can be rewritten as

(6.12) $\underline{\underline{F}}_{xx}z_t + \underline{\underline{F}}_{xu}v_t - \phi_t + \underline{\underline{P}}'\phi_{t+1} = 0, \qquad t\geq 0;$

(6.13) $\underline{\underline{F}}_{uu}v_t + \underline{\underline{F}}_{xu}'z_t + \underline{\underline{Q}}'\phi_{t+1} = 0, \qquad t\geq 0,$

(6.14) $z_{t+1} = \underline{\underline{P}}\, z_t + \underline{\underline{Q}}\, v_t, \qquad t\geq 0.$

The system is now on a form which lends itself to the standard linear-quadratic derivation. Thus, the optimal policy is

(6.15) $u_t = u^* - g'(x_t - x^*)$, where

(6.16) $(\underline{\underline{P}} - \underline{\underline{I}})\, x^* + \underline{\underline{Q}}\, u^* = -a,$

(6.17) $\underline{\underline{F}}_{xx}\, x^* + \underline{\underline{F}}_{xu}\, u^* + (\underline{\underline{P}}' - \underline{\underline{I}})\, \psi^* = -h_x;$

(6.18) $\underline{\underline{F}}_{xu}'\, x^* + \underline{\underline{F}}_{uu}\, u^* + \underline{\underline{Q}}'\, \psi^* = -h_u,$

and the gain vector, g, is defined by

(6.19) $g = (\underline{\underline{F}}_{uu} + \underline{\underline{Q}}'\underline{\underline{H}}\,\underline{\underline{Q}})^{-1}(\underline{\underline{Q}}'\underline{\underline{H}}\,\underline{\underline{P}} + \underline{\underline{F}}_{xu}'),$

where $\underline{\underline{H}}$ is the solution to the Matrix-Riccati equation

(6.20) $\underline{\underline{H}} = \underline{\underline{F}}_{xx} + \underline{\underline{P}}'\underline{\underline{H}}\,\underline{\underline{P}} - (\underline{\underline{F}}_{xu} + \underline{\underline{P}}'\underline{\underline{H}}\,\underline{\underline{Q}})(\underline{\underline{F}}_{u} + \underline{\underline{Q}}'\underline{\underline{H}}\,\underline{\underline{Q}})^{-1}(\underline{\underline{Q}}'\underline{\underline{H}}\,\underline{\underline{P}} + \underline{\underline{F}}_{xu}').$

7. Note regarding non-linear aggregates versus arithmetic averages

In the experimental software, firms observed not the non-linear aggregates $\tilde{X}$ and $\tilde{P}$ but the simple arithmetic average sales, X, and the trade-weighted average price, P, defined by

$$(7.1) \qquad X = \frac{1}{K}[x_1 + \ldots + x_K];$$

$$(7.2) \qquad P = \frac{x_1 p_1 + \ldots + x_K p_K}{x_1 + \ldots + x_K}.$$

From (1.4) and (1.8), it follows that

$$(7.3) \qquad X P = \tilde{X} \tilde{P}.$$

Moreover, since $\tilde{X}$ is a convex, homogenous function of its individual components, it must always be less than or equal to the average X. There is therefore a systematic bias between the two variables. X will generally overestimate $\tilde{X}$. Conversely, it follows from (7.3) that P will underestimate $\tilde{P}$. It is important, therefore, to determine just how important this bias may be. Accordingly, a set of Monte Carlo simulations were performed, with normally or log-normally independently distributed individual prices (with mean 1), and the aggregate and average price were compared. A similar analysis was done for sales. The results are shown in Figure 7.1. As can be seen from the figure, the correlation between the two measures is very high and the bias very small except for very large variances (σ=.5).

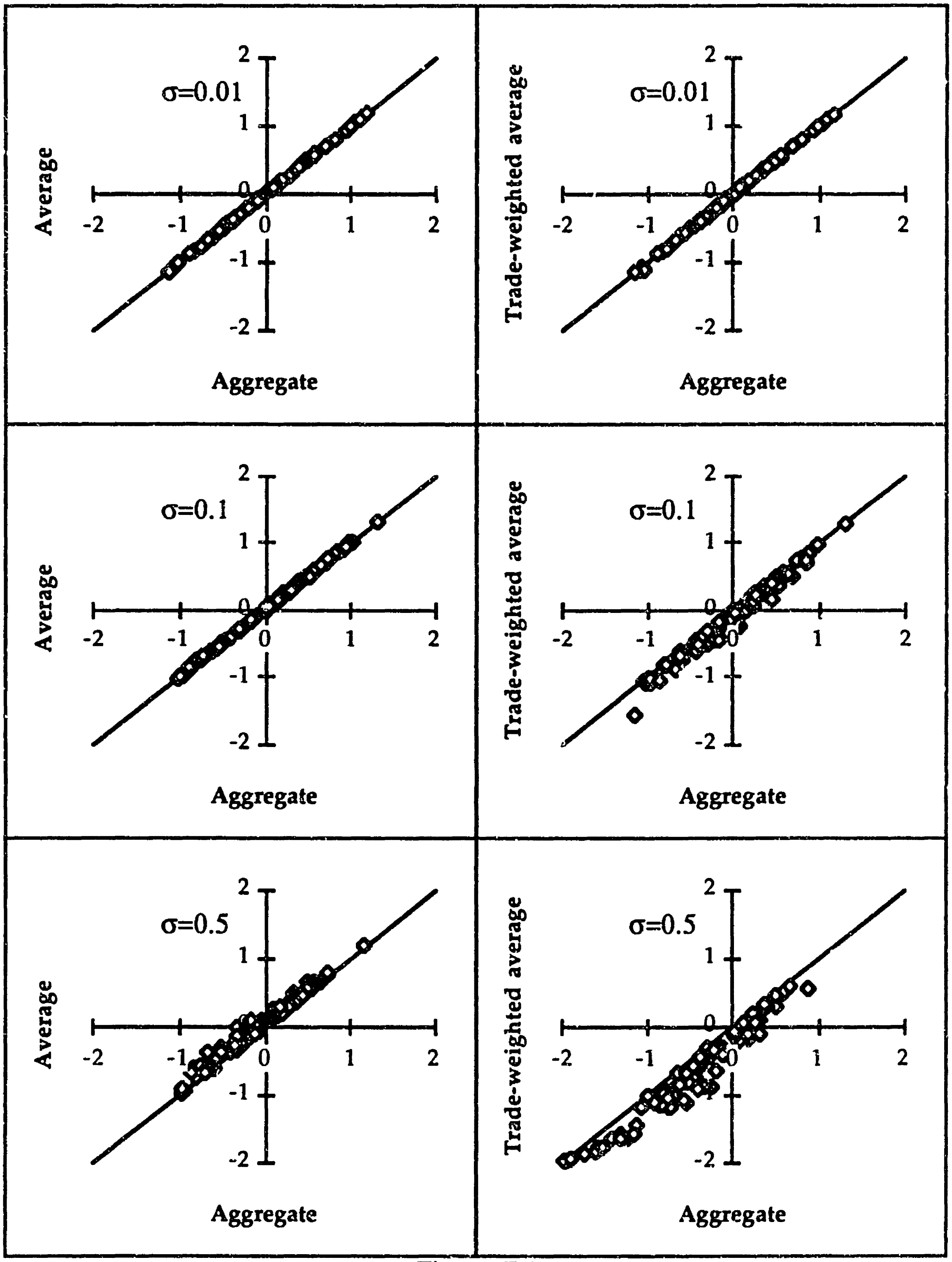

Figure 7.1

Plot of aggregate versus average price and demand

The figure shows the aggregates X and P plotted against the corresponding averages X and P, assuming that the individual x's and p's are normally distributed with mean 1 and variance σ^2. The results have been normalized to "Z-scores", i.e., $(X - 1)/\sigma$, etc.

8. Note regarding E{|n|}.

If a **random variable, n,** with a **cumulative distribution function, $\tilde{F}(n)$** and an **expected value of** μ, one can write the expected absolute value of n as

$$(8.1) \qquad E\{\,|n|\,\} = \int_{-\infty}^{0} -n\, d\tilde{F}(n) + \int_{0}^{\infty} n\, d\tilde{F}(n).$$

Suppose further that n can be written as

$$(8.2) \qquad n = \mu + t,$$

where the **distribution of t, F(t), is independent of** μ. Then,

$$(8.3) \qquad E\{\,|n|\,\} = \int_{-\infty}^{-\mu} -(\mu+t)\, dF(t) + \int_{-\mu}^{\infty} (\mu+t)\, dF(t),$$

$$= -\mu \int_{-\infty}^{-\mu} dF(t) + \mu \int_{-\mu}^{\infty} dF(t) - \int_{-\infty}^{-\mu} t\, dF(t) + \int_{-\mu}^{\infty} t\, dF(t)$$

$$= \mu(1 - 2F(-\mu)) + \int_{-\mu}^{\infty} t\, dF(t) - \int_{-\infty}^{-\mu} t\, dF(t).$$

Moreover, by differentiating with respect to μ, one finds

$$(8.4) \qquad \frac{dE\{|n|\}}{d\mu} = 1 - 2F(-\mu) + 2\mu\, dF(-\mu) - \mu\, dF(-\mu) - \mu\, dF(-\mu),$$

$$= 1 - 2F(-\mu).$$

The expected absolute value of n finds its minimum when

$$(8.5) \qquad \frac{dE\{|n|\}}{d\mu} = 0, \Rightarrow$$

$$F(-\mu) = 1/2,$$

i.e. when μ is equal to minus the median of the distribution of the random error, t. Note that if the distribution of t is symmetric, the mean should also be zero.

If, in particular, n, is normally distributed with mean μ **and variance** σ^2, one gets

(8.6) $$dF(t) = \phi(t/\sigma)/\sigma = \frac{1}{\sigma\sqrt{2\pi}} e^{-(t/\sigma)^2/2}, \text{ and}$$

(8.5) $$F(t) = \Phi(t/\sigma) = \int_{-\infty}^{t/\sigma} \frac{1}{\sqrt{2\pi}} e^{-s^2/2} ds,$$

where Φ and ϕ are the cumulative and frequency distribution, respectively, of the standard normal distribution.

Inserting this in (8.3) yields

(8.6) $$E\{\,|n|\,\} = \mu(1 - 2\Phi(-\mu/\sigma)) + 2\sigma\phi(-\mu/\sigma).$$

If, in particular, μ is zero, (8.6) becomes

(8.7) $$E\{\,|n|\,\} = 2\sigma\phi(0) = \sqrt{\frac{2}{\pi}}\sigma.$$

One can also approximate the expression (8.6) with a second-order Taylor expansion around the point μ=0. One gets

(8.8) $$E\{|n|\} \approx \sqrt{\frac{2}{\pi}}\sigma[1 + \frac{1}{2}(\mu/\sigma)^2].$$

Figure 8.1 shows a plot of $E\{|n|\}/\sigma$ and the approximation (8.8) as a function of μ/σ. It is evident that, within one standard deviation, the approximation is very good.

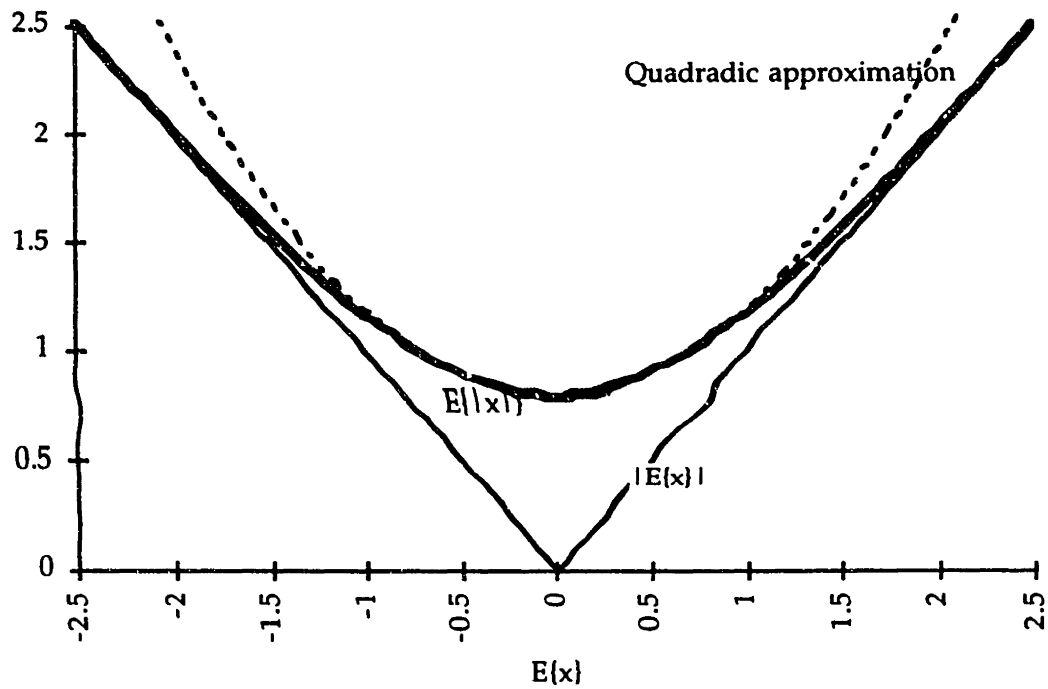

Figure 8.1
Plot of the expected absolute value of a normally distributed variable, x, as a function of its mean. Also shown is the quadratic Taylor approximation around the point x=0. All measures have been normalized by the standard deviation of x.

System Structure	Price regime		
	Fixed • Prices are constant • Demand met via inventory/backlog fluctuations	**Posted** • Sellers set prices • Demand met via inventory/backlog fluctuations	**Clearing** • Market-clearing prices found by computer • No inventories or backlogs needed
Simple • No production lags • No multiplier effect on demand	Condition 1	Condition 3	Condition 5
Complex • 3-period production lag • Multiplier effect on demand	Condition 2	Condition 4	Condition 6

Table 3.1.1: Experimental treatment design

Symbol	Text	Value
α	Marginal propensity to consume	0 or 0.5
δ	Production lag	0 or 3
γ/ω	Unit inventory cost, relative to unit production cost	0.5
ε	Price elasticity of individual-firm demand	2.5
μ	Price elasticity of industry demand around competitive equilibrium	0.75
π_0	Ratio of price to "reference" price at which aggregate demand is zero	4
χ_0	Ratio of demand to "reference" demand when aggregate price is zero	3
ω	Unit production cost	Arbitrary
G	Equilibrium output level	Arbitrary
σ	Standard deviation of random error in demand (selected cases only)	.10
ρ	Cross-firm correlation of random error in demand (selected cases only)	.50

Table 3.1.2: System parameter values.

The parameters for random errors apply only in those cases where such errors were introduced (not in the primary data set).

Educational status				
Undergraduate	Master's	Ph.D.	Other	Total
43	45	7	2	97
Affiliation				
MIT Sloan	Other MIT	Harvard	Other	Total
43	20	29	5	97
Background in...	None	Elementary	Intermediate	Advanced
Economics	19	41	26	11
Stat./OR	28	29	30	10
System dynamics	72	21	4	

Table 3.3.1: Summary of subject backgrounds.

Financial		**Production and sales**		**Prices, etc.**	**Forecasts**
Revenue	Mkt. avg. revenue	Production starts	Mkt. avg. prod. starts	Price	Sales forecast
Cost of goods sold	Mkt. avg. cost of goods sold	Units in production	Mkt. avg. units in prod.	Market avg. Price	Sales forecast error
Gross profits	Mkt. avg. gross profits	Finished production	Mkt. avg. fin. prod.	Market highest Price	Sales forecast score
Inventory costs	Mkt. avg. invent. costs	Inventory	Mkt. avg. inventory	Market lowest Price	Sales forecast
Net profits	Mkt. avg. net profits	Sales	Mkt. avg. sales	Price/mkt. avg. price	Sales forecast error
Cumulative profits	Mkt. avg. cum. profits			Sales/mkt. avg. sales	Sales forecast score

Table 3.3.2: Historical data available to subjects during experiment.

Table number				
1	2	3	4	5
Variables				
Revenue	Production starts	Price	Sales	<empty>
Cost of goods sold	Units in production	Mkt. avg. Price	Mkt. avg. Sales	<empty>
Inventory costs	Finished production	Price forecast	Sales forecast	<empty>
Net profits	Inventory	Price forecast error	Sales forecast error	<empty>
Cumulative profits	Sales	Price forecast score	Sales forecast score	<empty>

Table 3.3.3.: Initial definition of historical data tables available to subjects

Graph no. 1	2	3	4	5	6
Horizontal axis					
Time	Time	Time	Price/mkt. avg. price	Mkt. avg. sales	Mkt.avg. units in prod.
Vertical axis					
Gross profits	Production starts	Price	Sales/mkt. avg. sales	Mkt. avg. price	Mkt. avg. sales
Inventory costs	Inventory	Mkt. avg. price			
Net profits	Sales	Mkt. lowest price			
Mkt. avg. net profits	Mkt. avg. sales	Mkt. highest price			

Table 3.3.4: Historical data plots available to subjects.

Appendix B: Simulations

In connection with the formation of the experimental hypotheses, a large number of simulations were performed to chart the range of possible behaviors one could expect. For each condition, two kinds of simulations were performed: A set of "optimal" simulations, based on the assumption that firms correctly estimate the structural parameters of the system and, once these parameters have become known, switch to the optimal decision rule, and a set of "behavioral" simulations, based on a simple set of decision rules with unchanging parameters. This appendix documents the simulation models in detail and tabulates the results for each of the six conditions. The first part lists the complete equations for each simulation model. The second part is a series of tables of the simulation outcomes.

Simulation models

Fixed-price simple condition

Optimal model

For this condition, the optimal model is functionally equivalent to the behavioral model with the parameters

$\tau = 1; a_1 = 1; a_2 = 0; \sigma_x = 0$ (see below).

Behavioral model

$y_{i,t} = \mathrm{Max}\{0, x_{i,t}^e - n_{i,t}/\tau + u_{i,t}\}, t>0;$

$u_{i,t} \sim N(0, \sigma_y^2);$

$y_{i,o} = x_o;$

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$x_{i,t}{}^{e} = x_{i,t-1}{}^{e} + a_1(x_{i,t-1} - x_{i,t-1}{}^{e}) + a_2(x_{i,t-1} - x_{i,t-2}) + w_{i,t}, \ t>0;$

$x_{i,o}{}^{e} = x_o + w_{i,t};$

$w_{i,t} \sim N(0, \sigma_x{}^2);$

$x_o = \frac{2}{3}G; G = 1; p_o = p_{i,t} = p^* = 1; K = 4.$

Fixed-price complex condition

Optimal model

$y_{i,t} = \text{Max}\{\ 0, \hat{G}_t + \hat{\alpha}_t(Z_t - 3\hat{G}_t - N_t - S_t)/(1-\hat{\alpha}_t) + Z_t + 3\hat{G}_t - n_{i,t} - s_{i,t} + u_{i,t}\ \}, \ t>0;$

$u_{i,t} \sim N(0, \sigma_y{}^2);$

$\hat{G}_t = G, \hat{\alpha} = \alpha$ **for Est false;**

$\hat{G}_t = x_o, t \le 3, \hat{G}_t = G, t > 3,$ **for Est true;**

$\hat{\alpha}_t = 0, t \le 3, \hat{\alpha}_t = \alpha, t>3,$ **for Est true;**

$Z_t = \hat{\alpha}_t{}^*(\frac{3}{4}Y_{t-1} + \frac{1}{2}Y_{t-2} + \frac{1}{4}Y_{t-3} - \frac{3}{2}\hat{G}_t), t \ge 0;$

$x_o = \frac{2}{3}G; G = 1; p_o = p_{i,t} = p^* = 1; K = 4.$

Behavioral model

$y_{i,t} = \text{Max}\{\ 0, x_{i,t}{}^{e} + (n^* - n_{i,t})/\tau + \beta\ (s_{i,t}{}^{d} - s_{i,t})/\tau\ \}, \ t>0;$

$u_{i,t} \sim N(0, \sigma_y{}^2);$

$s_{i,t}{}^{d} = 3\theta x_{i,t}{}^{e} + (1-\theta)\ s_c, \ t>0;$

$y_{i,o} = x_o;$

$x_{i,t}^e = x_{i,t-1}^e + a_1(x_{i,t-1} - x_{i,t-1}^e) + a_2(x_{i,t-1} - x_{i,t-2}) + w_{i,t}$, t>0;

$x_{i,o}^e = x_o + w_{i,t}$;

$w_{i,t} \sim N(0, \sigma_x^2)$;

$x_o = \frac{2}{3}G$; $G = 1$; $n^* = 0$; $p_o = p_{i,t} = p^* = 1$; $K = 4$.

Clearing-price simple condition

Optimal model

$y_{i,o} = x_o$.

$u_{i,t} \sim N(0, \sigma_y^2)$;

$x_o = \frac{2}{3}G$; $G = 1$; $n^* = 0$; $K = 4$; $p^* = 1$; p_o = steady-state price, given x_o.

Parameters known (**Est** false), no collusion (**M** false):

$y_{i,t} = G + u_{i,t}$, t>0.

Parameters known (**Est** false), with collusion (**M** true):

$y_{i,t} = x_K^M + u_{i,t}$, t>0.

Parameters unknown (**Est** true):

$y_{i,t} = x_o + u_{i,t}$, $0<t\leq3$;

$y_{i,t} = \hat{a}_o + \hat{a}_1 p_{i,t}^d + u_{i,t}$, t>3;

$\hat{a}_o$ and $\hat{a}_1$ are estimated from the equation $X = a_o + a_1 P$, using data from the previous N_s periods.

No collusion (**M** false):

$p_{i,t}^d = \omega\, \hat{\varepsilon}_{i,t} / (1 - \hat{\varepsilon}_{i,t})$, t>3;

$\hat{\varepsilon}_{i,t}$ is estimated from the equation $\ln(x_i/X) = \varepsilon \ln(p_i/P)$, using data from the previous N_s periods.

With collusion (**M** true):

$$p_{i,t}{}^d = \frac{1}{2}(\omega - \hat{a}_0/\hat{a}_1).$$

<u>Behavioral model</u>

$$\ln(y_{i,t}) = (1-a_0)\ln(y_{i,t-1}) + a_0\ln(Y_{t-1}) + a_1\,\Delta v_{i,t-1} + a_2\,\Delta V_{t-1} + a_3\,dv_{i,t-1} + u_{i,t};$$

$$y_{i,o} = x_o;$$

$$u_{i,t} \sim N(0, \sigma_y{}^2);$$

$$\Delta v_{i,t} = (v_{i,t}-V_t)/(x_{i,t}-X_t) \text{ if } |x_{i,t}-X_t| \geq \sigma_y/10, \text{ else } \Delta v_{i,t-1};$$

$$\Delta V_t = (V_t-V_{t-1})/(X_t-X_{t-1}) \text{ if } |X_t-X_{t-1}| \geq \sigma_y/10, \text{ else } \Delta V_{t-1};$$

$$dv_{i,t} = (v_{i,t}-v_{i,t-1} + b(V_t-V_{t-1}))/(x_{i,t}-x_{i,t-1} + b(X_t-X_{t-1}))$$

if $|x_{i,t}-x_{i,t-1} + b(X_t-X_{t-1})| \geq \sigma_y$, else $dv_{i,t-1}$.

$$\Delta v_{i,o} = \Delta V_o = dv_{i,o} = p_o - \omega.$$

If "Discrete" is true, then all the profit gradients, Δv, are replaced by $\text{sgn}(\Delta v)$.

$x_o = \frac{2}{3}G$; $G = 1$; $n^* = 0$; $K = 4$; $p^* = 1$; p_o = steady-state price, given x_o.

Clearing-price complex condition

<u>Optimal model</u>

$$y_{i,o} = x_o.$$

$$u_{i,t} \sim N(0, \sigma_y{}^2);$$

$x_o = \frac{2}{3}G$; $G = 1$; $n^* = 0$; $K = 4$; $p^* = 1$; p_o = steady-state price, given x_o.

Appendix B

Parameters known (Est false), no collusion (M false):

$y_{i,t} = G + u_{i,t}$, $t>0$.

Parameters known (Est false), with collusion (M true):

$y_{i,t} = x_K{}^M + u_{i,t}$, $t>0$.

Parameters unknown (Est true):

No collusion (M false):

$y_{i,t} = x_o + u_{i,t}$, $0<t\leq4$;

$y_{i,t} = \hat{G}_{i,t} + u_{i,t}$, $t>4$;

$\hat{G}_{i,t} = (\omega\, \hat{p}_{i,t}{}^* - \hat{a}_o)/(\hat{a}_1 + \hat{a}_2)$;

$\hat{p}_{i,t}{}^* = \omega\, \hat{\varepsilon}_{i,t} / (1 - \hat{\varepsilon}_{i,t})$, $t>4$;

$\hat{\varepsilon}_{i,t}$ is estimated from the equation $\ln(x_i/X) = \varepsilon \ln(p_i/P)$, using data from the previous N_s periods.

$\hat{a}_o$, $\hat{a}_1$, and $\hat{a}_2$ are estimated from the equation $P = a_o + a_1(S + Y)/4 + a_2X$, using data from the previous N_s periods.

With collusion (M true):

$y_{i,t} = \hat{x}_t{}^M + u_{i,t}$;

$\hat{x}_t{}^M = \frac{1}{2}(\omega-\hat{a}_o)/(\hat{a}_1+\hat{a}_2)$;

Behavioral model

$\ln(y_{i,t}) = (1-a_o)\ln(y_{i,t-1}) + a_o\ln(Y_{t-1}) + a_1\, \Delta v_{i,t-1} + a_2\, \Delta V_{t-1} + a_3\, dv_{i,t-1} + u_{i,t}$, $t>0$;

$y_{i,o} = x_o$;

$u_{i,t} \sim N(0, \sigma_y^2)$;

$\Delta v_{i,t} = (v_{i,t}-V_t)/(x_{i,t}-X_t)$ if $|x_{i,t}-X_t| \geq \sigma_y/10$, else $\Delta v_{i,t-1}$;

$\Delta V_t = (V_t-V_{t-1})/(X_t-X_{t-1})$ if $|X_t-X_{t-1}| \geq \sigma_y/10$, else ΔV_{t-1};

$dv_{i,t} = (v_{i,t}-v_{i,t-1} + b(V_t-V_{t-1}))/(x_{i,t}-x_{i,t-1} + b(X_t-X_{t-1}))$

if $|x_{i,t}-x_{i,t-1} + b(X_t-X_{t-1})| \geq \sigma_y$, else $dv_{i,t-1}$.

$\Delta v_{i,t} = \Delta V_t = P_t - \omega$, $t \leq 3$;

$dv_{i,o} = p_o - \omega$;

If "Discrete" is true, then all the profit gradients, Δv, are replaced by $sgn(\Delta v)$.

$x_o = \frac{2}{3}G$; $G = 1$; $n^* = 0$; $K = 4$; $p^* = 1$; p_o = steady-state price, given x_o.

Posted-price simple condition

Optimal model

$y_{i,o} = x_o$.

$u_{i,t} \sim N(0, \sigma_y^2)$;

$x_o = \frac{2}{3}G$; $G = 1$; $n^* = 0$; $K = 4$; $p^* = 1$; p_o = steady-state price, given x_o.

$p_{i,t} = p_{i,t}^d$, $t>0$.

Parameters known (**Est** false), no collusion (**M** false):

$y_{i,t} = Max\{ 0, G - n_{i,t} + u_{i,t}\}$, $t>0$.

Parameters known (**Est** false), with collusion (**M** true):

$y_{i,t} = Max\{0, x_K^M - n_{i,t} + u_{i,t}\}$, $t>0$.

Parameters unknown (**Est** true):

$y_{i,t} = \mathrm{Max}\{0, x_o - n_{i,t} + u_{i,t}\}$, $0<t\leq 3$;

$y_{i,t} = \mathrm{Max}\{0, \hat{a}_o + \hat{a}_1\, p_{i,t}{}^d + u_{i,t}\}$, $t>3$;

$\hat{a}_o$ and $\hat{a}_1$ are estimated from the equation $X = a_o + a_1 P$, using data from the previous N_s periods.

No collusion (**M** false):

$p_{i,t}{}^d = \omega\, \hat{\varepsilon}_{i,t} / (1 - \hat{\varepsilon}_{i,t})$, $t>3$;

$\hat{\varepsilon}_{i,t}$ is estimated from the equation $\ln(x_i/X) = \varepsilon \ln(p_i/P)$, using data from the previous N_s periods.

With collusion (**M** true):

$p_{i,t}{}^d = \frac{1}{2}(\omega - \hat{a}_o/\hat{a}_1)$.

<u>Behavioral model</u>

$y_{i,t} = \mathrm{Max}\{0, x_{i,t}{}^e - n_{i,t}/\tau + u_{i,t}\}$, $t>0$;

$x_{i,t}{}^e = x_{i,t-1}{}^e + \mathbf{a}_1(X_{t-1} - x_{i,t-1}{}^e) + \mathbf{a}_2(X_{t-1}-X_{t-2}) + w_{i,t}$, $t>0$;

$x_{i,o} = x_o + w_{i,o}$;

$w_{i,t} \sim N(0,\sigma_x{}^2)$;

$p_{i,t} = [\mathbf{b}_o \ln(P_{t-1}) + (1-\mathbf{b}_o)\ln(p_{i,t-1})]\, \mathrm{Exp}(\mathbf{b}_1 \Delta v_{i,t-1} + \mathbf{b}_2 \Delta V_{t-1} + r_{i,t})$, $t>0$;

$r_{i,t} \sim N(0,\sigma_p{}^2)$;

$p_{i,o} = p_o$;

$\Delta v_{i,t} = (v_{i,t}-V_t)/(p_{i,t}-P_t)$ if $|p_{i,t}-P_t| \geq \mathbf{d}$, else $\Delta v_{i,t-1}$;

$\Delta V_t = (V_t-V_{t-1})/(P_t-P_{t-1})$ if $|P_t-P_{t-1}| \geq \mathbf{d}$, else ΔV_{t-1};

$\Delta v_{i,o} = 0$; $\Delta V_o = 0$;

Appendix B

If "Discrete" is true, then all the profit gradients, Δv, are replaced by $\mathrm{sgn}(\Delta v)$.

Posted-price complex condition

Optimal model

$y_{i,o} = x_o$.

$u_{i,t} \sim N(0, \sigma_y^2)$;

$x_o = \frac{2}{3}G$; $G = 1$; $n^* = 0$; $K = 4$; $p^* = 1$; p_o = steady-state price, given x_o.

$y_{i,t} = \mathrm{Max}\{0, x_{i,t}^d + (1-\mathbf{a})(3 - 1.5\hat{\alpha}_t)x_{i,t}^d/(1-\hat{\alpha}_t) - Z_t - n_{i,t} - s_{i,t}) + u_{i,t}\}$, $t>0$;

known (**Est** false), no collusion (**M** false):

$x_{i,t}^d = G$, $\hat{\alpha}_t = \alpha$.

Parameters known (**Est** false), with collusion (**M** true):

$x_{i,t}^d = x_M$, $\hat{\alpha}_t = \alpha$.

Parameters unknown (**Est** true):

$x_{i,t}^d = X_{t-1}$, $0<t\leq 3$;

$x_{i,t}^d = (\hat{a}_o + \hat{a}_1 p_{i,t}^d)/(1-\hat{a}_2)$, $t>3$;

$\hat{a}_o$, $\hat{a}_1$ and $\hat{a}_2$ are estimated from the equation $X = a_o + a_1 P + a_2 (S+Y)/4$, using data from the previous $\mathbf{N_s}$ periods.

No collusion (**M** false):

$p_{i,t}^d = \omega\, \hat{\varepsilon}_{i,t} / (1 - \hat{\varepsilon}_{i,t})$, $t>3$;

$\hat{\varepsilon}_{i,t}$ is estimated from the equation $\ln(x_i/X) = \varepsilon \ln(p_i/P)$, using data from the previous $\mathbf{N_s}$ periods.

Appendix B

With collusion (**M** true):

$$p_{i,t}^{d} = \frac{1}{2}(\omega - \hat{a}_o/\hat{a}_1).$$

$$p_{i,t} = a\, p_{i,t}^{c}\, [(N_t + Y_{t-\delta})/(n_{i,t}+y_{i,t-\delta})]^{1/\varepsilon} + (1-a)p_{i,t}^{d} + w_{i,t};$$

$$w_{i,t} \sim N(0, \sigma_p^{2});$$

Parameters known (Est false):

$$p_{i,t}^{c} = P_t^{c} = \text{true aggregate clearing price.}$$

Parameters estimated (Est true):

$$p_{i,t}^{c} = p_{i,t}^{d} \text{ for } 0 \le t \le 3;$$

$$p_{i,t}^{c} = (N_t + Y_{t-\delta} - \hat{a}_o - \hat{a}_2(S_t + x_{i,t}^{d})/4)/\hat{a}_1;$$

Z_t is defined in the optimal rule in Appendix A, but replacing α with $\hat{\alpha}$ and G with X_{t-1}.

Behavioral model

$$y_{i,t} = \mathrm{Max}\{\, 0,\, x_{i,t}^{e} + (n^{*} - n_{i,t})/\tau + \beta\,(s_{i,t}^{d} - s_{i,t})/\tau\,\},\ t>0;$$

$$u_{i,t} \sim N(0, \sigma_y^{2});$$

$$s_{i,t}^{d} = 3\theta x_{i,t}^{e} + (1-\theta)\, s_c,\ t>0;$$

$$y_{i,o} = x_o;$$

$$x_{i,t}^{e} = x_{i,t-1}^{e} + a_1(x_{i,t-1} - x_{i,t-1}^{e}) + a_2(x_{i,t-1} - x_{i,t-2}) + w_{i,t},\ t>0;$$

$$x_{i,o}^{e} = x_o + w_{i,t};$$

$$w_{i,t} \sim N(0, \sigma_x^{2});$$

$$x_o = \frac{2}{3}G;\ G = 1;\ n^{*} = 0;\ p_o = p_{i,t} = p^{*} = 1;\ K = 4.$$

Appendix B

$p_{i,t} = P_{t-1}\ \mathrm{Exp}(\mathbf{b}_1 \Delta v_{i,t-1} + \mathbf{b}_2 \Delta V_{t-1} + \mathbf{b}_3(n_{i,t} + y_{i,t-\delta} - x_{i,t}^{e}) + r_{i,t}),\ t>0;$

$r_{i,t} \sim N(0,\sigma_p^{2});$

$p_{i,o} = p_o;$

$\Delta v_{i,t} = (v_{i,t}-V_t)/(p_{i,t}-P_t)$ if $|p_{i,t}-P_t| \geq \mathbf{d}$, else $\Delta v_{i,t-1};$

$\Delta V_t = (V_t-V_{t-1})/(P_t-P_{t-1})$ if $|P_t-P_{t-1}| \geq \mathbf{d}$, else $\Delta V_{t-1};$

$\Delta v_{i,o} = 0;\ \Delta V_o = 0;$

If "Discrete" is true, then all the profit gradients, Δv, are replaced by sgn(Δv).

Simulation results

The results are based on 20 simulations of 40 time periods each for each combination of parameters. The symbol ### means that the values exceed 1000, indicating that the system is unstable. The column "Crash" indicates how many out of the 20 simulations that led to infinite or undefined quantities (as a results of instability.) Otherwise, the following results are calculated:

S{Y}, t>10	standard deviation in Y_t, t>10;
S{P}, t>10	standard deviation in P_t, t>10;
Rev., t>10	average revenue, t>10;
P.C., t>10	average production cost, t>10;
G.P., t>10	average gross profits, t>10;
In.C., t>10	average inventory costs, t>10;
Prof., t>10	average net profits, t>10;
S{Y}, t≤20	standard deviation in Y_t, t≤20;
P, t≤20	average P_t, t≤ 20;
S{P}, t≤20	standard deviation in P_t, t≤ 20;
S{Y}, t>20	standard deviation in Y_t, t> 20;
P, t>20	average P_t, t>20;
S{P}, t>20	standard deviation in P_t, t>20.

Appendix B

Fixed-price simple condition, behavioral and optimal model

a_1	a_2	τ	σ_ν	σ_v	Crashes	S(Y) t>10	S(P) t>10	Rev. t>10	P.C. t>10	G.P. t>10	In.C. t>10	Prof. t>10	S(Y) t≤20	P t≤20	S(P) t≤20	S(Y) t>20	P t>20	S(P) t>20
0.5	0	1	0	0	0	.000	.000	1.000	.600	.400	.000	.400	.088	1.000	.000	.000	1.000	.000
						±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000
0.5	0	1	0	0.1	0	.060	.000	1.000	.600	.400	.028	.372	.108	1.000	.000	.060	1.000	.000
						±.008	±.000	±.000	±.000	±.000	±.002	±.002	±.011	±.000	±.000	±.009	±.000	±.000
0.5	0	1	0.1	0	0	.070	.000	1.000	.600	.400	.024	.376	.110	1.000	.000	.069	1.000	.000
						±.013	±.000	±.000	±.000	±.000	±.002	±.002	±.011	±.000	±.000	±.016	±.000	±.000
0.5	0	1	0.1	0.1	0	.096	.000	1.000	.600	.400	.037	.363	.128	1.000	.000	.096	1.000	.000
						±.012	±.000	±.000	±.000	±.000	±.003	±.003	±.017	±.000	±.000	±.015	±.000	±.000
0.5	0	2	0	0	0	.001	.000	1.000	.600	.400	.000	.400	.098	1.000	.000	.000	1.000	.000
						±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000
0.5	0	2	0	0.1	0	.054	.000	1.000	.600	.400	.041	.359	.112	1.000	.000	.054	1.000	.000
						±.008	±.000	±.000	±.000	±.000	±.005	±.005	±.016	±.000	±.000	±.008	±.000	±.000
0.5	0	2	0.1	0	0	.059	.000	1.000	.600	.400	.027	.373	.116	1.000	.000	.058	1.000	.000
						±.007	±.000	±.000	±.000	±.000	±.003	±.003	±.013	±.000	±.000	±.008	±.000	±.000
0.5	0	2	0.1	0.1	0	.078	.000	1.000	.600	.400	.048	.352	.123	1.000	.000	.080	1.000	.000
						±.007	±.000	±.000	±.000	±.000	±.005	±.005	±.020	±.000	±.000	±.012	±.000	±.000
1	0	1	0	0	0	.000	.000	1.000	.600	.400	.000	.400	.108	1.000	.000	.000	1.000	.000
						±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000
1	0	1	0	0.1	0	.070	.000	1.000	.600	.400	.024	.376	.128	1.000	.000	.073	1.000	.000
						±.010	±.000	±.000	±.000	±.000	±.002	±.002	±.011	±.000	±.000	±.014	±.000	±.000
1	0	1	0.1	0	0	.073	.000	1.000	.600	.400	.025	.375	.132	1.000	.000	.074	1.000	.000
						±.010	±.000	±.000	±.000	±.000	±.001	±.001	±.012	±.000	±.000	±.010	±.000	±.000
1	0	1	0.1	0.1	0	.101	.000	1.000	.600	.400	.034	.366	.146	1.000	.000	.103	1.000	.000
						±.017	±.000	±.000	±.000	±.000	±.003	±.003	±.016	±.000	±.000	±.024	±.000	±.000
1	0	2	0	0	0	.000	.000	1.000	.600	.400	.000	.400	.108	1.000	.000	.000	1.000	.000
						±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000
1	0	2	0	0.1	0	.058	.000	1.000	.600	.400	.027	.373	.124	1.000	.000	.057	1.000	.000
						±.010	±.000	±.000	±.000	±.000	±.002	±.002	±.010	±.000	±.000	±.012	±.000	±.000
1	0	2	0.1	0	0	.060	.000	1.000	.600	.400	.028	.372	.123	1.000	.000	.059	1.000	.000
						±.010	±.000	±.000	±.000	±.000	±.003	±.003	±.011	±.000	±.000	±.011	±.000	±.000
1	0	2	0.1	0.1	0	.079	.000	1.000	.600	.400	.040	.360	.138	1.000	.000	.078	1.000	.000
						±.013	±.000	±.000	±.000	±.000	±.005	±.005	±.019	±.000	±.000	±.011	±.000	±.000
1	0.5	1	0	0	0	.000	.000	1.000	.600	.400	.000	.400	.094	1.000	.000	.000	1.000	.000
						±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000
1	0.5	1	0	0.1	0	.069	.000	1.000	.600	.400	.023	.377	.118	1.000	.000	.068	1.000	.000
						±.010	±.000	±.000	±.000	±.000	±.001	±.001	±.011	±.000	±.000	±.014	±.000	±.000
1	0.5	1	0.1	0	0	.070	.000	1.000	.600	.400	.024	.376	.115	1.000	.000	.072	1.000	.000
						±.012	±.000	±.000	±.000	±.000	±.002	±.002	±.014	±.000	±.000	±.014	±.000	±.000
1	0.5	1	0.1	0.1	0	.095	.000	1.000	.600	.400	.033	.367	.135	1.000	.000	.091	1.000	.000
						±.013	±.000	±.000	±.000	±.000	±.002	±.002	±.020	±.000	±.000	±.013	±.000	±.000
1	0.5	2	0	0	0	.000	.000	1.000	.600	.400	.000	.400	.088	1.000	.000	.000	1.000	.000
						±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000
1	0.5	2	0	0.1	0	.059	.000	1.000	.600	.400	.027	.373	.106	1.000	.000	.060	1.000	.000
						±.007	±.000	±.000	±.000	±.000	±.002	±.002	±.012	±.000	±.000	±.010	±.000	±.000
1	0.5	2	0.1	0	0	.060	.000	1.000	.600	.400	.028	.372	.101	1.000	.000	.059	1.000	.000
						±.009	±.000	±.000	±.000	±.000	±.002	±.002	±.012	±.000	±.000	±.010	±.000	±.000
1	0.5	2	0.1	0.1	0	.082	.000	1.000	.600	.400	.039	.361	.123	1.000	.000	.083	1.000	.000
						±.011	±.000	±.000	±.000	±.000	±.004	±.004	±.017	±.000	±.000	±.014	±.000	±.000
1	1	1	0	0	0	.000	.000	1.000	.600	.400	.000	.400	.108	1.000	.000	.000	1.000	.000
						±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000
1	1	1	0	0.1	0	.068	.000	1.000	.600	.400	.024	.376	.132	1.000	.000	.068	1.000	.000
						±.012	±.000	±.000	±.000	±.000	±.002	±.002	±.016	±.000	±.000	±.011	±.000	±.000
1	1	1	0.1	0	0	.071	.000	1.000	.600	.400	.023	.377	.132	1.000	.000	.069	1.000	.000
						±.013	±.000	±.000	±.000	±.000	±.002	±.002	±.017	±.000	±.000	±.016	±.000	±.000
1	1	1	0.1	0.1	0	.101	.000	1.000	.600	.400	.035	.365	.150	1.000	.000	.105	1.000	.000
						±.015	±.000	±.000	±.000	±.000	±.003	±.003	±.021	±.000	±.000	±.020	±.000	±.000
1	1	2	0	0	0	.000	.000	1.000	.600	.400	.000	.400	.088	1.000	.000	.000	1.000	.000
						±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000
1	1	2	0	0.1	0	.055	.000	1.000	.600	.400	.028	.372	.104	1.000	.000	.053	1.000	.000
						±.010	±.000	±.000	±.000	±.000	±.003	±.003	±.013	±.000	±.000	±.012	±.000	±.000
1	1	2	0.1	0	0	.059	.000	1.000	.600	.400	.027	.373	.101	1.000	.000	.060	1.000	.000
						±.008	±.000	±.000	±.000	±.000	±.002	±.002	±.015	±.000	±.000	±.010	±.000	±.000
1	1	2	0.1	0.1	0	.081	.000	1.000	.600	.400	.040	.360	.121	1.000	.000	.081	1.000	.000
						±.010	±.000	±.000	±.000	±.000	±.003	±.003	±.018	±.000	±.000	±.011	±.000	±.000
1	0	1	0	0	0	.000	.000	1.000	.600	.400	.000	.400	.108	1.000	.000	.000	1.000	.000
						±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000
1	0	1	0.05	0	0	.037	.000	1.000	.600	.400	.012	.388	.116	1.000	.000	.038	1.000	.000
						±.005	±.000	±.000	±.000	±.000	±.001	±.001	±.005	±.000	±.000	±.007	±.000	±.000
1	0	1	0.1	0	0	.077	.000	1.000	.600	.400	.024	.376	.136	1.000	.000	.077	1.000	.000
						±.011	±.000	±.000	±.000	±.000	±.001	±.001	±.014	±.000	±.000	±.013	±.000	±.000
1	0	1	0.01	0	0	.007	.000	1.000	.600	.400	.002	.398	.109	1.000	.000	.008	1.000	.000
						±.001	±.000	±.000	±.000	±.000	±.000	±.000	±.001	±.000	±.000	±.001	±.000	±.000

Appendix B

Fixed-price complex condition optimal model

σ_y	Est	Crashes	S(Y) t>10	S(P) t>10	Rev. t>10	P.C. t>10	G.P. t>10	In.C. t>10	Prof. t>10	S(Y) t≤20	P t≤20	S(P) t≤20	S(Y) t>20	P t>20	S(P) t>20
0.01	###	0	.007	.000	1.000	.600	.400	.002	.398	.335	1.000	.000	.007	1.000	.000
			±.001	±.000	±.000	±.000	±.000	±.000	±.000	±.001	±.000	±.000	±.001	±.000	±.000
0.05	###	0	.035	.000	1.000	.600	.400	.012	.388	.337	1.000	.000	.034	1.000	.000
			±.006	±.000	±.000	±.000	±.000	±.001	±.001	±.006	±.000	±.000	±.007	±.000	±.000
0.1	###	0	.071	.000	1.000	.600	.400	.023	.377	.344	1.000	.000	.075	1.000	.000
			±.008	±.000	±.001	±.000	±.000	±.002	±.002	±.013	±.000	±.000	±.012	±.000	±.000
0.01	###	0	.007	.000	1.000	.600	.400	.002	.398	.495	1.000	.000	.006	1.000	.000
			±.001	±.000	±.000	±.000	±.000	±.000	±.000	±.001	±.000	±.000	±.001	±.000	±.000
0.05	###	0	.035	.000	1.000	.600	.400	.012	.388	.497	1.000	.000	.035	1.000	.000
			±.005	±.000	±.000	±.000	±.000	±.001	±.001	±.009	±.000	±.000	±.006	±.000	±.000
0.1	###	0	.071	.000	1.000	.600	.400	.024	.376	.499	1.000	.000	.072	1.000	.000
			±.010	±.000	±.000	±.000	±.000	±.002	±.002	±.021	±.000	±.000	±.012	±.000	±.000

Appendix B

Fixed-price condition, behavioral model

τ	β	θ	S_c	σ_y	a_1	a_2	σ_x	Crash	S(Y) t>10	S(P) t>10	Rev. t>10	P.C. t>10	G.P. t>10	In.C t>10	Prof. t>10	S(Y) t≤20	P t≤20	S(P) t≤20	S(Y) t>20	P t>20	S(P) t>20
1	0	0.5	3	0	0.5	0	0	0	1.838	.000	1.194	.717	.478	.962	-.485	1.051	1.000	.000	2.053	1.000	.000
									±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000
1	0.5	0.5	3	0.05	0.5	0	0	0	.169	.000	.980	.588	.392	.095	.297	.284	1.000	.000	.141	1.000	.000
									±.006	±.000	±.001	±.000	±.000	±.003	±.003	±.005	±.000	±.000	±.007	±.000	±.000
1	1	0.5	3	0.1	0.5	0	0	0	.073	.000	1.002	.601	.401	.033	.368	.130	1.000	.000	.071	1.000	.000
									±.011	±.000	±.001	±.000	±.000	±.002	±.003	±.017	±.000	±.000	±.014	±.000	±.000
1	1	1	3	0	0.5	0	0	0	.101	.000	1.008	.605	.403	.069	.334	.202	1.000	.000	.073	1.000	.000
									±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000
1	1	1	3	0.05	0.5	0	0	0	.107	.000	1.008	.605	.403	.070	.333	.205	1.000	.000	.081	1.000	.000
									±.004	±.000	±.000	±.000	±.000	±.002	±.002	±.005	±.000	±.000	±.005	±.000	±.000
1	1	1	3	0.1	0.5	0	0	0	.117	.000	1.008	.605	.403	.073	.330	.213	1.000	.000	.097	1.000	.000
									±.007	±.000	±.001	±.001	±.000	±.003	±.003	±.006	±.000	±.000	±.008	±.000	±.000
1	1	1	3	0	0.5	0	0.05	0	.157	.000	1.009	.605	.404	.090	.314	.222	1.000	.000	.142	1.000	.000
									±.022	±.000	±.004	±.003	±.002	±.012	±.012	±.025	±.000	±.000	±.023	±.000	±.000
1	1	1	3	0	0.5	0	0.1	0	.257	.000	1.010	.606	.404	.133	.271	.295	1.000	.000	.256	1.000	.000
									±.034	±.000	±.009	±.006	±.004	±.019	±.017	±.043	±.000	±.000	±.039	±.000	±.000
1	1	1	3	0.1	0.4	0	0.1	0	.247	.000	1.012	.607	.405	.144	.261	.289	1.000	.000	.226	1.000	.000
									±.033	±.000	±.007	±.004	±.003	±.020	±.020	±.040	±.000	±.000	±.041	±.000	±.000
1	1	0	3	0	0.5	0	0	0	.006	.000	1.002	.601	.401	.004	.396	.153	1.000	.000	.001	1.000	.000
									±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000
1	1	0	3	0.05	0.5	0	0	0	.036	.000	1.002	.601	.401	.014	.387	.158	1.000	.000	.033	1.000	.000
									±.005	±.000	±.000	±.000	±.000	±.001	±.001	±.006	±.000	±.000	±.006	±.000	±.000
1	1	0	3	0.1	0.5	0	0	0	.069	.000	1.002	.601	.401	.025	.375	.163	1.000	.000	.067	1.000	.000
									±.014	±.000	±.001	±.000	±.000	±.001	±.001	±.013	±.000	±.000	±.016	±.000	±.000
1	1	0	3	0.05	1	0	0	0	.034	.000	1.001	.600	.400	.013	.388	.160	1.000	.000	.034	1.000	.000
									±.006	±.000	±.000	±.000	±.000	±.001	±.001	±.006	±.000	±.000	±.008	±.000	±.000
1	1	0	3	0.05	1	1	0	0	.034	.000	1.000	.600	.400	.012	.388	.161	1.000	.000	.034	1.000	.000
									±.006	±.000	±.000	±.000	±.000	±.001	±.001	±.005	±.000	±.000	±.007	±.000	±.000

Appendix B

Clearing-price simple condition, optimal model

σ_y	N_s	M	Est	Crash	S(Y) t>10	S(P) t>10	Rev. t>10	P.C. t>10	G.P. t>10	In.C. t>10	Prof. t>10	S(Y) t≤20	P t≤20	S(P) t≤20	S(Y) t>20	P t>20	S(P) t>20
0	10	No	No	0	.000	.000	1.000	.600	.400	.000	.400	.005	1.000	.007	.005	1.000	.006
					±.000	±.000	±.000	±.000	±.001	±.000	±.001	±.001	±.002	±.001	±.001	±.002	±.001
0	10	No	Yes	0	.006	.008	1.000	.599	.401	.000	.401	.119	1.130	.256	.005	1.000	006
					±.001	±.001	±.000	±.001	±.001	±.000	±.001	±.001	±.003	±.004	±.001	±.001	±.001
0	10	Yes	No	0	.005	.017	1.142	.298	.844	.000	.844	.005	2.300	.018	.005	2.301	.017
					±.001	±.002	±.000	±.000	±.000	±.000	±.000	±.001	±.004	±.003	±.001	±.003	±.002
0	10	Yes	Yes	0	.005	.017	1.142	.298	.844	.000	.844	.062	2.212	.214	.005	2.300	.016
					±.001	±.002	±.000	±.000	±.000	±.000	±.000	±.001	±.004	±.004	±.001	±.003	±.003
0.1	10	No	No	0	.024	.033	1.001	.599	.401	.000	.401	.025	1.002	.034	.024	1.002	.032
					±.003	±.005	±.002	±.003	±.004	±.000	±.004	±.004	±.007	±.006	±.004	±.006	±.006
0.1	10	No	Yes	0	.025	.034	1.001	.599	.402	.000	.402	.121	1.127	.258	.026	1.002	.035
					±.002	±.003	±.002	±.003	±.004	±.000	±.004	±.007	±.014	±.023	±.003	±.007	±.005
0.1	10	Yes	No	0	.025	.085	1.139	.298	.841	.000	.841	.025	2.299	.085	.024	2.296	.083
					±.003	±.011	±.003	±.002	±.001	±.000	±.001	±.004	±.019	±.015	±.004	±.018	±.015
0.1	10	Yes	Yes	0	.025	.084	1.139	.298	.841	.000	.841	.065	2.211	.223	.025	2.298	.085
					±.003	±.009	±.002	±.002	±.000	±.000	±.000	±.005	±.023	±.018	±.003	±.014	±.012
0.1	10	No	No	0	.052	.071	1.000	.602	.397	.000	.397	.049	1.000	.066	.052	1.000	.070
					±.008	±.010	±.004	±.007	±.011	±.000	±.011	±.013	±.017	±.019	±.009	±.018	±.012
0.1	10	No	Yes	0	.047	.065	1.002	.599	.402	.000	.402	.124	1.134	.262	.047	1.004	.064
					±.007	±.010	±.003	±.005	±.007	±.000	±.007	±.010	±.028	±.030	±.008	±.011	±.012
0.1	10	Yes	No	0	.052	.175	1.131	.298	.832	000	.832	.053	2.306	.178	.051	2.290	.171
					±.007	±.022	±.007	±.006	±.002	±.000	±.002	±.007	±.039	±.022	±.009	±.041	±.029
0.1	10	Yes	Yes	0	.051	.170	1.133	.300	.833	.000	.833	.080	2.195	.268	.050	2.283	.168
					±.005	±.017	±.005	±.005	±.002	±.000	±.002	±.011	±.035	±.035	±.006	±.045	±.020
1	1	0	0	0	.034	.000	1.001	.600	.400	.013	.388	.160	1.000	.000	.034	1.000	.000
					±.006	±.000	±.000	±.000	±.000	±.001	±.001	±.006	±.000	±.000	±.008	±.000	±.000
1	1	1	0	0	.034	.000	1.000	.600	.400	.012	.388	.161	1.000	.000	.034	1.000	.000
					±.006	±.000	±.000	±.000	±.000	±.001	±.001	±.005	±.000	±.000	±.007	±.000	±.000

Clearing-price simple condition, behavioral model

a1	a2	a3	a0	Discrete	b	σy	Crash	S[Y] t>10	S[P] t>10	Rev. t>10	P.C. t>10	G.P. t>10	In.C. t>10	Prof. t>10	S[Y] t≤20	P t≤20	S[P] t≤20	S[Y] t>20	P t>20	S[P] t>20
0.25	0	0	1	No	1	0.01	0	.418	.009	1.000	.600	.400	.000	.400	.032	1.030	.049	.006	.999	.008
								±.004	±.001	±.001	±.002	±.002	±.000	±.002	±.004	±.013	±.006	±.001	±.004	±.002
0.25	0	0	1	No	1	0.05	0	.417	.047	.999	.602	.398	.000	.398	.047	1.031	.067	.030	.999	.040
								±.017	±.007	±.007	±.013	±.020	±.000	±.020	±.012	±.032	±.018	±.007	±.035	±.009
0.25	0	0	1	No	1	0.1	0	.396	.093	.994	.616	.378	.000	.378	.077	1.002	.103	.070	.983	.091
								±.043	±.022	±.017	±.032	±.049	±.000	±.049	±.020	±.071	±.022	±.018	±.072	±.025
0.5	0	0	1	No	1	0.01	0	.397	.008	1.000	.601	.399	.000	.399	.050	.970	.056	.006	.999	.008
								±.005	±.001	±.001	±.001	±.002	±.000	±.002	±.017	±.011	±.015	±.001	±.004	±.002
0.5	0	0	1	No	1	0.05	0	.390	.040	.999	.602	.397	.000	.397	.077	.963	.078	.030	.996	.039
								±.017	±.007	±.004	±.006	±.010	±.000	±.010	±.068	±.034	±.039	±.006	±.018	±.008
0.5	0	0	1	No	1	0.1	0	.380	.083	.998	.606	.392	.000	.392	.094	.941	.107	.061	.995	.079
								±.018	±.015	±.007	±.013	±.020	±.000	±.020	±.031	±.038	±.033	±.016	±.029	±.018
1	0	0	1	No	1	0.01	0	.365	.007	1.000	.600	.400	.000	.400	.291	.930	.177	.005	1.000	.007
								±.024	±.001	±.000	±.001	±.001	±.000	±.001	±.113	±.025	±.038	±.001	±.002	±.001
1	0	0	1	No	1	0.05	0	.366	.035	.999	.601	.398	.000	.398	.281	.931	.178	.027	.998	.036
								±.018	±.005	±.002	±.004	±.006	±.000	±.006	±.082	±.023	±.033	±.004	±.011	±.006
1	0	0	1	No	1	0.1	0	.357	.074	.997	.608	.388	.000	.388	.299	.932	.194	.058	.981	.074
								±.021	±.011	±.005	±.009	±.014	±.000	±.014	±.079	±.025	±.033	±.011	±.023	±.013
2	0	0	1	No	1	0.01	0	.273	.013	1.000	.600	.400	.000	.400	1.292	.917	.268	.008	1.000	.010
								±.073	±.005	±.000	±.000	±.001	±.000	±.001	±.522	±.024	±.030	±.003	±.002	±.003
2	0	0	1	No	1	0.05	0	.270	.053	1.000	.601	.398	.000	.398	1.317	.918	.277	.036	1.000	.047
								±.066	±.019	±.001	±.003	±.004	±.000	±.004	±.496	±.021	±.028	±.009	±.006	±.012
2	0	0	1	No	1	0.1	0	.277	.128	.996	.615	.381	.000	.381	1.147	.923	.286	.137	.992	.125
								±.029	±.047	±.010	±.027	±.037	±.000	±.037	±.055	±.014	±.023	±.194	±.026	±.057
0.25	0	0	1	Yes	1	0.01	0	.396	.189	.997	.615	.382	.000	.382	.149	1.012	.197	.146	1.001	.189
								±.021	±.011	±.008	±.015	±.023	±.000	±.023	±.011	±.040	±.013	±.006	±.033	±.010
0.25	0	0	1	Yes	1	0.05	0	.398	.186	.997	.615	.382	.000	.382	.140	1.015	.186	.142	1.000	.186
								±.008	±.011	±.004	±.008	±.012	±.000	±.012	±.011	±.031	±.011	±.015	±.023	±.016
0.25	0	0	1	Yes	1	0.1	0	.401	.196	1.000	.611	.388	.000	.388	.145	1.014	.194	.146	1.013	.196
								±.014	±.016	±.005	±.008	±.013	±.000	±.013	±.013	±.032	±.015	±.014	±.036	±.024
0.5	0	0	1	Yes	1	0.01	0	.360	.375	.992	.654	.338	.000	.338	.302	1.038	.388	.305	1.017	.376
								±.031	±.025	±.014	±.028	±.042	±.000	±.042	±.021	±.069	±.035	±.019	±.068	±.036
0.5	0	0	1	Yes	1	0.05	0	.361	.370	.993	.651	.342	.000	.342	.287	1.043	.376	.302	1.006	.370
								±.032	±.034	±.013	±.024	±.037	±.000	±.037	±.026	±.079	±.030	±.022	±.073	±.046
0.5	0	0	1	Yes	1	0.1	0	.360	.378	.992	.653	.340	.000	.340	.300	1.031	.396	.293	1.018	.376
								±.039	±.033	±.015	±.030	±.044	±.000	±.044	±.030	±.065	±.025	±.026	±.063	±.032
1	0	0	1	Yes	1	0.01	0	.118	.707	.894	.822	.072	.000	.072	.721	1.059	.742	.754	1.011	.714
								±.077	±.070	±.035	±.061	±.093	±.000	±.093	±.097	±.111	±.064	±.081	±.140	±.072
1	0	0	1	Yes	1	0.05	0	.161	.732	.919	.783	.135	.000	.135	.716	1.103	.761	.700	1.054	.720
								±.106	±.068	±.051	±.081	±.129	±.000	±.129	±.082	±.140	±.067	±.095	±.145	±.086
1	0	0	1	Yes	1	0.1	0	.164	.733	.924	.773	.150	.000	.150	.678	1.047	.708	.710	1.079	.742
								±.062	±.082	±.033	±.054	±.080	±.000	±.080	±.061	±.096	±.049	±.110	±.151	±.113
2	0	0	1	Yes	1	0.01	0	-.526	1.092	.488	.973	-.486	.000	-.486	2.245	1.481	1.275	1.745	2.127	.865
								±.249	±.347	±.269	±.586	±.355	±.000	±.355	±.435	±.602	±.209	±1.236	±1.293	±.591
2	0	0	1	Yes	1	0.05	0	-.618	1.042	.430	1.041	-.611	.000	-.611	2.401	1.403	1.253	1.785	2.261	.910
								±.348	±.516	±.254	±.682	±.452	±.000	±.452	±.456	±.659	±.238	±1.349	±1.294	±.588
2	0	0	1	Yes	1	0.1	0	-.679	1.169	.619	1.280	-.661	.000	-.661	2.288	1.095	1.113	2.035	1.442	1.084
								±.212	±.146	±.128	±.352	±.284	±.000	±.284	±.375	±.137	±.095	±.761	±.850	±.263
0	0.1	0	1	No	1	0.01	0	.833	.012	1.142	.298	.844	.000	.844	.067	2.199	.224	.003	2.300	.011
								±.000	±.003	±.001	±.001	±.000	±.000	±.000	±.002	±.005	±.005	±.001	±.007	±.003
0	0.1	0	1	No	1	0.05	0	.832	.059	1.140	.298	.843	.000	.843	.070	2.200	.233	.016	2.304	.055
								±.001	±.012	±.004	±.004	±.001	±.000	±.001	±.007	±.028	±.023	±.003	±.027	±.011
0	0.1	0	1	No	1	0.1	0	.829	.114	1.143	.304	.839	.000	.839	.070	2.172	.233	.032	2.267	.109
								±.003	±.026	±.007	±.008	±.002	±.000	±.002	±.011	±.063	±.034	±.008	±.055	±.029
0	0.25	0	1	No	1	0.01	0	.830	.011	1.142	.298	.844	.000	.844	.096	2.224	.289	.003	2.301	.010
								±.000	±.002	±.000	±.000	±.000	±.000	±.000	±.001	±.003	±.003	±.001	±.002	±.002
0	0.25	0	1	No	1	0.05	0	.830	.052	1.141	.298	.843	.000	.843	.096	2.221	.290	.015	2.301	.051
								±.001	±.010	±.002	±.002	±.000	±.000	±.000	±.007	±.013	±.018	±.003	±.014	±.011
0	0.25	0	1	No	1	0.1	0	.826	.105	1.140	.299	.840	.000	.840	.102	2.229	.310	.031	2.287	.106
								±.003	±.014	±.004	±.004	±.001	±.000	±.001	±.013	±.031	±.033	±.005	±.026	±.017
0	0.5	0	1	No	1	0.01	0	.815	.105	1.138	.297	.841	.000	.841	.171	2.213	.440	.019	2.303	.065
								±.001	±.006	±.001	±.001	±.000	±.000	±.000	±.002	±.005	±.005	±.002	±.004	±.006
0	0.5	0	1	No	1	0.05	0	.813	.132	1.136	.297	.839	.000	.839	.174	2.211	.446	.030	2.303	.102
								±.003	±.032	±.003	±.002	±.002	±.000	±.002	±.008	±.006	±.019	±.010	±.013	±.035
0	0.5	0	1	No	1	0.1	0	.809	.183	1.132	.298	.834	.000	.834	.178	2.206	.459	.048	2.303	.164
								±.004	±.035	±.005	±.002	±.004	±.000	±.004	±.015	±.024	±.031	±.013	±.016	±.044
0	1	0	1	No	1	0.01	0	.547	1.006	.924	.358	.566	.000	.566	.478	2.070	1.048	.388	2.093	1.021
								±.001	±.002	±.001	±.001	±.001	±.000	±.001	±.002	±.001	±.001	±.002	±.002	±.002
0	1	0	1	No	1	0.05	0	.548	1.005	.925	.359	.567	.000	.567	.478	2.067	1.047	.385	2.092	1.017
								±.003	±.007	±.003	±.002	±.004	±.000	±.004	±.010	±.007	±.008	±.009	±.008	±.009
0	1	0	1	No	1	0.1	0	.549	.996	.927	.357	.570	.000	.570	.490	2.073	1.042	.388	2.106	1.008
								±.007	±.015	±.010	±.010	±.007	±.000	±.007	±.018	±.063	±.019	±.016	±.041	±.017

Appendix B

0	0.1	0	1	Yes	1	0.01	0	.703 ±.018	.111 ±.004	1.124 ±.010	.423 ±.012	.701 ±.022	.000 ±.000	.701 ±.022	.037 ±.001	1.604 ±.036	.112 ±.003	.037 ±.001	1.602 ±.068	.112 ±.005
0	0.1	0	1	Yes	1	0.05	0	.696 ±.057	.142 ±.028	1.118 ±.030	.428 ±.042	.690 ±.071	.000 ±.000	.690 ±.071	.047 ±.008	1.670 ±.126	.139 ±.022	.048 ±.011	1.589 ±.226	.134 ±.029
0	0.1	0	1	Yes	1	0.1	0	.645 ±.087	.225 ±.109	1.076 ±.088	.449 ±.094	.627 ±.114	.000 ±.000	.627 ±.114	.068 ±.019	1.575 ±.182	.185 ±.043	.071 ±.018	1.546 ±.548	.169 ±.054
0	0.25	0	1	Yes	1	0.01	0	.646 ±.009	.250 ±.005	1.100 ±.005	.455 ±.007	.644 ±.012	.000 ±.000	.644 ±.012	.098 ±.001	1.469 ±.020	.249 ±.004	.097 ±.001	1.485 ±.034	.253 ±.006
0	0.25	0	1	Yes	1	0.05	0	.613 ±.074	.251 ±.032	1.082 ±.036	.479 ±.058	.603 ±.094	.000 ±.000	.603 ±.094	.108 ±.009	1.426 ±.114	.257 ±.019	.108 ±.016	1.416 ±.229	.247 ±.036
0	0.25	0	1	Yes	1	0.1	0	.558 ±.181	.280 ±.065	1.054 ±.081	.528 ±.151	.527 ±.232	.000 ±.000	.527 ±.232	.130 ±.028	1.457 ±.278	.294 ±.032	.148 ±.059	1.326 ±.479	.256 ±.076
0	0.5	0	1	Yes	1	0.01	0	.531 ±.015	.424 ±.012	1.056 ±.007	.531 ±.012	.526 ±.019	.000 ±.000	.526 ±.019	.223 ±.003	1.292 ±.023	.427 ±.008	.222 ±.005	1.300 ±.040	.429 ±.013
0	0.5	0	1	Yes	1	0.05	0	.533 ±.049	.438 ±.038	1.056 ±.021	.528 ±.038	.529 ±.059	.000 ±.000	.529 ±.059	.228 ±.012	1.306 ±.096	.438 ±.032	.225 ±.017	1.318 ±.138	.437 ±.039
0	0.5	0	1	Yes	1	0.1	0	.488 ±.161	.464 ±.142	1.006 ±.150	.536 ±.165	.470 ±.202	.000 ±.000	.470 ±.202	.241 ±.043	1.318 ±.260	.444 ±.071	.243 ±.101	1.436 ±.689	.413 ±.102
0	1	0	1	Yes	1	0.01	0	.234 ±.017	.632 ±.014	.973 ±.007	.756 ±.013	.218 ±.020	.000 ±.000	.218 ±.020	.592 ±.007	1.072 ±.019	.643 ±.011	.599 ±.011	1.057 ±.028	.636 ±.015
0	1	0	1	Yes	1	0.05	0	.245 ±.090	.657 ±.072	.973 ±.036	.739 ±.075	.233 ±.110	.000 ±.000	.233 ±.110	.591 ±.035	1.090 ±.095	.658 ±.053	.586 ±.068	1.101 ±.166	.663 ±.083
0	1	0	1	Yes	1	0.1	0	.205 ±.207	.669 ±.128	.933 ±.093	.756 ±.172	.177 ±.260	.000 ±.000	.177 ±.260	.601 ±.080	1.105 ±.200	.677 ±.098	.618 ±.156	1.101 ±.347	.671 ±.135
0	0	0.25	0	No	1	0.01	0	-1.904 ±10.201	.091 ±.162	.968 ±.107	3.742 ±13.844	-2.774 ±13.942	.000 ±.000	-2.774 ±13.942	.038 ±.013	1.041 ±.031	.058 ±.024	7.687 ±33.457	.943 ±.188	.087 ±.152
0	0	0.25	0	No	1	0.1	0	.442 ±.043	.215 ±.121	1.027 ±.026	.588 ±.032	.439 ±.051	.000 ±.000	.439 ±.051	.097 ±.040	1.025 ±.077	.132 ±.053	.132 ±.073	1.124 ±.117	.222 ±.148
0	0	0.25	0	Yes	1	0.01	0	.747 ±.024	.357 ±.067	1.099 ±.016	.314 ±.025	.785 ±.016	.000 ±.000	.785 ±.016	.228 ±.023	1.971 ±.163	.595 ±.038	.109 ±.022	2.188 ±.174	.325 ±.055
0	0	0.25	0	Yes	1	0.1	0	.516 ±.072	.340 ±.108	1.066 ±.035	.528 ±.055	.539 ±.088	.000 ±.000	.539 ±.088	.150 ±.034	1.120 ±.143	.242 ±.098	.179 ±.038	1.365 ±.227	.347 ±.106
0	0	0.5	0	No	1	0.01	0	###	.478 ±.241	.895 ±.175	###	###	.000 ±.000	###	4.267 ±13.800	.991 ±.143	.388 ±.279	###	1.045 ±.253	.455 ±.247
0	0	0.5	0	No	1	0.1	0	-2.236 ±6.851	.395 ±.164	.980 ±.132	4.177 ±9.240	-3.197 ±9.355	.000 ±.000	-3.197 ±9.355	4.075 ±17.199	.984 ±.070	.237 ±.137	14.745 ±35.893	1.118 ±.334	.399 ±.167
0	0	0.5	0	Yes	1	0.01	0	.696 ±.036	.497 ±.056	1.008 ±.035	.287 ±.038	.722 ±.034	.000 ±.000	.722 ±.034	.289 ±.069	2.110 ±.219	.689 ±.090	.175 ±.022	2.292 ±.216	.500 ±.067
0	0	0.5	0	Yes	1	0.1	0	.594 ±.034	.527 ±.081	1.042 ±.027	.434 ±.031	.607 ±.038	.000 ±.000	.607 ±.038	.301 ±.039	1.620 ±.200	.617 ±.114	.301 ±.037	1.597 ±.144	.478 ±.089
0	0	1	0	No	1	0.01	0	###	.429 ±.249	.760 ±.330	###	###	.000 ±.000	###	###	1.077 ±.261	.701 ±.164	###	.879 ±.426	.344 ±.273
0	0	1	0	No	1	0.1	0	###	.510 ±.169	.724 ±.292	###	###	.000 ±.000	###	###	1.001 ±.307	.627 ±.145	###	.846 ±.425	.372 ±.249
0	0	1	0	Yes	1	0.01	0	.484 ±.037	.823 ±.046	.939 ±.049	.439 ±.080	.499 ±.039	.000 ±.000	.499 ±.039	.580 ±.057	1.774 ±.207	.880 ±.057	.517 ±.087	1.863 ±.214	.832 ±.046
0	0	1	0	Yes	1	0.1	0	.444 ±.060	.703 ±.112	.883 ±.042	.440 ±.092	.443 ±.072	.000 ±.000	.443 ±.072	.553 ±.091	1.854 ±.290	.824 ±.104	.579 ±.123	1.611 ±.288	.651 ±.122
0	0	0.1	0	No	0	0.01	0	###	.540 ±.272	.692 ±.396	###	###	.000 ±.000	###	###	1.554 ±.507	.639 ±.291	###	.873 ±.758	.387 ±.316
0	0	0.1	0	No	0	0.1	0	.406 ±.621	.389 ±.178	1.027 ±.168	.706 ±.683	.321 ±.844	.000 ±.000	.321 ±.844	.184 ±.224	1.484 ±.375	.315 ±.118	.451 ±.946	1.305 ±.485	.349 ±.130
0	0	0.1	0	Yes	0	0.01	0	.829 ±.001	.130 ±.009	1.140 ±.003	.302 ±.004	.838 ±.001	.000 ±.000	.838 ±.001	.073 ±.002	2.190 ±.020	.247 ±.006	.038 ±.003	2.280 ±.027	.131 ±.011
0	0	0.1	0	Yes	0	0.1	0	.692 ±.071	.268 ±.082	1.119 ±.031	.427 ±.054	.692 ±.085	.000 ±.000	.692 ±.085	.088 ±.024	1.608 ±.247	.239 ±.061	.088 ±.035	1.658 ±.263	.239 ±.086
0	0	0.2	0	No	0	0.01	0	###	.459 ±.343	.520 ±.433	###	###	.000 ±.000	###	###	1.280 ±.467	.861 ±.188	###	.631 ±.699	.305 ±.320
0	0	0.2	0	No	0	0.1	0	-5.089 ±17.524	.597 ±.207	.872 ±.257	7.589 ±23.826	-6.717 ±23.963	.000 ±.000	-6.717 ±23.963	3.906 ±9.562	1.454 ±.550	.626 ±.211	14.361 ±41.494	1.106 ±.590	.478 ±.215
0	0	0.2	0	Yes	0	0.01	0	.814 ±.003	.248 ±.011	1.127 ±.006	.303 ±.008	.824 ±.003	.000 ±.000	.824 ±.003	.104 ±.004	2.176 ±.043	.338 ±.014	.073 ±.002	2.270 ±.048	.248 ±.008
0	0	0.2	0	Yes	0	0.1	0	.786 ±.012	.318 ±.030	1.136 ±.006	.346 ±.013	.790 ±.011	.000 ±.000	.790 ±.011	.111 ±.017	2.005 ±.091	.351 ±.047	.097 ±.015	2.035 ±.079	.306 ±.042
0	0	0.5	0	No	0	0.01	0	###	.672 ±.247	.480 ±.308	###	###	.000 ±.000	###	###	1.525 ±.300	.962 ±.211	###	.509 ±.469	.406 ±.349
0	0	0.5	0	No	0	0.1	0	###	.622 ±.323	.497 ±.328	###	###	.000 ±.000	###	###	1.268 ±.405	.933 ±.125	###	.635 ±.628	.366 ±.348
0	0	0.5	0	Yes	0	0.01	0	.738 ±.001	.523 ±.010	1.003 ±.006	.255 ±.005	.749 ±.001	.000 ±.000	.749 ±.001	.215 ±.001	2.437 ±.017	.646 ±.003	.156 ±.003	2.520 ±.031	.533 ±.011
0	0	0.5	0	Yes	0	0.1	0	.717 ±.017	.583 ±.028	1.056 ±.014	.333 ±.028	.723 ±.023	.000 ±.000	.723 ±.023	.234 ±.016	2.110 ±.105	.655 ±.025	.209 ±.033	2.110 ±.138	.578 ±.036
0	0	0.25	0	No	1	0.05	0	.420 ±.044	.106 ±.098	1.006 ±.020	.603 ±.040	.403 ±.058	.000 ±.000	.403 ±.058	.057 ±.033	1.045 ±.057	.079 ±.032	.119 ±.252	1.025 ±.063	.111 ±.108

Appendix B

0	0	0.25	0	Yes	1	0.05	0	.580 ±.080	.424 ±.096	1.079 ±.033	.479 ±.059	.601 ±.090	.000 ±.000	.601 ±.090	.176 ±.045	1.365 ±.327	.373 ±.169	.184 ±.036	1.495 ±.253	.401 ±.096
0	0	0.5	0	No	1	0.05	0	###	.362 ±.219	.949 ±.159	###	###	.000 ±.000	###	1.493 ±6.067	.979 ±.069	.187 ±.181	###	1.070 ±.315	.376 ±.223
0	0	0.5	0	Yes	1	0.05	0	.644 ±.057	.517 ±.090	1.029 ±.028	.375 ±.061	.654 ±.067	.000 ±.000	.654 ±.067	.288 ±.042	1.874 ±.222	.659 ±.061	.246 ±.057	1.818 ±.290	.474 ±.103
0	0	1	0	No	1	0.05	0	###	.523 ±.140	.828 ±.208	###	###	.000 ±.000	###	###	1.075 ±.267	.636 ±.170	###	1.005 ±.394	.380 ±.216
0	0	1	0	Yes	1	0.05	0	.481 ±.044	.722 ±.126	.887 ±.072	.397 ±.093	.490 ±.052	.000 ±.000	.490 ±.052	.555 ±.073	1.831 ±.212	.855 ±.072	.461 ±.122	1.825 ±.345	.690 ±.157
0	0	0.1	0	No	0	0.05	0	-31.915 ±101.552	.465 ±.243	.748 ±.385	44.239 ±13[illegible].826	-43.491 ±138.873	.000 ±.000	-43.491 ±138.873	14.352 ±35.830	1.393 ±.611	.537 ±.223	93.335 ±380.293	.890 ±.748	.302 ±.224
0	0	0.1	0	Yes	0	0.05	0	.813 ±.008	.169 ±.028	1.155 ±.004	.334 ±.010	.821 ±.008	.000 ±.000	.821 ±.008	.070 ±.010	2.049 ±.077	.232 ±.034	.049 ±.008	2.089 ±.065	.163 ±.027
0	0	0.2	0	No	0	0.05	0	###	.594 ±.277	.836 ±.307	###	###	.000 ±.000	###	###	1.725 ±.444	.634 ±.253	###	1.137 ±.658	.499 ±.293
0	0	0.2	0	Yes	0	0.05	0	.806 ±.006	.274 ±.024	1.136 ±.005	.322 ±.011	.813 ±.008	.000 ±.000	.813 ±.008	.104 ±.006	2.135 ±.061	.337 ±.018	.082 ±.008	2.139 ±.080	.270 ±.027
0	0	0.5	0	No	0	0.05	0	###	.715 ±.268	.516 ±.298	###	###	.000 ±.000	###	###	1.375 ±.496	.958 ±.196	###	.741 ±.622	.563 ±.346
0	0	0.5	0	Yes	0	0.05	0	.727 ±.009	.581 ±.031	1.042 ±.022	.307 ±.031	.735 ±.012	.000 ±.000	.735 ±.012	.224 ±.010	2.270 ±.149	.650 ±.020	.197 ±.023	2.231 ±.155	.587 ±.033

Clearing-price complex condition, optimal model

Sy	Collude	Est	Ns	Crash	S(Y) t>10	S(P) t>10	Rev. t>10	P.C. t>10	G.P. t>10	In.C. t>10	Prof. t>10	S(Y) t≤20	P t≤20	S(P) t≤20	S(Y) t>20	P t>20	S(P) t>20
0	No	No	10	0	.005 ±.001	.006 ±.001	1.000 ±.000	.600 ±.001	.400 ±.000	.000 ±.000	.400 ±.000	.005 ±.001	1.079 ±.000	.197 ±.001	.005 ±.001	1.000 ±.001	.006 ±.001
0	No	Yes	10	0	.017 ±.002	.031 ±.003	.998 ±.000	.590 ±.001	.408 ±.001	.000 ±.000	.408 ±.001	.121 ±.001	1.17[illegible] ±.002	.174 ±.003	.005 ±.001	1.001 ±.001	.006 ±.001
0	Yes	No	10	0	.005 ±.001	.022 ±.003	.823 ±.001	.245 ±.001	.577 ±.000	.000 ±.000	.577 ±.000	.005 ±.001	1.892 ±.003	.296 ±.002	.005 ±.001	2.011 ±.005	.023 ±.004
0	Yes	Yes	10	1	.041 ±.002	.154 ±.004	.840 ±.001	.278 ±.002	.562 ±.001	.000 ±.000	.562 ±.001	.067 ±.001	1.511 ±.006	.153 ±.005	.014 ±.002	1.921 ±.007	.074 ±.005
0.1	No	No	10	0	.026 ±.003	.031 ±.005	1.000 ±.002	.599 ±.003	.401 ±.002	.000 ±.000	.401 ±.002	.024 ±.004	1.081 ±.002	.196 ±.003	.025 ±.004	1.002 ±.006	.030 ±.006
0.1	No	Yes	10	0	.030 ±.005	.048 ±.007	.998 ±.002	.590 ±.004	.408 ±.002	.000 ±.000	.406 ±.002	.125 ±.007	1.178 ±.009	.180 ±.016	.025 ±.005	1.002 ±.004	.032 ±.006
0.1	Yes	No	10	0	.025 ±.004	.114 ±.020	.820 ±.003	.245 ±.003	.575 ±.001	.000 ±.000	.575 ±.001	.023 ±.004	1.894 ±.017	.312 ±.009	.026 ±.005	2.010 ±.021	.119 ±.024
0.1	Yes	Yes	10	0	.051 ±.008	.191 ±.017	.842 ±.005	.285 ±.008	.557 ±.003	.000 ±.000	.557 ±.003	.071 ±.006	1.508 ±.021	.177 ±.025	.034 ±.007	1.882 ±.051	.148 ±.027
0.1	No	No	10	0	.049 ±.006	.062 ±.008	1.000 ±.003	.599 ±.005	.401 ±.003	.000 ±.000	.401 ±.003	.052 ±.007	1.083 ±.006	.206 ±.008	.049 ±.006	1.003 ±.007	.060 ±.008
0.1	No	Yes	10	2	.052 ±.005	.073 ±.010	1.001 ±.003	.595 ±.006	.406 ±.004	.000 ±.000	.406 ±.004	.141 ±.016	1.174 ±.020	.207 ±.029	.050 ±.007	1.003 ±.006	.062 ±.009
0.1	Yes	Yes	10	2	.073 ±.025	.263 ±.045	.846 ±.015	.297 ±.025	.548 ±.011	.000 ±.000	.548 ±.011	.098 ±.025	1.507 ±.068	.237 ±.052	.062 ±.025	1.808 ±.144	.245 ±.067
0.1	Yes	No	10	0	.048 ±.005	.215 ±.029	.813 ±.005	.245 ±.004	.568 ±.003	.000 ±.000	.568 ±.003	.048 ±.009	1.897 ±.031	.355 ±.025	.048 ±.006	2.009 ±.032	.221 ±.032

Clearing-price complex condition, behavioral model

a1	a2	a3	a0	Discrete	b	Sy	Crash	S(Y) t>10	S(P) t>10	Rev. t>10	P.C. t>10	G.P. t>10	In.C. t>10	Prof. t>10	S(Y) t≤20	P t≤20	S(P) t≤20	S(Y) t>20	P t>20	S(P) t>20
0	0	0.1	0	No	0	0.01	2	###	1.068 ±.439	21.248 ±52.226	###	###	.000 ±.000	###	###	1.639 ±.309	.775 ±.430	###	1.790 ±.529	1.096 ±.451
0	0	0.1	0	No	0	0.05	0	###	.655 ±.375	1.?07 ±.656	###	###	.000 ±.000	###	1.899 ±6.133	1.401 ±.114	.394 ±.247	###	1.560 ±.331	.699 ±.415
0	0	0.1	0	No	0	0.1	0	.173 ±.121	.340 ±.214	.922 ±.031	.443 ±.047	.480 ±.030	.000 ±.000	.480 ±.030	.090 ±.043	1.313 ±.055	.187 ±.110	.190 ±.147	1.360 ±.149	.374 ±.251
0	0	0.1	0	Yes	0	0.01	0	.129 ±.015	.417 ±.033	.851 ±.005	.335 ±.012	.516 ±.007	.000 ±.000	.516 ±.007	.082 ±.006	1.340 ±.017	.203 ±.020	.106 ±.020	1.733 ±.050	.433 ±.055
0	0	0.1	0	Yes	0	0.05	0	.097 ±.025	.290 ±.101	.881 ±.025	.370 ±.034	.511 ±.012	.000 ±.000	.511 ±.012	.068 ±.015	1.330 ±.029	.160 ±.045	.091 ±.025	1.541 ±.153	.310 ±.110
0	0	0.1	0	Yes	0	0.1	0	.094 ±.017	.211 ±.071	.925 ±.029	.440 ±.050	.485 ±.022	.000 ±.000	.485 ±.022	.070 ±.013	1.287 ±.042	.142 ±.041	.098 ±.023	1.312 ±.146	.224 ±.082
0	0	0.2	0	No	0	0.01	4	###	1.300 ±.415	###	###	###	.000 ±.000	###	###	1.911 ±.334	1.150 ±.361	###	2.020 ±.587	1.252 ±.471
0	0	0.2	0	No	0	0.05	1	###	.817 ±.447	3.795 ±6.224	###	###	.000 ±.000	###	###	1.480 ±.217	.631 ±.330	###	1.530 ±.475	.810 ±.487
0	0	0.2	0	No	0	0.1	0	25.265 ±77.721	.678 ±.320	1.282 ±.651	3.730 ±10.092	-2.448 ±9.838	.000 ±.000	-2.448 ±9.838	1.082 ±3.401	1.330 ±.128	.375 ±.218	30.108 ±93.481	1.411 ±.318	.736 ±.368
0	0	0.2	0	Yes	0	0.01	0	.199 ±.032	.688 ±.072	.814 ±.007	.330 ±.022	.484 ±.017	.000 ±.000	.484 ±.017	.150 ±.013	1.355 ±.028	.384 ±.033	.197 ±.043	1.844 ±.074	.741 ±.094
0	0	0.2	0	Yes	0	0.05	0	.181 ±.030	.618 ±.119	.822 ±.035	.326 ±.034	.496 ±.020	.000 ±.000	.496 ±.020	.152 ±.020	1.359 ±.036	.377 ±.050	.168 ±.039	1.850 ±.149	.642 ±.145
0	0	0.2	0	Yes	0	0.1	0	.168 ±.032	.445 ±.214	.899 ±.079	.416 ±.087	.483 ±.022	.000 ±.000	.483 ±.022	.124 ±.027	1.305 ±.076	.274 ±.097	.172 ±.040	1.496 ±.302	.475 ±.243
0	0	0.25	0	No	1	0.01	0	.033 ±.007	.039 ±.012	.966 ±.003	.532 ±.005	.434 ±.002	.000 ±.000	.434 ±.002	.029 ±.005	1.179 ±.005	.109 ±.004	.027 ±.012	1.077 ±.007	.035 ±.020
0	0	0.25	0	No	1	0.05	0	.059 ±.020	.085 ±.033	.970 ±.007	.535 ±.013	.435 ±.007	.000 ±.000	.435 ±.007	.047 ±.012	1.181 ±.013	.123 ±.013	.059 ±.026	1.081 ±.023	.086 ±.042
0	0	0.25	0	No	1	0.1	0	.108 ±.037	.155 ±.064	.979 ±.014	.544 ±.024	.435 ±.011	.000 ±.000	.435 ±.011	.073 ±.025	1.180 ±.023	.144 ±.022	.111 ±.049	1.090 ±.048	.167 ±.084
0	0	0.25	0	Yes	1	0.01	0	.220 ±.052	.287 ±.106	1.005 ±.021	.592 ±.031	.413 ±.014	.000 ±.000	.413 ±.014	.166 ±.018	1.115 ±.030	.237 ±.018	.223 ±.065	1.098 ±.081	.301 ±.139
0	0	0.25	0	Yes	1	0.05	0	.226 ±.049	.267 ±.084	1.035 ±.039	.630 ±.049	.406 ±.015	.000 ±.000	.406 ±.015	.154 ±.024	1.108 ±.021	.225 ±.018	.238 ±.058	1.049 ±.074	.293 ±.107
0	0	0.25	0	Yes	1	0.1	0	.228 ±.071	.276 ±.111	1.035 ±.042	.625 ±.050	.410 ±.015	.000 ±.000	.410 ±.015	.168 ±.040	1.108 ±.026	.235 ±.034	.239 ±.081	1.069 ±.104	.298 ±.133
0	0	0.5	0	No	0	0.01	18	###	1.234 ±.372	###	###	###	.000 ±.000	###	###	2.122 ±.178	1.374 ±.111	###	1.576 ±1.238	1.054 ±.701
0	0	0.5	0	No	0	0.05	7	###	1.041 ±.281	841.887 ±1613.026	###	###	.000 ±.000	###	###	1.624 ±.222	.973 ±.274	###	1.334 ±.504	.844 ±.415
0	0	0.5	0	No	0	0.1	4	###	1.003 ±.361	290.210 ±1035.321	###	###	.000 ±.000	###	###	1.543 ±.248	.950 ±.268	###	1.298 ±.466	.855 ±.498
0	0	0.5	0	Yes	1	0.01	0	.577 ±.257	.500 ±.097	1.160 ±.145	.780 ±.220	.380 ±.086	.000 ±.000	.380 ±.086	.424 ±.167	1.201 ±.073	.407 ±.067	.598 ±.297	1.060 ±.196	.499 ±.139
0	0	0.5	0	Yes	1	0.05	0	.664 ±.439	.540 ±.165	1.173 ±.242	.762 ±.290	.411 ±.079	.000 ±.000	.411 ±.079	.367 ±.131	1.219 ±.061	.427 ±.058	.680 ±.426	1.136 ±.186	.545 ±.227
0	0	0.5	0	Yes	1	0.1	0	.444 ±.090	.496 ±.126	1.107 ±.075	.691 ±.092	.416 ±.042	.000 ±.000	.416 ±.042	.352 ±.095	1.181 ±.096	.416 ±.084	.439 ±.103	1.114 ±.144	.480 ±.165
0	0	1	0	No	1	0.01	0	###	.310 ±.314	1.167 ±.402	###	###	.000 ±.000	###	.198 ±.091	1.010 ±.008	.342 ±.013	###	1.118 ±.220	.351 ±.371
0	0	1	0	No	1	0.05	0	###	.322 ±.407	###	###	###	.000 ±.000	###	2.958 ±12.493	1.034 ±.087	.379 ±.174	###	1.112 ±.354	.344 ±.418
0	0	1	0	No	1	0.1	0	51.279 ±166.867	.490 ±.425	3.016 ±6.377	11.087 ±36.440	-8.071 ±30.074	.000 ±.000	-8.071 ±30.074	40.644 ±180.953	1.031 ±.075	.379 ±.132	39.951 ±109.445	1.200 ±.354	.561 ±.481
0	0	1	0	Yes	1	0.01	0	2.733 ±2.902	.911 ±.206	1.547 ±.764	1.491 ±1.245	.056 ±.535	.000 ±.000	.056 ±.535	1.357 ±1.068	1.291 ±.101	.758 ±.113	2.986 ±3.425	1.309 ±.362	.922 ±.275
0	0	1	0	Yes	1	0.05	0	5.969 ±18.243	.914 ±.140	2.509 ±5.546	2.728 ±7.380	-.220 ±1.845	.000 ±.000	-.220 ±1.845	1.001 ±.572	1.263 ±.063	.741 ±.083	6.429 ±19.278	1.344 ±.246	.945 ±.190

0	0	1	0	Yes	1	0.1	0	1.999 ±1.992	.900 ±.158	1.467 ±.936	1.278 ±1.234	.190 ±.331	.000 ±.000	.190 ±.331	1.160 ±.965	1.283 ±.061	.772 ±.084	2.087 ±2.197	1.332 ±.266	.899 ±.217
0	0.1	0	1	No	1	0.01	0	###	1.114 ±.497	12.452 ±43.906	###	###	.000 ±.000	###	###	1.828 ±.423	.972 ±.489	###	2.111 ±.468	1.077 ±.506
0	0.1	0	1	No	1	0.05	0	.084 ±.011	.306 ±.025	.826 ±.007	.280 ±.011	.547 ±.005	.000 ±.000	.547 ±.005	.095 ±.008	1.415 ±.031	.176 ±.031	.045 ±.014	1.995 ±.065	.211 ±.049
0	0.1	0	1	No	1	0.1	0	.084 ±.013	.306 ±.046	.828 ±.012	.283 ±.014	.545 ±.006	.000 ±.000	.545 ±.006	.095 ±.010	1.416 ±.032	.189 ±.037	.047 ±.014	1.969 ±.079	.226 ±.060
0	0.1	0	1	Yes	1	0.01	0	.117 ±.014	.371 ±.027	.840 ±.010	.311 ±.019	.528 ±.010	.000 ±.000	.528 ±.010	.094 ±.009	1.357 ±.020	.203 ±.026	.077 ±.014	1.847 ±.098	.337 ±.050
0	0.1	0	1	Yes	1	0.05	0	.101 ±.013	.345 ±.037	.828 ±.008	.289 ±.015	.539 ±.008	.000 ±.000	.539 ±.008	.092 ±.007	1.390 ±.025	.177 ±.030	.064 ±.016	1.953 ±.079	.286 ±.066
0	0.1	0	1	Yes	1	0.1	0	.104 ±.016	.355 ±.040	.827 ±.011	.291 ±.019	.537 ±.009	.000 ±.000	.537 ±.009	.092 ±.010	1.389 ±.046	.176 ±.039	.072 ±.017	1.944 ±.094	.309 ±.067
0	0.25	0	1	No	1	0.01	0	###	1.147 ±.460	868.599 ±2573.545	###	###	.000 ±.000	###	###	2.166 ±.256	1.247 ±.309	###	2.496 ±.302	1.054 ±.503
0	0.25	0	1	No	1	0.05	0	###	.783 ±.441	351.248 ±1565.734	###	###	.000 ±.000	###	###	1.658 ±.310	.759 ±.360	###	2.177 ±.257	.675 ±.484
0	0.25	0	1	No	1	0.1	0	.088 ±.035	.398 ±.096	.776 ±.016	.236 ±.015	.540 ±.014	.000 ±.000	.540 ±.014	.188 ±.030	1.521 ±.090	.412 ±.107	.047 ±.021	2.250 ±.047	.254 ±.118
0	0.25	0	1	Yes	1	0.01	0	.210 ±.067	.688 ±.116	.779 ±.024	.296 ±.051	.483 ±.033	.000 ±.000	.483 ±.033	.233 ±.034	1.398 ±.071	.497 ±.055	.154 ±.062	2.061 ±.189	.658 ±.164
0	0.25	0	1	Yes	1	0.05	0	.147 ±.034	.577 ±.082	.769 ±.014	.258 ±.021	.511 ±.019	.000 ±.000	.511 ±.019	.220 ±.015	1.435 ±.051	.459 ±.030	.100 ±.024	2.202 ±.089	.500 ±.098
0	0.25	0	1	Yes	1	0.1	0	.132 ±.051	.531 ±.110	.769 ±.017	.251 ±.023	.518 ±.021	.000 ±.000	.518 ±.021	.214 ±.022	1.471 ±.055	.435 ±.064	.104 ±.021	2.208 ±.092	.475 ±.105
0	0.5	0	1	No	1	0.01	0	.114 ±.049	.531 ±.140	.715 ±.029	.205 ±.008	.509 ±.030	.000 ±.000	.509 ±.030	.308 ±.007	1.833 ±.037	.753 ±.038	.111 ±.061	2.290 ±.053	.515 ±.188
0	0.5	0	1	No	1	0.05	0	###	.814 ±.473	###	###	###	.000 ±.000	###	###	1.879 ±.192	.940 ±.298	###	2.258 ±.216	.791 ±.493
0	0.5	0	1	No	1	0.1	0	.497 ±1.264	.741 ±.421	.703 ±.057	.381 ±.520	.322 ±.563	.000 ±.000	.322 ±.563	.928 ±2.059	1.716 ±.171	.832 ±.266	.253 ±.483	2.211 ±.115	.691 ±.425
0	0.5	0	1	Yes	1	0.01	0	.431 ±.150	1.079 ±.154	.708 ±.059	.364 ±.098	.344 ±.066	.000 ±.000	.344 ±.066	.509 ±.100	1.464 ±.075	.828 ±.074	.375 ±.157	2.074 ±.283	1.091 ±.207
0	0.5	0	1	Yes	1	0.05	0	.318 ±.123	.933 ±.122	.711 ±.042	.302 ±.064	.409 ±.049	.000 ±.000	.409 ±.049	.451 ±.111	1.518 ±.095	.782 ±.080	.277 ±.079	2.144 ±.210	.924 ±.144
0	0.5	0	1	Yes	1	0.1	0	.264 ±.172	.840 ±.203	.698 ±.038	.267 ±.088	.431 ±.081	.000 ±.000	.431 ±.081	.423 ±.134	1.557 ±.110	.730 ±.103	.233 ±.083	2.274 ±.147	.838 ±.208
0	1	0	1	No	1	0.01	0	###	1.623 ±.232	###	###	###	.000 ±.000	###	###	1.843 ±.135	1.340 ±.205	###	2.225 ±.472	1.668 ±.212
0	1	0	1	No	1	0.05	0	###	1.245 ±.244	51.799 ±228.871	832.119 ±3719.592	-780.320 ±3490.721	.000 ±.000	-780.320 ±3490.721	###	1.728 ±.089	1.106 ±.161	708.723 ±3167.411	2.124 ±.163	1.279 ±.257
0	1	0	1	No	1	0.1	0	2.661 ±9.793	1.090 ±.196	.754 ±.366	.901 ±2.630	-.148 ±2.269	.000 ±.000	-.148 ±2.269	.867 ±.092	1.687 ±.072	1.002 ±.092	2.950 ±11.426	2.059 ±.126	1.114 ±.270
0	1	0	1	Yes	1	0.01	0	1.013 ±.699	1.338 ±.188	.634 ±.104	.543 ±.296	.091 ±.227	.000 ±.000	.091 ±.227	1.425 ±.609	1.713 ±.169	1.183 ±.101	.814 ±.483	2.098 ±.335	1.356 ±.201
0	1	0	1	Yes	1	0.05	0	.924 ±.497	1.345 ±.163	.632 ±.106	.546 ±.283	.085 ±.234	.000 ±.000	.085 ±.234	1.312 ±.203	1.707 ±.152	1.189 ±.083	.934 ±.582	2.093 ±.349	1.349 ±.190
0	1	0	1	Yes	1	0.1	0	1.197 ±1.094	1.343 ±.180	.680 ±.124	.647 ±.475	.033 ±.368	.000 ±.000	.033 ±.368	1.363 ±.634	1.633 ±.123	1.145 ±.105	1.117 ±1.142	2.054 ±.312	1.379 ±.182
0.25	0	0	1	No	1	0.01	0	.025 ±.002	.027 ±.003	.969 ±.002	.538 ±.004	.431 ±.002	.000 ±.000	.431 ±.002	.028 ±.003	1.171 ±.005	.113 ±.004	.015 ±.003	1.069 ±.004	.018 ±.004
0.25	0	0	1	No	1	0.05	0	.045 ±.010	.059 ±.013	.972 ±.005	.542 ±.010	.430 ±.004	.000 ±.000	.430 ±.004	.040 ±.008	1.173 ±.010	.119 ±.008	.041 ±.010	1.065 ±.015	.057 ±.015
0.25	0	0	1	No	1	0.1	0	.077 ±.021	.111 ±.027	.976 ±.015	.546 ±.025	.429 ±.011	.000 ±.000	.429 ±.011	.067 ±.018	1.172 ±.022	.145 ±.020	.076 ±.021	1.070 ±.036	.113 ±.029
0.25	0	0	1	Yes	1	0.01	0	.131 ±.009	.164 ±.008	.998 ±.007	.586 ±.011	.412 ±.005	.000 ±.000	.412 ±.005	.124 ±.005	1.101 ±.004	.211 ±.002	.131 ±.010	1.052 ±.017	.165 ±.010
0.25	0	0	1	Yes	1	0.05	0	.150 ±.015	.168 ±.011	1.007 ±.011	.608 ±.021	.399 ±.013	.000 ±.000	.399 ±.013	.139 ±.020	1.104 ±.013	.214 ±.010	.150 ±.019	1.014 ±.030	.167 ±.015
0.25	0	0	1	Yes	1	0.1	0	.156 ±.019	.162 ±.014	1.021 ±.016	.631 ±.026	.389 ±.013	.000 ±.000	.389 ±.013	.147 ±.022	1.103 ±.022	.216 ±.011	.155 ±.021	.983 ±.025	.159 ±.016
0.5	0	0	1	No	1	0.01	0	.015 ±.005	.022 ±.005	1.005 ±.002	.611 ±.005	.394 ±.003	.000 ±.000	.394 ±.003	.037 ±.010	1.065 ±.006	.207 ±.004	.008 ±.003	.991 ±.004	.011 ±.003

0.5	0	0	1	No	1	0.05	0	.035 ±.006	.044 ±.008	1.006 ±.006	.612 ±.011	.394 ±.005	.000 ±.000	.394 ±.005	.044 ±.008	1.064 ±.008	.210 ±.006	.033 ±.006	.992 ±.013	.041 ±.008
0.5	0	0	1	No	1	0.1	0	.061 ±.013	.073 ±.016	1.016 ±.014	.627 ±.024	.389 ±.011	.000 ±.000	.389 ±.011	.067 ±.017	1.063 ±.018	.217 ±.017	.062 ±.015	.978 ±.026	.073 ±.019
0.5	0	0	1	Yes	1	0.01	0	.355 ±.062	.335 ±.025	1.058 ±.048	.655 ±.075	.402 ±.031	.000 ±.000	.402 ±.031	.266 ±.070	1.192 ±.060	.364 ±.014	.361 ±.060	1.029 ±.080	.312 ±.039
0.5	0	0	1	Yes	1	0.05	0	.343 ±.049	.360 ±.046	1.055 ±.041	.651 ±.061	.404 ±.024	.000 ±.000	.404 ±.024	.267 ±.060	1.182 ±.049	.361 ±.021	.342 ±.056	1.042 ±.087	.349 ±.065
0.5	0	0	1	Yes	1	0.1	0	.336 ±.046	.344 ±.044	1.061 ±.036	.678 ±.062	.383 ±.031	.000 ±.000	.383 ±.031	.307 ±.050	1.119 ±.065	.360 ±.027	.343 ±.051	1.032 ±.072	.340 ±.053
1	0	0	1	No	1	0.01	0	.175 ±.034	.116 ±.009	1.051 ±.013	.734 ±.030	.318 ±.017	.000 ±.000	.318 ±.017	.529 ±.056	.928 ±.015	.450 ±.009	.040 ±.012	.949 ±.010	.041 ±.006
1	0	0	1	No	1	0.05	0	.180 ±.036	.119 ±.012	1.055 ±.014	.741 ±.033	.314 ±.019	.000 ±.000	.314 ±.019	.532 ±.063	.927 ±.016	.450 ±.013	.054 ±.015	.944 ±.015	.054 ±.008
1	0	0	1	No	1	0.1	0	.194 ±.045	.126 ±.014	1.065 ±.025	.755 ±.052	.309 ±.027	.000 ±.000	.309 ±.027	.524 ±.079	.927 ±.023	.452 ±.020	.077 ±.015	.934 ±.027	.074 ±.013
1	0	0	1	Yes	1	0.01	0	.920 ±.259	.637 ±.045	1.202 ±.099	.887 ±.145	.315 ±.067	.000 ±.000	.315 ±.067	.698 ±.214	1.239 ±.055	.654 ±.031	.978 ±.299	1.073 ±.078	.623 ±.068
1	0	0	1	Yes	1	0.05	0	.906 ±.225	.642 ±.047	1.197 ±.093	.890 ±.156	.307 ±.074	.000 ±.000	.307 ±.074	.716 ±.210	1.228 ±.064	.657 ±.027	.924 ±.270	1.079 ±.095	.632 ±.059
1	0	0	1	Yes	1	0.1	0	.901 ±.252	.646 ±.038	1.201 ±.065	.894 ±.108	.307 ±.058	.000 ±.000	.307 ±.058	.708 ±.242	1.219 ±.070	.662 ±.031	.909 ±.265	1.081 ±.067	.636 ±.046
2	0	0	1	No	1	0.01	0	12.61 8 ±4.55 9	.370 ±.005	2.155 ±.561	10.19 3 ±3.10 5	- 8.038 ±2.55 2	.000 ±.000	- 8.038 ±2.55 2	103.6 96 ±16.2 14	.984 ±.014	1.420 ±.015	.830 ±.283	.724 ±.036	.277 ±.010
2	0	0	1	No	1	0.05	0	14.21 8 ±7.20 1	.367 ±.010	2.378 ±1.00 4	11.23 7 ±4.68 3	- 8.860 ±3.70 0	.000 ±.000	- 8.860 ±3.70 0	108.5 23 ±23.6 54	.987 ±.021	1.423 ±.018	.948 ±.464	.710 ±.052	.280 ±.015
2	0	0	1	No	1	0.1	0	###	.437 ±.319	###	###	###	.000 ±.000	###	###	1.079 ±.383	1.435 ±.062	###	.728 ±.131	.350 ±.277
2	0	0	1	Yes	1	0.01	0	10.66 8 ±3.36 2	1.069 ±.065	3.178 ±.844	4.054 ±1.33 3	-.876 ±.846	.000 ±.000	-.876 ±.846	4.461 ±4.31 0	1.457 ±.060	1.176 ±.045	11.10 2 ±4.34 9	1.177 ±.132	1.024 ±.066
2	0	0	1	Yes	1	0.05	0	14.20 9 ±13.6 66	1.026 ±.119	4.321 ±3.23 7	5.545 ±4.75 3	- 1.225 ±1.63 2	.000 ±.000	- 1.225 ±1.63 2	11.84 6 ±17.8 05	1.427 ±.078	1.145 ±.074	10.28 0 ±6.18 6	1.183 ±.139	.989 ±.126
2	0	0	1	Yes	1	0.1	0	55.98 7 ±136. 480	.996 ±.132	11.84 6 ±24.9 26	17.46 5 ±43.1 84	- 5.619 ±18.8 13	.000 ±.000	- 5.619 ±18.8 13	14.65 4 ±19.3 56	1.406 ±.082	1.117 ±.080	62.23 3 ±162. 430	1.142 ±.207	.953 ±.171

Posted-price simple condition, optimal model

σy	σp	M	Est	Ns	Crash	S(Y) t>10	S(P) t>10	Rev. t>10	P.C. t>10	G.P. t>10	In.C. t>10	Prof. t>10	S(Y) t≤20	P t≤20	S(P) t≤20	S(Y) t>20	P t>20	S(P) t>20
0	0	No	Yes	10	0	.004	.005	1.000	.600	.400	.005	.395	.129	1.107	.262	.004	1.000	.005
						±.001	±.001	±.000	±.001	±.001	±.000	±.001	±.001	±.001	±.002	±.001	±.001	±.001
0	0	Yes	Yes	10	0	.003	.012	1.142	.298	.844	.003	.841	.062	2.212	.214	.003	2.299	.012
						±.001	±.002	±.000	±.000	±.000	±.000	±.000	±.000	±.002	±.001	±.001	±.003	±.002
0	0.1	No	Yes	10	0	.038	.050	1.001	.605	.396	.054	.342	.135	1.097	.260	.038	.993	.049
						±.005	±.007	±.003	±.005	±.007	±.003	±.008	±.008	±.012	±.016	±.007	±.013	±.009
0	0.1	Yes	Yes	10	0	.035	.118	1.142	.305	.837	.027	.810	.067	2.170	.233	.035	2.249	.114
						±.004	±.015	±.005	±.005	±.001	±.002	±.002	±.007	±.026	±.025	±.005	±.035	±.016
0	0	No	No	10	0	.008	.005	1.000	.600	.400	.006	.394	.008	1.000	.005	.008	1.000	.005
						±.001	±.001	±.000	±.001	±.001	±.000	±.001	±.001	±.001	±.001	±.002	±.001	±.001
0	0	No	Yes	10	0	.008	.005	1.000	.600	.400	.006	.394	.129	1.108	.263	.008	1.000	.005
						±.001	±.001	±.000	±.000	±.001	±.000	±.001	±.001	±.001	±.002	±.002	±.001	±.001
0	0	Yes	No	10	0	.008	.012	1.142	.298	.844	.004	.840	.007	2.299	.011	.008	2.300	.012
						±.001	±.001	±.000	±.000	±.000	±.000	±.000	±.002	±.003	±.002	±.001	±.004	±.001
0	0	Yes	Yes	10	0	.008	.011	1.142	.298	.844	.004	.840	.063	2.213	.214	.008	2.299	.011
						±.001	±.001	±.000	±.000	±.000	±.000	±.000	±.001	±.002	±.002	±.001	±.003	±.002
0	0.1	No	No	10	0	.020	.025	.999	.603	.397	.027	.370	.019	.996	.024	.021	.996	.026
						±.002	±.003	±.001	±.002	±.003	±.002	±.004	±.003	±.006	±.004	±.003	±.005	±.004
0	0.1	No	Yes	10	0	.020	.024	1.001	.600	.400	.027	.374	.130	1.106	.262	.019	1.000	.024
						±.002	±.002	±.001	±.002	±.003	±.002	±.004	±.003	±.006	±.010	±.003	±.004	±.003
0	0.1	Yes	No	10	0	.018	.057	1.141	.299	.842	.014	.829	.017	2.295	.055	.017	2.295	.057
						±.002	±.008	±.002	±.002	±.000	±.001	±.001	±.003	±.010	±.010	±.002	±.015	±.008
0	0.1	Yes	Yes	10	0	.018	.060	1.143	.300	.842	.014	.828	.063	2.194	.213	.019	2.287	.062
						±.002	±.007	±.002	±.002	±.000	±.001	±.001	±.002	±.014	±.009	±.003	±.011	±.008
0.1	0	No	No	10	0	.033	.005	1.000	.600	.400	.013	.387	.034	1.000	.005	.033	1.000	.005
						±.005	±.001	±.000	±.000	±.001	±.001	±.001	±.008	±.001	±.001	±.006	±.001	±.001
0.1	0	No	Yes	10	0	.035	.005	1.000	.600	.400	.013	.387	.134	1.108	.263	.035	1.000	.005
						±.006	±.001	±.000	±.000	±.000	±.001	±.001	±.003	±.002	±.002	±.007	±.001	±.001
0.1	0	Yes	No	10	0	.037	.011	1.142	.298	.844	.012	.832	.039	2.299	.011	.035	2.299	.011
						±.007	±.002	±.000	±.000	±.000	±.001	±.001	±.008	±.003	±.002	±.007	±.003	±.002
0.1	0	Yes	Yes	10	0	.036	.012	1.142	.298	.844	.012	.832	.072	2.211	.213	.037	2.299	.012
						±.005	±.001	±.000	±.000	±.000	±.001	±.001	±.004	±.003	±.002	±.005	±.002	±.002
0.1	0.1	No	No	10	0	.040	.025	1.000	.602	.398	.029	.369	.039	.997	.026	.041	.997	.025
						±.005	±.003	±.001	±.002	±.003	±.002	±.005	±.007	±.006	±.004	±.008	±.006	±.004
0.1	0.1	No	Yes	10	0	.038	.024	1.001	.600	.401	.030	.370	.134	1.109	.264	.039	1.000	.023
						±.004	±.002	±.001	±.002	±.002	±.002	±.003	±.004	±.008	±.012	±.008	±.006	±.003
0.1	0.1	Yes	No	10	0	.040	.058	1.142	.300	.842	.018	.824	.038	2.289	.058	.041	2.289	.058
						±.005	±.006	±.002	±.002	±.000	±.001	±.002	±.007	±.013	±.012	±.006	±.011	±.007
0.1	0.1	Yes	Yes	10	0	.039	.058	1.142	.300	.842	.018	.825	.072	2.202	.216	.038	2.286	.058
						±.007	±.007	±.002	±.002	±.000	±.001	±.002	±.006	±.013	±.011	±.008	±.014	±.009
0.1	0	No	No	10	0	.068	.005	1.000	.600	.400	.025	.375	.067	1.000	.005	.071	1.000	.005
						±.008	±.001	±.000	±.000	±.001	±.002	±.002	±.016	±.001	±.001	±.010	±.001	±.001
0.1	0	No	Yes	10	0	.069	.005	1.000	.600	.400	.025	.375	.144	1.107	.262	.072	1.000	.005
						±.013	±.000	±.000	±.000	±.001	±.002	±.002	±.007	±.001	±.002	±.017	±.001	±.001
0.1	0	Yes	Yes	10	0	.075	.012	1.142	.298	.844	.024	.819	.098	2.212	.214	.074	2.299	.012
						±.011	±.001	±.000	±.000	±.000	±.002	±.002	±.010	±.003	±.001	±.012	±.003	±.002
0.1	0.1	No	No	10	0	.075	.024	.999	.603	.397	.036	.361	.073	.999	.024	.076	.994	.024
						±.010	±.003	±.001	±.002	±.002	±.003	±.003	±.015	±.004	±.004	±.012	±.006	±.003
0.1	0.1	Yes	Yes	10	0	.074	.058	1.142	.300	.842	.028	.814	.095	2.204	.214	.075	2.287	.058
						±.011	±.007	±.002	±.002	±.000	±.003	±.003	±.012	±.010	±.008	±.012	±.012	±.009
0.1	0.1	No	Yes	10	0	.083	.053	1.001	.606	.395	.059	.336	.150	1.098	.257	.080	.994	.054
						±.012	±.006	±.002	±.004	±.006	±.006	±.009	±.013	±.014	±.022	±.016	±.011	±.008
0.1	0.1	Yes	Yes	10	0	.081	.113	1.141	.303	.837	.036	.802	.098	2.171	.237	.081	2.268	.110
						±.014	±.017	±.004	±.004	±.001	±.002	±.003	±.013	±.022	±.023	±.015	±.027	±.021
0.1	0	Yes	No	10	0	.070	.011	1.142	.298	.844	.024	.820	.075	2.300	.011	.067	2.300	.012
						±.009	±.001	±.000	±.001	±.000	±.001	±.001	±.014	±.003	±.001	±.012	±.003	±.002
0.1	0.1	No	Yes	10	0	.069	.027	1.000	.602	.398	.036	.362	.148	1.102	.263	.070	.999	.027
						±.012	±.004	±.002	±.003	±.004	±.003	±.004	±.010	±.008	±.013	±.015	±.005	±.005
0.1	0.1	Yes	No	10	0	.067	.058	1.141	.299	.842	.027	.815	.070	2.291	.054	.066	2.292	.060
						±.012	±.010	±.002	±.002	±.000	±.002	±.002	±.014	±.011	±.011	±.013	±.014	±.012
0.1	0.1	No	No	10	0	.079	.051	.999	.608	.391	.057	.334	.079	.994	.049	.078	.985	.052
						±.013	±.008	±.002	±.004	±.005	±.004	±.007	±.014	±.011	±.010	±.016	±.008	±.010
0.1	0.1	Yes	No	10	0	.076	.116	1.139	.302	.837	.035	.802	.075	2.266	.111	.078	2.265	.116
						±.010	±.014	±.004	±.004	±.001	±.002	±.003	±.010	±.026	±.013	±.014	±.029	±.017

Posted-price simple condition, behavioral model

τ	σy	bo	b1	b2	σp	Discrete	a1	Crash	S[Y] t>10	S[P] t>10	Rev. t>10	P.C. t>10	G.P. t>10	In.C. t>10	Prof. t>10	S[Y] t≤20	P t≤20	S[P] t≤20	S[Y] t>20	P t>20	S[P] t>20
1	0	1	0.25	0	0.01	No	0.5	0	.007 ±.002	.013 ±.004	1.000 ±.001	.600 ±.001	.400 ±.002	.006 ±.000	.394 ±.002	.123 ±.003	-.105 ±.011	.146 ±.015	.006 ±.002	.001 ±.005	.011 ±.003
1	0	1	0.25	0	0.05	No	0.5	0	.034 ±.009	.057 ±.021	.998 ±.002	.603 ±.004	.394 ±.006	.027 ±.003	.367 ±.007	.129 ±.010	-.094 ±.019	.159 ±.027	.032 ±.007	.017 ±.022	.052 ±.013
1	0	1	0.25	0	0.1	No	0.5	0	.073 ±.012	.131 ±.027	.993 ±.007	.619 ±.012	.374 ±.019	.056 ±.004	.319 ±.021	.151 ±.017	-.038 ±.054	.206 ±.044	.070 ±.016	.061 ±.046	.123 ±.043
1	0	1	0.25	0	0.01	Yes	0.5	0	.253 ±.052	.682 ±.425	.979 ±.014	.665 ±.030	.314 ±.044	.103 ±.006	.211 ±.048	.278 ±.049	.220 ±.186	.652 ±.333	.241 ±.060	.279 ±.278	.598 ±.443
1	0	1	0.25	0	0.05	Yes	0.5	0	.305 ±.121	.908 ±.555	.943 ±.140	.723 ±.180	.220 ±.319	.120 ±.010	.100 ±.317	.292 ±.093	.245 ±.233	.688 ±.392	.274 ±.091	.438 ±.382	.892 ±.736
1	0	1	0.25	0	0.1	Yes	0.5	0	.273 ±.139	.673 ±.329	.955 ±.132	.706 ±.170	.248 ±.302	.137 ±.010	.111 ±.307	.283 ±.069	.236 ±.203	.646 ±.448	.232 ±.052	.312 ±.227	.624 ±.381
1	0	1	0.5	0	0.01	Yes	0.5	0	.544 ±.291	3.149 ±3.314	.584 ±.417	1.187 ±.447	-.604 ±.864	.153 ±.084	-.756 ±.781	.712 ±.157	1.268 ±1.505	3.115 ±2.850	.390 ±.332	.965 ±1.828	2.721 ±3.495
1	0	1	0.5	0	0.05	Yes	0.5	0	.656 ±.360	4.836 ±6.765	.580 ±.397	1.269 ±.473	-.689 ±.868	.293 ±.108	-.982 ±.909	.844 ±.174	3.495 ±6.326	5.344 ±5.868	.536 ±.418	5.486 ±14.680	2.851 ±4.220
1	0	1	0.5	0	0.1	Yes	0.5	0	.335 ±.340	1.753 ±2.893	.191 ±.340	1.821 ±.458	-1.630 ±.792	.526 ±.238	-2.156 ±.948	1.000 ±.182	4.692 ±6.615	4.469 ±4.850	.155 ±.224	9.136 ±11.835	.299 ±.780
1	0	1	1	0	0.01	No	0.5	4	1.995 ±1.166	19.791 ±30.974	.361 ±.342	3.073 ±1.350	-2.712 ±1.688	1.999 ±1.177	-4.711 ±2.860	1.934 ±1.468	3.753 ±7.136	17.478 ±26.947	.968 ±1.304	-.052 ±8.342	8.119 ±29.516
1	0	1	1	0	0.05	No	0.5	1	.989 ±1.075	5.630 ±9.873	.583 ±.450	2.030 ±1.540	-1.448 ±1.974	1.196 ±1.257	-2.643 ±3.226	1.248 ±1.363	1.457 ±3.676	6.515 ±11.656	.543 ±1.045	-.051 ±2.124	1.837 ±5.317
1	0	1	1	0	0.1	No	0.5	0	.322 ±.528	.882 ±1.879	.837 ±.363	1.142 ±1.149	-.306 ±1.502	.507 ±.917	-.813 ±2.410	.595 ±.965	.241 ±.331	.643 ±.764	.260 ±.548	.376 ±.768	.942 ±2.316
1	0	1	1	0	0.01	Yes	0.5	0	.108 ±.211	.000 ±.000	.008 ±.025	2.079 ±.490	-2.071 ±.497	.460 ±.695	-2.531 ±1.189	1.247 ±.385	20.268 ±62.744	25.631 ±57.120	.003 ±.005	33.152 ±116.945	.000 ±.000
1	0	1	1	0	0.05	Yes	0.5	1	.103 ±.172	1.257 ±5.478	.003 ±.011	2.489 ±.444	-2.486 ±.444	1.063 ±.626	-3.549 ±1.070	1.455 ±.272	20.939 ±26.574	18.357 ±17.766	.025 ±.020	29.538 ±40.420	.000 ±.000
1	0	1	1	0	0.1	Yes	0.5	0	.111 ±.076	.000 ±.000	.000 ±.000	2.860 ±.615	-2.860 ±.615	1.571 ±.836	-4.432 ±1.451	1.547 ±.397	13.030 ±17.268	7.750 ±9.866	.102 ±.076	17.984 ±23.393	.000 ±.000
1	0	1	0	0.1	0.01	No	0.5	0	.014 ±.004	.013 ±.002	1.148 ±.002	.306 ±.002	.843 ±.001	.004 ±.000	.838 ±.001	.055 ±.003	-.417 ±.017	.056 ±.050	.006 ±.002	-.422 ±.004	.012 ±.002
1	0	1	0	0.1	0.05	No	0.5	0	.036 ±.008	.064 ±.024	1.152 ±.009	.317 ±.012	.836 ±.004	.015 ±.001	.820 ±.005	.055 ±.010	-.407 ±.029	.077 ±.038	.032 ±.007	-.431 ±.023	.057 ±.010
1	0	1	0	0.1	0.1	No	0.5	0	.062 ±.016	.125 ±.022	1.154 ±.007	.347 ±.022	.807 ±.020	.032 ±.004	.775 ±.024	.057 ±.024	-.415 ±.032	.120 ±.028	.056 ±.016	-.410 ±.032	.129 ±.027
1	0	1	0	0.1	0.01	Yes	0.5	0	.067 ±.004	.023 ±.003	1.132 ±.007	.297 ±.007	.835 ±.001	.017 ±.001	.818 ±.002	.093 ±.009	-.406 ±.020	.050 ±.047	.067 ±.004	-.416 ±.007	.022 ±.004
1	0	1	0	0.1	0.05	Yes	0.5	0	.076 ±.005	.062 ±.008	1.132 ±.005	.300 ±.004	.833 ±.002	.023 ±.001	.810 ±.003	.097 ±.009	-.423 ±.026	.069 ±.021	.076 ±.006	-.419 ±.012	.061 ±.007
1	0	1	0	0.1	0.1	Yes	0.5	0	.088 ±.009	.123 ±.016	1.134 ±.006	.307 ±.007	.827 ±.003	.034 ±.002	.793 ±.004	.103 ±.009	-.410 ±.036	.131 ±.032	.087 ±.011	-.407 ±.023	.124 ±.024
1	0	1	0	0.2	0.01	No	0.5	0	.006 ±.001	.012 ±.003	1.143 ±.002	.299 ±.002	.844 ±.000	.003 ±.000	.840 ±.000	.063 ±.006	-.420 ±.011	.034 ±.037	.006 ±.002	-.422 ±.003	.012 ±.003
1	0	1	0	0.2	0.05	No	0.5	0	.032 ±.006	.061 ±.010	1.144 ±.008	.304 ±.009	.840 ±.001	.014 ±.001	.826 ±.002	.069 ±.010	-.418 ±.018	.082 ±.038	.030 ±.007	-.431 ±.023	.059 ±.009
1	0	1	0	0.2	0.1	No	0.5	0	.058 ±.014	.121 ±.021	1.151 ±.011	.327 ±.018	.824 ±.010	.031 ±.003	.793 ±.012	.077 ±.012	-.418 ±.042	.126 ±.025	.058 ±.016	-.414 ±.033	.119 ±.028
1	0	1	0	0.2	0.01	Yes	0.5	0	.136 ±.015	.039 ±.012	1.114 ±.024	.309 ±.017	.805 ±.007	.033 ±.003	.772 ±.010	.135 ±.019	-.401 ±.021	.055 ±.037	.136 ±.016	-.407 ±.013	.038 ±.014
1	0	1	0	0.2	0.05	Yes	0.5	0	.146 ±.009	.074 ±.010	1.105 ±.007	.299 ±.007	.805 ±.003	.039 ±.002	.767 ±.005	.154 ±.012	-.399 ±.019	.078 ±.019	.146 ±.012	-.406 ±.020	.074 ±.015
1	0	1	0	0.2	0.1	Yes	0.5	0	.155 ±.012	.123 ±.021	1.101 ±.010	.302 ±.007	.799 ±.005	.046 ±.003	.753 ±.007	.153 ±.013	-.408 ±.037	.126 ±.031	.155 ±.015	-.395 ±.038	.123 ±.030
1	0	1	0	0.5	0.01	No	0.5	0	.006 ±.001	.012 ±.002	1.142 ±.001	.298 ±.001	.844 ±.000	.003 ±.000	.841 ±.000	.068 ±.006	-.412 ±.014	.039 ±.042	.006 ±.001	-.421 ±.003	.012 ±.002
1	0	1	0	0.5	0.05	No	0.5	0	.030 ±.006	.063 ±.011	1.142 ±.003	.300 ±.003	.841 ±.001	.015 ±.001	.827 ±.002	.072 ±.008	-.413 ±.021	.077 ±.037	.030 ±.007	-.424 ±.025	.064 ±.016

Appendix B

1	0	1	0	0.5	0.1	No	0.5	0	.061 ±.012	.119 ±.019	1.141 ±.006	.307 ±.007	.834 ±.003	.030 ±.002	.804 ±.004	.090 ±.014	-.409 ±.031	.124 ±.026	.060 ±.014	-.411 ±.030	.118 ±.024
1	0	1	0	0.5	0.01	Yes	0.5	20	###	###	###	###	###	###	###	###	###	###	###	###	###
1	0	1	0	0.5	0.05	Yes	0.5	20	###	###	###	###	###	###	###	###	###	###	###	###	###
1	0	1	0	0.5	0.1	Yes	0.5	20	###	###	###	###	###	###	###	.319 ±.011	-.359 ±.018	.163 ±.011	###	###	###
1	0	0	0.25	0	0.01	No	0.5	0	.015 ±.021	.040 ±.070	.997 ±.013	.625 ±.017	.372 ±.026	.105 ±.048	.267 ±.051	.141 ±.012	-.097 ±.033	.159 ±.073	.013 ±.022	-.003 ±.010	.026 ±.030
1	0	0	0.25	0	0.05	No	0.5	0	.071 ±.039	.189 ±.193	1.024 ±.025	.634 ±.040	.390 ±.044	.207 ±.093	.183 ±.110	.162 ±.040	-.089 ±.050	.249 ±.186	.066 ±.035	.003 ±.057	.158 ±.152
1	0	0	0.25	0	0.1	No	0.5	1	.146 ±.060	.321 ±.255	1.052 ±.021	.640 ±.028	.412 ±.020	.281 ±.083	.131 ±.090	.178 ±.030	-.083 ±.108	.247 ±.151	.154 ±.080	.015 ±.082	.356 ±.313
1	0	0	0.25	0	0.01	Yes	0.5	0	.049 ±.012	.312 ±.273	1.052 ±.021	.647 ±.036	.405 ±.036	.297 ±.091	.107 ±.108	.162 ±.017	.004 ±.068	.344 ±.200	.043 ±.010	.027 ±.119	.306 ±.286
1	0	0	0.25	0	0.05	Yes	0.5	0	.147 ±[illegible]	.229 ±.275	1.062 ±.015	.698 ±.064	.365 ±.052	.371 ±.100	-.006 ±.149	.196 ±.027	.042 ±.071	.269 ±.155	.132 ±.046	.038 ±.060	.220 ±.317
1	0	0	0.25	0	0.1	Yes	0.5	0	.379 ±.548	.384 ±.471	1.042 ±.050	.809 ±.294	.233 ±.336	.438 ±.234	-.205 ±.560	.219 ±.057	.078 ±.087	.396 ±.388	.395 ±.589	.074 ±.082	.320 ±.480
1	0	0	0.5	0	0.01	No	0.5	4	.258 ±.801	.086 ±.203	.939 ±.261	.917 ±1.002	.022 ±1.262	.384 ±.802	-.362 ±2.060	.369 ±.888	-.017 ±.102	.288 ±.393	.235 ±.913	.026 ±.088	.084 ±.240
1	0	0	0.5	0	0.05	No	0.5	2	.274 ±.823	.259 ±.461	.946 ±.266	.919 ±.983	.027 ±1.248	.434 ±.793	-.407 ±2.038	.331 ±.733	-.015 ±.139	.396 ±.703	.261 ±.861	.046 ±.131	.258 ±.540
1	0	0	0.5	0	0.1	No	0.5	1	1.066 ±1.428	.403 ±.370	.847 ±.326	1.370 ±1.209	-.524 ±1.533	.793 ±.895	-1.317 ±2.427	.554 ±.897	-.099 ±.258	.517 ±.447	.651 ±1.171	-.196 ±.873	.271 ±.292
1	0	0	0.5	0	0.01	Yes	0.5	0	.131 ±.090	.440 ±.417	1.029 ±.037	.684 ±.080	.345 ±.096	.268 ±.106	.078 ±.166	.249 ±.077	.075 ±.044	.562 ±.546	.112 ±.086	-.004 ±.269	.438 ±.419
1	0	0	0.5	0	0.05	Yes	0.5	0	1.053 ±1.196	3.256 ±4.890	.918 ±.251	1.404 ±.978	-.486 ±1.229	.911 ±.700	-1.397 ±1.927	.410 ±.150	.708 ±1.947	1.890 ±3.788	.750 ±.967	.639 ±2.186	3.421 ±5.204
1	0	0	0.5	0	0.1	Yes	0.5	0	1.816 ±1.158	1.201 ±2.243	.503 ±.374	2.798 ±1.198	-2.295 ±1.558	1.890 ±.909	-4.185 ±2.459	1.229 ±1.068	.918 ±1.737	2.793 ±4.554	1.248 ±1.240	-.557 ±1.919	.300 ±.895
1	0	0	1	0	0.01	No	0.5	7	.062 ±.172	.021 ±.016	.788 ±.449	1.494 ±1.734	-.706 ±2.183	.873 ±1.427	-1.579 ±3.610	.868 ±1.395	-.235 ±.927	.325 ±.351	.004 ±.002	-.262 ±.960	.015 ±.008
1	0	0	1	0	0.05	No	0.5	5	.889 ±1.312	.103 ±.166	.284 ±.424	3.277 ±1.555	-2.993 ±1.962	2.243 ±1.317	-5.236 ±3.265	1.979 ±1.558	-.043 ±.766	1.145 ±1.269	.771 ±1.506	-.336 ±1.141	.106 ±.196
1	0	0	1	0	0.1	No	0.5	1	1.322 ±1.206	.263 ±.521	.101 ±.167	4.184 ±.640	-4.083 ±.807	2.994 ±.557	-7.077 ±1.362	2.964 ±1.179	-.249 ±.884	.783 ±.607	.340 ±.925	-.759 ±1.532	.068 ±.226
1	0	0	1	0	0.01	Yes	0.5	0	.854 ±1.034	5.909 ±24.624	.586 ±.467	2.175 ±1.627	-1.589 ±2.065	1.417 ±1.243	-3.006 ±3.276	1.166 ±1.195	3.207 ±8.415	12.001 ±27.906	.512 ±.817	1.841 ±9.864	6.996 ±29.893
1	0	0	1	0	0.05	Yes	0.5	0	.460 ±.717	.025 ±.059	.024 ±.052	4.199 ±.667	-4.175 ±.671	2.975 ±.782	-7.150 ±1.451	2.739 ±.675	1.217 ±5.093	4.584 ±13.004	.031 ±.063	-.474 ±.897	.000 ±.000
1	0	0	1	0	0.1	Yes	0.5	0	.541 ±.866	.056 ±.208	.038 ±.126	3.638 ±.818	-3.600 ±.859	2.383 ±.905	-5.983 ±1.749	2.091 ±.859	-.181 ±.538	.262 ±.231	.317 ±.674	-.255 ±.564	.049 ±.220
1	0	1	0.5	0	0.01	No	0.5	0	.016 ±.039	.021 ±.039	1.000 ±.001	.601 ±.004	.399 ±.005	.008 ±.010	.391 ±.015	.136 ±.069	-.041 ±.039	.155 ±.142	.008 ±.005	.002 ±.005	.013 ±.009
1	0	1	0.5	0	0.05	No	0.5	0	.033 ±.007	.059 ±.012	1.000 ±.002	.603 ±.005	.397 ±.007	.029 ±.003	.368 ±.008	.128 ±.008	-.034 ±.020	.133 ±.036	.033 ±.008	.010 ±.019	.059 ±.015
1	0	1	0.5	0	0.1	No	0.5	0	.068 ±.018	.114 ±.037	1.000 ±.005	.607 ±.010	.393 ±.015	.060 ±.006	.333 ±.018	.140 ±.015	-.015 ±.029	.168 ±.042	.070 ±.021	.035 ±.039	.114 ±.047
1	0.05	1	0.5	0	0.01	No	0.5	0	.036 ±.007	.011 ±.003	1.000 ±.000	.600 ±.000	.400 ±.001	.014 ±.001	.386 ±.001	.126 ±.005	-.052 ±.014	.129 ±.030	.035 ±.009	.000 ±.003	.011 ±.003
1	0.05	1	0.5	0	0.05	No	0.5	0	.047 ±.007	.059 ±.012	.999 ±.002	.604 ±.004	.395 ±.006	.032 ±.002	.363 ±.007	.130 ±.007	-.039 ±.017	.143 ±.026	.047 ±.008	.009 ±.016	.054 ±.013
1	0.05	1	0.5	0	0.1	No	0.5	0	.076 ±.013	.127 ±.032	.998 ±.003	.612 ±.006	.385 ±.009	.061 ±.004	.324 ±.010	.149 ±.015	-.004 ±.037	.189 ±.057	.074 ±.013	.029 ±.034	.128 ±.032
1	0.1	1	0.5	0	0.01	No	0.5	0	.076 ±.011	.012 ±.003	1.000 ±.000	.600 ±.001	.400 ±.001	.025 ±.002	.376 ±.002	.139 ±.008	-.052 ±.020	.134 ±.034	.079 ±.014	.000 ±.003	.012 ±.003
1	0.1	1	0.5	0	0.05	No	0.5	0	.081 ±.009	.058 ±.008	1.000 ±.002	.603 ±.004	.397 ±.006	.039 ±.003	.358 ±.006	.150 ±.007	-.045 ±.027	.151 ±.038	.081 ±.011	.009 ±.017	.056 ±.011

1	0.1	1	0.5	0	0.1	No	0.5	0	.101	.128	.997	.613	.384	.066	.318	.158	-.008	.183	.102	.051	.126
									±.011	±.036	±.003	±.006	±.010	±.007	±.013	±.019	±.038	±.041	±.017	±.045	±.051
2	0	1	0.5	0	0.01	No	0.5	0	.008	.011	1.000	.601	.399	.007	.392	.134	-.055	.142	.006	.002	.010
									±.004	±.003	±.000	±.001	±.001	±.001	±.002	±.004	±.017	±.036	±.002	±.004	±.002
2	0	1	0.5	0	0.05	No	0.5	0	.030	.058	.999	.604	.396	.032	.364	.139	-.044	.146	.030	.014	.057
									±.006	±.012	±.002	±.003	±.005	±.002	±.004	±.007	±.019	±.034	±.007	±.015	±.015
2	0	1	0.5	0	0.1	No	0.5	0	.066	.129	.999	.609	.390	.065	.325	.153	-.007	.174	.066	.039	.132
									±.014	±.031	±.004	±.008	±.012	±.005	±.015	±.022	±.036	±.043	±.015	±.030	±.042
2	0.05	1	0.5	0	0.01	No	0.5	0	.030	.012	1.000	.600	.400	.015	.384	.136	-.051	.126	.029	.001	.011
									±.003	±.003	±.000	±.001	±.001	±.001	±.001	±.004	±.016	±.033	±.005	±.003	±.003
2	0.05	1	0.5	0	0.05	No	0.5	0	.042	.060	1.000	.603	.397	.035	.362	.144	-.036	.141	.039	.010	.059
									±.007	±.014	±.002	±.004	±.006	±.003	±.006	±.008	±.016	±.035	±.006	±.012	±.011
2	0.05	1	0.5	0	0.1	No	0.5	0	.064	.119	.999	.610	.389	.065	.324	.152	-.026	.188	.063	.042	.125
									±.015	±.028	±.005	±.008	±.013	±.007	±.014	±.018	±.038	±.050	±.016	±.037	±.034
2	0.1	1	0.5	0	0.01	No	0.5	0	.059	.013	1.000	.600	.400	.029	.371	.142	-.048	.129	.061	.000	.014
									±.010	±.004	±.000	±.001	±.001	±.003	±.004	±.008	±.014	±.027	±.013	±.003	±.005
2	0.1	1	0.5	0	0.05	No	0.5	0	.068	.058	.999	.603	.396	.043	.353	.151	-.030	.148	.069	.007	.055
									±.010	±.016	±.002	±.004	±.005	±.004	±.007	±.009	±.026	±.040	±.010	±.014	±.014
2	0.1	1	0.5	0	0.1	No	0.5	0	.080	.126	.997	.614	.382	.073	.310	.167	-.011	.185	.081	.053	.124
									±.012	±.032	±.003	±.006	±.008	±.008	±.012	±.019	±.045	±.066	±.013	±.033	±.026
1	0.1	1	0	0.2	0.01	No	0.5	0	.069	.013	1.143	.299	.844	.023	.820	.093	-.419	.033	.068	-.421	.012
									±.012	±.002	±.001	±.001	±.000	±.001	±.001	±.014	±.016	±.042	±.014	±.004	±.003
1	0.1	1	0	0.2	0.1	No	0.5	0	.100	.305	1.146	.320	.827	.039	.788	.107	-.427	.289	.095	-.419	.277
									±.014	±.226	±.010	±.014	±.007	±.003	±.008	±.022	±.087	±.231	±.016	±.064	±.217

Note: a_2 and σ_x are zero and $d=\sigma_p/2$ in all simulations.

Posted-price complex condition, optimal model

σy	σp	a	M	Est	Ns	Crash	S(Y) t>10	S(P) t>10	Rev. t>10	P.C. t>10	G.P. t>10	In.C. t>10	Prof. t>10	S(Y) t≤20	P t≤20	S(P) t≤20	S(Y) t>20	P t>20	S(P) t>20
0	0	1	No	No	10	0	.005	.010	1.000	.600	.400	.005	.395	.005	1.080	.197	.005	1.000	.010
							±.001	±.001	±.000	±.001	±.000	±.000	±.000	±.001	±.001	±.001	±.001	±.001	±.001
0	0	1	No	Yes	10	0	.009	.037	.999	.600	.400	.006	.394	.122	1.154	.199	.005	1.000	.010
							±.001	±.005	±.001	±.002	±.001	±.000	±.002	±.003	±.002	±.005	±.001	±.002	±.002
0	0	1	Yes	No	10	0	.005	.023	.823	.246	.577	.001	.576	.005	1.891	.295	.005	2.011	.023
							±.001	±.004	±.001	±.001	±.000	±.000	±.000	±.001	±.004	±.002	±.001	±.003	±.005
0	0	1	Yes	Yes	10	0	.040	.147	.840	.276	.563	.001	.562	.064	1.527	.156	.011	1.929	.068
							±.001	±.003	±.001	±.001	±.000	±.000	±.000	±.002	±.003	±.004	±.001	±.008	±.007
0	0.1	1	No	No	10	0	.005	.040	1.002	.600	.402	.026	.375	.005	1.082	.199	.005	1.003	.041
							±.001	±.006	±.001	±.001	±.001	±.002	±.001	±.001	±.002	±.005	±.001	±.002	±.009
0	0.1	1	No	Yes	10	3	.016	.060	.999	.594	.405	.026	.379	.120	1.166	.201	.009	1.005	.039
							±.008	±.013	±.002	±.007	±.005	±.003	±.007	±.012	±.026	±.028	±.005	±.003	±.007
0	0.1	1	Yes	No	10	0	.005	.045	.823	.246	.577	.005	.572	.005	1.890	.297	.005	2.011	.045
							±.001	±.006	±.000	±.000	±.000	±.000	±.000	±.001	±.004	±.006	±.001	±.004	±.010
0	0.1	1	Yes	Yes	10	5	.044	.160	.842	.281	.561	.007	.554	.072	1.518	.171	.015	1.908	.085
							±.005	±.014	±.004	±.008	±.004	±.001	±.005	±.025	±.032	±.027	±.005	±.039	±.026
0.1	0	1	No	No	10	0	.025	.032	1.000	.600	.400	.005	.395	.025	1.081	.199	.024	1.000	.032
							±.003	±.005	±.002	±.004	±.002	±.000	±.002	±.004	±.003	±.004	±.003	±.004	±.005
0.1	0	1	No	Yes	10	3	.036	.068	1.003	.606	.397	.008	.390	.139	1.155	.224	.035	.998	.046
							±.026	±.048	±.007	±.013	±.007	±.004	±.010	±.037	±.006	±.050	±.029	±.007	±.045
0.1	0	1	Yes	No	10	0	.025	.116	.820	.245	.575	.001	.574	.025	1.898	.314	.026	2.009	.119
							±.002	±.009	±.003	±.003	±.001	±.000	±.001	±.003	±.017	±.011	±.004	±.020	±.016
0.1	0	1	Yes	Yes	10	1	.058	.202	.842	.284	.559	.002	.556	.071	1.520	.174	.040	1.899	.161
							±.027	±.071	±.005	±.012	±.006	±.002	±.009	±.023	±.017	±.018	±.038	±.039	±.100
0.1	0.1	1	No	No	10	0	.026	.047	1.002	.600	.402	.025	.376	.025	1.083	.202	.025	1.003	.046
							±.004	±.006	±.002	±.002	±.002	±.002	±.002	±.004	±.003	±.007	±.004	±.003	±.008
0.1	0.1	1	No	Yes	10	3	.046	.079	1.006	.608	.399	.028	.370	.139	1.156	.222	.044	1.001	.068
							±.042	±.041	±.016	±.030	±.014	±.005	±.019	±.046	±.008	±.057	±.043	±.017	±.036
0.1	0.1	1	Yes	No	10	0	.025	.117	.821	.246	.575	.006	.569	.024	1.893	.314	.024	2.007	.116
							±.003	±.014	±.003	±.003	±.001	±.000	±.001	±.003	±.018	±.013	±.004	±.020	±.019
0.1	0.1	1	Yes	Yes	10	4	.050	.184	.843	.282	.560	.007	.553	.113	1.540	.195	.032	1.886	.149
							±.008	±.024	±.006	±.009	±.004	±.001	±.005	±.180	±.021	±.030	±.010	±.055	±.030
0.1	0	1	No	No	10	0	.050	.063	1.001	.600	.401	.006	.396	.048	1.083	.202	.051	1.005	.065
							±.007	±.008	±.003	±.005	±.003	±.000	±.003	±.010	±.007	±.009	±.007	±.008	±.011
0.1	0	1	No	Yes	10	8	.064	.103	1.006	.608	.398	.008	.390	.163	1.161	.243	.058	1.000	.077
							±.021	±.040	±.009	±.020	±.011	±.003	±.014	±.050	±.015	±.060	±.012	±.012	±.016
0.1	0	1	Yes	No	10	0	.051	.231	.812	.246	.566	.002	.564	.049	1.895	.359	.052	2.010	.239
							±.006	±.027	±.008	±.006	±.003	±.000	±.003	±.007	±.027	±.022	±.009	±.043	±.034
0.1	0	1	Yes	Yes	10	0	.068	.252	.844	.290	.554	.004	.550	.079	1.530	.217	.060	1.854	.236
							±.013	±.047	±.014	±.017	±.005	±.002	±.006	±.010	±.034	±.045	±.019	±.101	±.061
0.1	0.1	1	No	No	10	0	.049	.074	1.003	.600	.403	.026	.376	.051	1.086	.205	.049	1.006	.072
							±.006	±.010	±.003	±.006	±.004	±.002	±.004	±.007	±.007	±.010	±.008	±.010	±.013
0.1	0.1	1	No	Yes	10	3	.055	.092	1.004	.602	.402	.027	.376	.141	1.161	.231	.052	1.007	.080
							±.012	±.017	±.007	±.015	±.009	±.002	±.010	±.041	±.021	±.048	±.011	±.015	±.016
0.1	0.1	1	Yes	No	10	1	.048	.218	.811	.245	.566	.006	.561	.048	1.875	.354	.048	2.027	.215
							±.007	±.030	±.009	±.007	±.003	±.001	±.003	±.011	±.034	±.033	±.008	±.051	±.034
0.1	0.1	1	Yes	Yes	10	2	.063	.241	.838	.281	.557	.008	.549	.078	1.544	.225	.054	1.890	.229
							±.011	±.034	±.005	±.006	±.003	±.001	±.003	±.010	±.028	±.035	±.010	±.033	±.045

Appendix B

Posted-price complex condition, behavioral model

τ	σy	θ	b1	b2	b3	σp	a1	σx	Discrete	Crash	S[Y] t>10	S[P] t>10	Rev. t>10	P.C. t>10	G.P. t>10	In.C. t>10	Prof. t>10	S[Y] t≤20	P t≤20	S[P] t≤20	S[Y] t>20	P t>20	S[P] t>20
2	0	0	0.5	0	0	0.01	0.5	0	No	0	.023 ±.002	.006 ±.002	1.008 ±.001	.605 ±.001	.403 ±.001	.024 ±.002	.379 ±.002	.062 ±.003	1.032 ±.004	.090 ±.006	.005 ±.002	.999 ±.002	.006 ±.001
2	0	0	0.5	0	0	0.05	0.5	0	No	0	.040 ±.008	.034 ±.005	1.012 ±.004	.610 ±.006	.402 ±.003	.065 ±.010	.336 ±.011	.071 ±.010	1.029 ±.011	.099 ±.010	.029 ±.008	.996 ±.009	
2	0	0	0.5	0	0	0.1	0.5	0	No	0	.069 ±.016	.063 ±.008	1.027 ±.008	.630 ±.016	.398 ±.009	.134 ±.013	.264 ±.017	.086 ±.018	1.005 ±.019	.107 ±.016	.060 ±.018	.986 ±.020	.000 ±.000
2	0.1	0	0.5	0	0	0.01	0.5	0	No	0	.060 ±.009	.006 ±.001	1.007 ±.002	.605 ±.002	.403 ±.001	.038 ±.004	.365 ±.004	.082 ±.009	1.034 ±.006	.093 ±.007	.056 ±.011	1.000 ±.002	
2	0.1	0	0.5	0	0	0.05	0.5	0	No	0	.066 ±.010	.032 ±.004	1.011 ±.003	.610 ±.005	.401 ±.003	.074 ±.008	.327 ±.008	.092 ±.011	1.028 ±.011	.097 ±.010	.060 ±.013	.994 ±.008	
2	0.1	0	0.5	0	0	0.1	0.5	0	No	0	.093 ±.015	.068 ±.010	1.025 ±.011	.626 ±.013	.399 ±.006	.128 ±.022	.271 ±.023	.111 ±.025	1.025 ±.022	.124 ±.022	.084 ±.019	.986 ±.016	
2	0	0	1	0	0	0.01	0.5	0	No	9	###	.230 ±.198	.394 ±.381	###	###	###	###	139.696 ±431.294	.832 ±.233	.283 ±.191	###	.248 ±.419	
2	0	0	1	0	0	0.05	0.5	0	No	3	###	.225 ±.173	.663 ±.674	###	###	###	###	50.834 ±112.364	.795 ±.216	.298 ±.175	###	.320 ±.415	
2	0	0	1	0	0	0.1	0.5	0	No	0	###	.236 ±.161	.700 ±.551	###	###	###	###	49.671 ±168.952	.846 ±.211	.262 ±.166	###	.458 ±.441	
2	0.1	0	1	0	0	0.01	0.5	0	No	6	###	.178 ±.199	.488 ±.436	###	###	###	###	271.221 ±789.005	.798 ±.274	.291 ±.196	###	.382 ±.465	
2	0.1	0	1	0	0	0.05	0.5	0	No	1	###	.239 ±.188	.581 ±.709	###	###	###	###	50.524 ±125.934	.768 ±.218	.337 ±.194	###	.274 ±.409	
2	0.1	0	1	0	0	0.1	0.5	0	No	0	###	.227 ±.155	.543 ±.408	###	###	###	###	32.350 ±94.695	.798 ±.228	.283 ±.168	###	.350 ±.440	
2	0	0	0	0.1	0	0.01	0.5	0	No	1	.067 ±.034	.174 ±.053	.811 ±.051	.247 ±.048	.564 ±.017	.537 ±.074	.027 ±.077	.246 ±.062	1.765 ±.385	.280 ±.109	.031 ±.017	2.116 ±.187	
2	0	0	0	0.1	0	0.05	0.5	0	No	0	.054 ±.025	.167 ±.049	.836 ±.024	.265 ±.026	.571 ±.005	.508 ±.041	.063 ±.039	.199 ±.020	1.615 ±.150	.176 ±.035	.031 ±.011	2.018 ±.123	
2	0	0	0	0.1	0	0.1	0.5	0	No	0	.083 ±.042	.205 ±.076	.861 ±.027	.303 ±.039	.558 ±.014	.450 ±.060	.108 ±.048	.180 ±.040	1.459 ±.144	.164 ±.060	.049 ±.019	1.858 ±.183	
2	0.1	0	0	0.1	0	0.01	0.5	0	No	2	.100 ±.025	.227 ±.062	.831 ±.040	.269 ±.040	.562 ±.009	.504 ±.064	.058 ±.064	.233 ±.057	1.549 ±.173	.261 ±.137	.068 ±.015	2.033 ±.208	
2	0.1	0	0	0.1	0	0.05	0.5	0	No	0	.084 ±.017	.188 ±.050	.832 ±.023	.262 ±.025	.570 ±.005	.512 ±.039	.058 ±.037	.202 ±.023	1.632 ±.177	.161 ±.046	.069 ±.010	2.038 ±.099	
2	0.1	0	0	0.1	0	0.1	0.5	0	No	0	.097 ±.025	.207 ±.077	.851 ±.029	.291 ±.038	.560 ±.012	.468 ±.060	.092 ±.051	.199 ±.027	1.551 ±.186	.165 ±.042	.082 ±.020	1.888 ±.205	
2	0	0	0	0.2	0	0.01	0.5	0	No	6	.083 ±.053	.249 ±.120	.782 ±.036	.226 ±.032	.557 ±.015	.573 ±.052	-.016 ±.054	.280 ±.030	1.917 ±.425	.404 ±.101	.048 ±.036	2.197 ±.129	
2	0	0	0	0.2	0	0.05	0.5	0	No	0	.046 ±.026	.154 ±.067	.784 ±.031	.219 ±.023	.564 ±.010	.582 ±.037	-.018 ±.045	.246 ±.034	1.859 ±.225	.291 ±.094	.027 ±.012	2.235 ±.085	
2	0	0	0	0.2	0	0.1	0.5	0	No	0	.067 ±.025	.218 ±.059	.802 ±.026	.238 ±.022	.565 ±.007	.554 ±.036	.011 ±.041	.225 ±.018	1.761 ±.180	.249 ±.064	.052 ±.020	2.143 ±.124	
2	0.1	0	0	0.2	0	0.01	0.5	0	No	8	.130 ±.057	.340 ±.141	.768 ±.071	.227 ±.047	.540 ±.031	.570 ±.076	-.030 ±.098	.321 ±.106	1.881 ±.457	.480 ±.194	.093 ±.037	2.202 ±.215	
2	0.1	0	0	0.2	0	0.05	0.5	0	No	0	.084 ±.014	.189 ±.049	.772 ±.031	.212 ±.023	.560 ±.010	.592 ±.037	-.033 ±.045	.260 ±.034	1.899 ±.211	.341 ±.100	.069 ±.011	2.262 ±.133	
2	0.1	0	0	0.2	0	0.1	0.5	0	No	0	.085 ±.017	.200 ±.057	.796 ±.031	.232 ±.023	.564 ±.009	.563 ±.037	.001 ±.045	.248 ±.024	1.819 ±.189	.285 ±.051	.075 ±.015	2.145 ±.163	
2	0	0	0	0.5	0	0.01	0.5	0	No	19	##	##	##	##	##	##	##	.339 ±.023	2.232 ±.016	.690 ±.070	##	##	
2	0	0	0	0.5	0	0.05	0.5	0	No	4	.051 ±.025	.187 ±.065	.753 ±.020	.201 ±.011	.552 ±.011	.613 ±.019	-.061 ±.029	.289 ±.033	2.052 ±.156	.445 ±.088	.040 ±.013	2.281 ±.058	
2	0	0	0	0.5	0	0.1	0.5	0	No	0	.054 ±.026	.195 ±.053	.767 ±.022	.210 ±.013	.557 ±.009	.595 ±.022	-.039 ±.030	.270 ±.023	2.028 ±.152	.380 ±.091	.048 ±.016	2.223 ±.082	
2	0.1	0	0	0.5	0	0.01	0.5	0	No	16	.285 ±.236	.522 ±.200	.774 ±.034	.260 ±.057	.514 ±.039	.596 ±.053	-.082 ±.078	1.049 ±1.587	1.637 ±.395	.713 ±.167	.119 ±.102	2.173 ±.221	
2	0.1	0	0	0.5	0	0.05	0.5	0	No	4	.089 ±.019	.241 ±.069	.736 ±.024	.193 ±.012	.543 ±.013	.623 ±.023	-.080 ±.034	.287 ±.030	2.098 ±.184	.442 ±.100	.084 ±.018	2.341 ±.085	
2	0.1	0	0	0.5	0	0.1	0.5	0	No	0	.086 ±.017	.219 ±.042	.761 ±.021	.208 ±.012	.554 ±.010	.598 ±.023	-.045 ±.032	.269 ±.027	2.064 ±.191	.378 ±.072	.079 ±.014	2.249 ±.070	

Appendix B

2	0	0	0.5	0	0.1	0.01	0.5	0	No	0	.009 ±.002	.008 ±.002	1.004 ±.000	.601 ±.001	.403 ±.001	.014 ±.001	.389 ±.001	.056 ±.005	1.049 ±.006	.091 ±.012	.005 ±.002	1.000 ±.001
2	0	0	0.5	0	0.1	0.05	0.5	0	No	0	.030 ±.007	.037 ±.008	1.007 ±.003	.607 ±.005	.401 ±.003	.048 ±.006	.352 ±.006	.066 ±.009	1.043 ±.008	.099 ±.009	.027 ±.007	.995 ±.008
2	0	0	0.5	0	0.1	0.1	0.5	0	No	0	.061 ±.015	.071 ±.014	1.018 ±.005	.617 ±.010	.401 ±.006	.095 ±.010	.306 ±.013	.082 ±.014	1.034 ±.014	.112 ±.021	.059 ±.019	.991 ±.014
2	0.1	0	0.5	0	0.1	0.01	0.5	0	No	0	.087 ±.155	.023 ±.057	1.022 ±.082	.621 ±.090	.401 ±.008	.063 ±.168	.338 ±.176	.091 ±.054	1.044 ±.016	.095 ±.028	.092 ±.190	.995 ±.022
2	0.1	0	0.5	0	0.1	0.05	0.5	0	No	0	.064 ±.012	.039 ±.009	1.006 ±.004	.603 ±.005	.403 ±.002	.050 ±.006	.353 ±.007	.085 ±.015	1.045 ±.010	.094 ±.011	.062 ±.012	.999 ±.006
2	0.1	0	0.5	0	0.1	0.1	0.5	0	No	0	.086 ±.015	.073 ±.018	1.021 ±.008	.623 ±.013	.398 ±.008	.102 ±.014	.296 ±.018	.100 ±.020	1.025 ±.015	.103 ±.019	.086 ±.018	.983 ±.020
2	0	0	0.5	0	0.2	0.01	0.5	0	No	0	.008 ±.002	.010 ±.002	1.002 ±.001	.600 ±.000	.402 ±.001	.010 ±.001	.392 ±.001	.055 ±.004	1.054 ±.003	.091 ±.007	.006 ±.002	1.000 ±.001
2	0	0	0.5	0	0.2	0.05	0.5	0	No	0	.031 ±.007	.040 ±.009	1.006 ±.003	.605 ±.004	.401 ±.002	.039 ±.004	.361 ±.005	.061 ±.008	1.052 ±.009	.097 ±.018	.030 ±.008	.995 ±.006
2	0	0	0.5	0	0.2	0.1	0.5	0	No	0	.070 ±.013	.087 ±.014	1.017 ±.007	.616 ±.007	.402 ±.004	.082 ±.009	.319 ±.010	.081 ±.016	1.044 ±.013	.120 ±.022	.068 ±.014	.995 ±.011
2	0.1	0	0.5	0	0.2	0.01	0.5	0	No	1	.059 ±.015	.022 ±.024	1.004 ±.009	.603 ±.011	.402 ±.002	.026 ±.024	.376 ±.025	.078 ±.010	1.053 ±.005	.092 ±.009	.060 ±.019	.998 ±.008
2	0.1	0	0.5	0	0.2	0.05	0.5	0	No	0	.063 ±.006	.039 ±.008	1.006 ±.002	.605 ±.003	.401 ±.003	.042 ±.005	.359 ±.005	.082 ±.014	1.050 ±.004	.098 ±.010	.062 ±.010	.997 ±.006
2	0.1	0	0.5	0	0.2	0.1	0.5	0	No	0	.088 ±.016	.083 ±.021	1.019 ±.007	.619 ±.010	.400 ±.005	.088 ±.009	.313 ±.012	.100 ±.019	1.045 ±.009	.125 ±.021	.089 ±.021	.990 ±.011
5	0	0	0.5	0	0.2	0.01	0.5	0	No	0	.008 ±.001	.021 ±.004	1.002 ±.001	.591 ±.001	.411 ±.001	.027 ±.003	.384 ±.003	.059 ±.003	1.088 ±.003	.081 ±.006	.004 ±.001	1.007 ±.001
5	0	0	0.5	0	0.2	0.05	0.5	0	No	0	.025 ±.005	.046 ±.009	1.005 ±.003	.594 ±.004	.411 ±.002	.048 ±.006	.364 ±.007	.062 ±.005	1.087 ±.006	.088 ±.008	.023 ±.007	1.006 ±.005
5	0	0	0.5	0	0.2	0.1	0.5	0	No	0	.050 ±.013	.084 ±.015	1.018 ±.006	.608 ±.008	.410 ±.005	.089 ±.012	.321 ±.014	.077 ±.012	1.076 ±.010	.106 ±.021	.051 ±.016	1.001 ±.010
5	0.1	0	0.5	0	0.2	0.01	0.5	0	No	1	.054 ±.007	.026 ±.006	1.001 ±.004	.590 ±.006	.412 ±.003	.035 ±.005	.377 ±.004	.077 ±.009	1.086 ±.011	.087 ±.021	.053 ±.008	1.009 ±.008
5	0.1	0	0.5	0	0.2	0.05	0.5	0	No	0	.053 ±.008	.043 ±.007	1.006 ±.003	.594 ±.005	.412 ±.003	.050 ±.005	.362 ±.006	.078 ±.008	1.085 ±.009	.088 ±.011	.052 ±.009	1.007 ±.007
5	0.1	0	0.5	0	0.2	0.1	0.5	0	No	0	.069 ±.011	.080 ±.020	1.018 ±.008	.[illegible]07 ±.008	.411 ±.006	.088 ±.011	.323 ±.011	.098 ±.015	1.083 ±.012	.116 ±.027	.066 ±.014	.997 ±.008
2	0	1	0.5	0	0.1	0.01	0.5	0	No	0	.021 ±.004	.025 ±.002	1.013 ±.001	.599 ±.002	.414 ±.001	.046 ±.003	.368 ±.002	.138 ±.005	1.087 ±.006	.080 ±.005	.012 ±.004	1.001 ±.002
2	0	1	0.5	0	0.1	0.05	0.5	0	No	0	.055 ±.012	.045 ±.008	1.018 ±.006	.60[illegible] ±.009	.412 ±.004	.073 ±.010	.339 ±.010	.149 ±.013	1.082 ±.012	.091 ±.012	.050 ±.013	.996 ±.011
2	0	1	0.5	0	0.1	0.1	0.5	0	No	0	.108 ±.028	.076 ±.016	1.042 ±.012	.637 ±.017	.405 ±.008	.116 ±.018	.288 ±.019	.192 ±.040	1.062 ±.027	.115 ±.017	.093 ±.024	.970 ±.017
2	0.1	1	0.5	0	0.1	0.01	0.5	0	No	0	.061 ±.007	.026 ±.003	1.012 ±.002	.599 ±.003	.414 ±.001	.052 ±.004	.361 ±.003	.148 ±.008	1.087 ±.008	.080 ±.005	.058 ±.009	1.001 ±.003
2	0.1	1	0.5	0	0.1	0.05	0.5	0	No	0	.077 ±.011	.047 ±.006	1.019 ±.005	.606 ±.008	.413 ±.003	.075 ±.006	.338 ±.008	.161 ±.011	1.066 ±.010	.090 ±.010	.075 ±.014	.994 ±.010
2	0.1	1	0.5	0	0.1	0.1	0.5	0	No	0	.112 ±.020	.075 ±.013	1.035 ±.013	.628 ±.016	.407 ±.007	.118 ±.016	.289 ±.018	.186 ±.024	1.071 ±.017	.108 ±.022	.112 ±.023	.975 ±.018
2	0	1	0.5	0	0.1	0.01	0.2	0	No	0	.009 ±.002	.030 ±.002	1.010 ±.001	.586 ±.002	.424 ±.001	.088 ±.003	.336 ±.002	.124 ±.004	1.101 ±.006	.073 ±.006	.006 ±.002	1.017 ±.002
2	0	1	0.5	0	0.1	0.05	0.2	0	No	0	.031 ±.008	.047 ±.006	1.013 ±.004	.589 ±.006	.424 ±.003	.098 ±.008	.325 ±.008	.128 ±.009	1.098 ±.012	.085 ±.013	.028 ±.009	1.015 ±.007
2	0	1	0.5	0	0.1	0.1	0.2	0	No	0	.061 ±.016	.079 ±.017	1.032 ±.010	.613 ±.011	.419 ±.004	.134 ±.013	.285 ±.013	.145 ±.017	1.081 ±.016	.100 ±.018	.061 ±.020	.995 ±.011
2	0.1	1	0.5	0	0.1	0.01	0.2	0	No	0	.055 ±.010	.032 ±.003	1.010 ±.002	.585 ±.003	.424 ±.002	.090 ±.005	.334 ±.004	.135 ±.006	1.101 ±.008	.073 ±.006	.056 ±.011	1.017 ±.003
2	0.1	1	0.5	0	0.1	0.05	0.2	0	No	0	.064 ±.008	.048 ±.008	1.016 ±.004	.593 ±.006	.422 ±.003	.104 ±.005	.318 ±.006	.138 ±.008	1.097 ±.011	.077 ±.006	.064 ±.009	1.009 ±.007
2	0.1	1	0.5	0	0.1	0.1	0.2	0	No	0	.082 ±.013	.077 ±.012	1.031 ±.013	.612 ±.017	.419 ±.005	.134 ±.015	.285 ±.01[illegible]	.158 ±.019	1.090 ±.021	.100 ±.023	.080 ±.015	.992 ±.018
2	0	1	0.5	0	0.2	0.01	0.2	0	No	0	.031 ±.002	.037 ±.002	.987 ±.001	.554 ±.002	.433 ±.001	.079 ±.002	.354 ±.002	.089 ±.003	1.146 ±.005	.067 ±.005	.015 ±.002	1.048 ±.002
2	0	1	0.5	0	0.2	0.05	0.2	0	No	0	.045 ±.009	.057 ±.009	.993 ±.003	.562 ±.006	.431 ±.003	.086 ±.004	.345 ±.006	.094 ±.009	1.141 ±.009	.083 ±.014	.032 ±.010	1.039 ±.008
2	0	1	0.5	0	0.2	0.1	0.2	0	No	0	.067 ±.015	.093 ±.021	1.010 ±.009	.563 ±.013	.427 ±.006	.112 ±.010	.315 ±.011	.118 ±.018	1.123 ±.018	.113 ±.034	.059 ±.014	1.022 ±.016
2	0.1	1	0.5	0	0.2	0.01	0.2	0	No	1	.083 ±.090	.049 ±.034	.991 ±.011	.561 ±.024	.430 ±.012	.085 ±.026	.345 ±.038	.105 ±.006	1.144 ±.008	.068 ±.005	.082 ±.109	1.042 ±.0[illegible]3
2	0.1	1	0.5	0	0.2	0.05	0.2	0	No	0	.070 ±.010	.057 ±.008	.992 ±.005	.560 ±.007	.431 ±.004	.088 ±.004	.343 ±.006	.107 ±.008	1.143 ±.009	.080 ±.016	.065 ±.012	1.041 ±.011
2	0.1	1	0.5	0	0.2	0.1	0.2	0	No	0	.089 ±.017	.089 ±.013	1.012 ±.017	.585 ±.021	.427 ±.006	.116 ±.018	.312 ±.021	.133 ±.031	1.123 ±.018	.108 ±.022	.078 ±.014	1.023 ±.020
2	0	0	0.5	0	0	0.01	0.5	0	Yes	0	208.073 ±347.794	.295 ±191	.766 ±572	100.946 ±168.390	- 100.180 ±168.827	119.580 ±200.744	- 219.759 ±369.571	4.599 ±7.295	.684 ±245	.394 ±057	225.367 ±376.331	.382 ±388

Appendix B

2	0	0	0.5	0	0	0.05	0.5	0	Yes	0	376.9 04 ±658. 821	.296 ±.185	.829 ±.634	173.5 79 ±298. 447	- 172.7 50 ±298. 890	198.7 03 ±339. 478	- 371.4 53 ±636. 290	5.522 ±8.58 5	.676 ±.253	.396 ±.060	415.4 01 ±729. 557	.325 ±.353
2	0	0	0.5	0	0	0.1	0.5	0	Yes	0	###	.227 ±.191	.584 ±.527	650.2 33 ±113 9.949	- 649.6 49 ±114 0.181	684.7 00 ±116 3.787	###	9.324 ±12.3 85	.584 ±.220	.409 ±.060	###	.166 ±.269
2	0.1	0	0.5	0	0	0.01	0.5	0	Yes	0	135.0 77 ±286. 412	.332 ±.156	.857 ±.530	65.59 6 ±138. 649	- 64.73 9 ±139. 017	77.43 4 ±165. 271	- 142.1 73 ±304. 288	3.057 ±5.98 9	.742 ±.213	.390 ±.054	146.3 47 ±309. 945	.400 ±.366
2	0.1	0	0.5	0	0	0.05	0.5	0	Yes	0	144.1 47 ±295. 763	.313 ±.136	.985 ±.521	68.60 9 ±141. 497	- 67.62 4 ±141. 917	79.94 7 ±167. 485	- 147.5 71 ±309. 384	2.875 ±5.81 3	.759 ±.208	.366 ±.056	157.4 00 ±321. 350	.480 ±.394
2	0.1	0	0.5	0	0	0.1	0.5	0	Yes	0	###	.178 ±.179	.527 ±.540	###	###	###	###	15.26 7 ±20.6 76	.530 ±.237	.404 ±.062	###	.082 ±.190
2	0	0	1	0	0	0.01	0.5	0	Yes	2	###	.242 ±.231	.224 ±.279	###	###	###	###	23.25 0 ±39.0 92	.534 ±.265	.569 ±.105	###	.003 ±.007
2	0	0	1	0	0	0.05	0.5	0	Yes	0	###	.021 ±.076	.025 ±.068	###	###	###	###	460.4 69 ±906. 022	.246 ±.150	.458 ±.162	###	.000 ±.000
2	0	0	1	0	0	0.1	0.5	0	Yes	0	###	.000 ±.000	.003 ±.004	###	###	###	###	###	.117 ±.041	.319 ±.059	###	.000 ±.000
2	0.1	0	1	0	0	0.01	0.5	0	Yes	2	###	.236 ±.245	.269 ±.326	###	###	###	###	22.37 2 ±32.6 40	.522 ±.271	.550 ±.144	###	.018 ±.050
2	0.1	0	1	0	0	0.05	0.5	0	Yes	0	###	.001 ±.002	.005 ±.007	###	###	###	###	635.3 01 ±103 5.409	.196 ±.097	.409 ±.098	###	.000 ±.000
2	0.1	0	1	0	0	0.1	0.5	0	Yes	0	###	.000 ±.000	.002 ±.002	###	###	###	###	###	.124 ±.042	.331 ±.050	###	.000 ±.000
2	0	0	0	0.1	0	0.01	0.5	0	Yes	0	.033 ±.004	.186 ±.013	.748 ±.008	.196 ±.004	.552 ±.004	.620 ±.006	-.068 ±.010	.266 ±.011	1.997 ±.073	.358 ±.043	.033 ±.005	2.343 ±.038
2	0	0	0	0.1	0	0.05	0.5	0	Yes	0	.040 ±.008	.194 ±.018	.750 ±.012	.198 ±.006	.553 ±.006	.617 ±.011	-.065 ±.017	.276 ±.015	1.969 ±.091	.391 ±.049	.038 ±.007	2.318 ±.039
2	0	0	0	0.1	0	0.1	0.5	0	Yes	0	.068 ±.030	.250 ±.044	.760 ±.025	.207 ±.015	.553 ±.010	.601 ±.026	-.049 ±.035	.268 ±.016	1.874 ±.180	.396 ±.042	.052 ±.012	2.284 ±.061
2	0.1	0	0	0.1	0	0.01	0.5	0	Yes	0	.074 ±.010	.203 ±.019	.753 ±.010	.199 ±.005	.553 ±.005	.614 ±.011	-.060 ±.015	.273 ±.015	1.983 ±.096	.370 ±.050	.074 ±.014	2.309 ±.040
2	0.1	0	0	0.1	0	0.05	0.5	0	Yes	0	.073 ±.006	.214 ±.020	.750 ±.012	.198 ±.007	.551 ±.006	.615 ±.012	-.063 ±.018	.275 ±.020	1.994 ±.094	.369 ±.061	.071 ±.009	2.315 ±.048
2	0.1	0	0	0.1	0	0.1	0.5	0	Yes	0	.086 ±.020	.246 ±.033	.762 ±.016	.209 ±.010	.554 ±.007	.600 ±.016	-.046 ±.022	.268 ±.022	1.923 ±.111	.373 ±.071	.082 ±.017	2.257 ±.044
2	0	0	0	0.2	0	0.01	0.5	0	Yes	0	.057 ±.005	.358 ±.015	.690 ±.018	.175 ±.009	.515 ±.009	.651 ±.010	-.136 ±.019	.291 ±.015	2.213 ±.112	.486 ±.037	.058 ±.007	2.438 ±.065
2	0	0	0	0.2	0	0.05	0.5	0	Yes	0	.074 ±.011	.375 ±.025	.721 ±.016	.194 ±.008	.527 ±.008	.625 ±.014	-.098 ±.022	.294 ±.027	2.141 ±.097	.488 ±.065	.070 ±.008	2.312 ±.057
2	0	0	0	0.2	0	0.1	0.5	0	Yes	0	.077 ±.009	.386 ±.025	.723 ±.012	.196 ±.006	.526 ±.007	.622 ±.010	-.095 ±.016	.296 ±.025	2.096 ±.098	.500 ±.059	.074 ±.009	2.281 ±.047
2	0.1	0	0	0.2	0	0.01	0.5	0	Yes	0	.085 ±.012	.370 ±.020	.699 ±.011	.181 ±.006	.519 ±.006	.644 ±.011	-.125 ±.016	.293 ±.019	2.144 ±.107	.484 ±.053	.086 ±.012	2.423 ±.039
2	0.1	0	0	0.2	0	0.05	0.5	0	Yes	0	.091 ±.011	.363 ±.018	.723 ±.011	.194 ±.006	.529 ±.006	.626 ±.010	-.097 ±.015	.297 ±.026	2.106 ±.074	.478 ±.043	.094 ±.013	2.319 ±.051
2	0.1	0	0	0.2	0	0.1	0.5	0	Yes	0	.095 ±.010	.388 ±.028	.727 ±.017	.199 ±.010	.527 ±.008	.615 ±.016	-.088 ±.023	.282 ±.025	2.135 ±.079	.449 ±.064	.095 ±.014	2.280 ±.071
2	0	0	0	0.5	0	0.01	0.5	0	Yes	0	.138 ±.002	.838 ±.014	.619 ±.015	.193 ±.007	.426 ±.009	.626 ±.011	-.200 ±.018	.336 ±.030	2.184 ±.106	.883 ±.014	.139 ±.001	2.343 ±.046
2	0	0	0	0.5	0	0.05	0.5	0	Yes	7	.145 ±.006	.794 ±.051	.672 ±.050	.223 ±.026	.449 ±.025	.579 ±.033	-.130 ±.057	.321 ±.030	2.194 ±.151	.863 ±.045	.140 ±.003	2.129 ±.149
2	0	0	0	0.5	0	0.1	0.5	0	Yes	18	.154 ±.024	##	.700 ±.031	.241 ±.012	.459 ±.019	.537 ±.033	-.078 ±.052	.303 ±.043	2.018 ±.120	.802 ±.088	.154 ±.017	##
2	0.1	0	0	0.5	0	0.01	0.5	0	Yes	0	.144 ±.011	.835 ±.021	.622 ±.022	.195 ±.010	.428 ±.012	.623 ±.015	-.195 ±.026	.328 ±.034	2.200 ±.118	.871 ±.022	.147 ±.011	2.337 ±.059
2	0.1	0	0	0.5	0	0.05	0.5	0	Yes	12	.153 ±.013	.758 ±.044	.702 ±.032	.236 ±.018	.465 ±.015	.561 ±.031	-.096 ±.044	.317 ±.032	2.115 ±.118	.832 ±.037	.143 ±.017	2.041 ±.063

2	0.1	0	0	0.5	0	0.1	0.5	0	Yes	20	##	##	##	##	##	##	##	.295 ±.012	2.057 ±.125	.795 ±.045	##	##
2	0	0	0.5	0	0.1	0.01	0.5	0	Yes	0	.280 ±.062	.321 ±.034	1.129 ±.037	.758 ±.049	.372 ±.017	.311 ±.062	.060 ±.073	.258 ±.084	.981 ±.036	.320 ±.044	.275 ±.054	.972 ±.034
2	0	0	0.5	0	0.1	0.05	0.5	0	Yes	0	.336 ±.084	.306 ±.043	1.251 ±.044	.888 ±.056	.363 ±.018	.531 ±.067	-.167 ±.080	.357 ±.106	.917 ±.047	.297 ±.038	.334 ±.103	.916 ±.043
2	0	0	0.5	0	0.1	0.1	0.5	0	Yes	0	.449 ±.123	.315 ±.050	1.313 ±.044	.981 ±.062	.332 ±.032	.676 ±.082	-.345 ±.102	.550 ±.251	.896 ±.046	.335 ±.069	.423 ±.142	.862 ±.041
2	0.1	0	0.5	0	0.1	0.01	0.5	0	Yes	0	.250 ±.049	.305 ±.031	1.138 ±.049	.763 ±.056	.375 ±.015	.327 ±.080	.049 ±.088	.267 ±.078	.965 ±.042	.302 ±.040	.250 ±.067	.962 ±.034
2	0.1	0	0.5	0	0.1	0.05	0.5	0	Yes	0	.357 ±.070	.312 ±.043	1.254 ±.039	.901 ±.058	.353 ±.029	.535 ±.070	-.183 ±.090	.339 ±.061	.916 ±.035	.300 ±.037	.365 ±.097	.908 ±.041
2	0.1	0	0.5	0	0.1	0.1	0.5	0	Yes	0	.519 ±.142	.319 ±.043	1.318 ±.044	1.004 ±.066	.314 ±.063	.699 ±.083	-.384 ±.118	.415 ±.166	.912 ±.031	.305 ±.048	.530 ±.165	.857 ±.055
2	0	0	0.5	0	0.2	0.01	0.5	0	Yes	0	.284 ±.050	.349 ±.056	1.128 ±.051	.736 ±.065	.392 ±.028	.272 ±.088	.121 ±.101	.260 ±.065	1.036 ±.045	.367 ±.054	.264 ±.061	.989 ±.054
2	0	0	0.5	0	0.2	0.05	0.5	0	Yes	0	.351 ±.076	.339 ±.064	1.226 ±.041	.844 ±.045	.382 ±.025	.437 ±.068	-.055 ±.074	.320 ±.061	.964 ±.037	.311 ±.057	.360 ±.086	.959 ±.055
2	0	0	0.5	0	0.2	0.1	0.5	0	Yes	1	.427 ±.161	.380 ±.144	1.250 ±.061	.883 ±.040	.367 ±.056	.522 ±.066	-.155 ±.103	.352 ±.067	.963 ±.061	.335 ±.083	.450 ±.197	.958 ±.063
2	0.1	0	0.5	0	0.2	0.01	0.5	0	Yes	0	.290 ±.056	.343 ±.042	1.153 ±.054	.764 ±.058	.390 ±.011	.317 ±.092	.073 ±.095	.267 ±.053	1.016 ±.052	.350 ±.056	.285 ±.074	.979 ±.030
2	0.1	0	0.5	0	0.2	0.05	0.5	0	Yes	0	.344 ±.079	.326 ±.077	1.219 ±.040	.833 ±.041	.386 ±.022	.434 ±.065	-.048 ±.066	.298 ±.057	.986 ±.035	.320 ±.052	.348 ±.103	.945 ±.055
2	0.1	0	0.5	0	0.2	0.1	0.5	0	Yes	0	.361 ±.065	.334 ±.048	1.248 ±.039	.861 ±.038	.387 ±.016	.491 ±.059	-.104 ±.058	.359 ±.077	.957 ±.040	.328 ±.066	.353 ±.087	.953 ±.037
5	0	0	0.5	0	0.2	0.01	0.5	0	Yes	0	.217 ±.041	.370 ±.072	1.107 ±.037	.711 ±.037	.396 ±.018	.274 ±.052	.122 ±.056	.210 ±.054	1.079 ±.054	.407 ±.089	.205 ±.054	1.013 ±.057
5	0	0	0.5	0	0.2	0.05	0.5	0	Yes	0	.223 ±.051	.327 ±.055	1.182 ±.030	.783 ±.035	.399 ±.020	.399 ±.058	.001 ±.062	.250 ±.045	1.023 ±.040	.331 ±.066	.220 ±.056	.958 ±.028
5	0	0	0.5	0	0.2	0.1	0.5	0	Yes	0	.273 ±.049	.345 ±.039	1.228 ±.022	.830 ±.033	[illegible] ±.025	.492 ±.047	-.094 ±.057	.276 ±.053	1.001 ±.031	.328 ±.045	.276 ±.057	.961 ±.030
5	0.1	0	0.5	0	0.2	0.01	0.5	0	Yes	0	.196 ±.032	.331 ±.043	1.125 ±.034	.730 ±.034	.394 ±.013	.286 ±.058	.108 ±.060	.202 ±.049	1.058 ±.046	.352 ±.037	.200 ±.035	.992 ±.034
5	0.1	0	0.5	0	0.2	0.05	0.5	0	Yes	0	.226 ±.032	.310 ±.031	1.188 ±.034	.800 ±.041	.388 ±.017	.410 ±.058	-.022 ±.066	.232 ±.040	1.010 ±.030	.313 ±.033	.231 ±.033	.948 ±.033
5	0.1	0	0.5	0	0.2	0.1	0.5	0	Yes	0	.292 ±.068	.348 ±.062	1.227 ±.030	.843 ±.026	.384 ±.022	.496 ±.041	-.112 ±.043	.295 ±.050	1.004 ±.039	.350 ±.062	.298 ±.081	.951 ±.040
2	0	1	0.5	0	0.1	0.01	0.5	0	Yes	0	.588 ±.119	.303 ±.040	1.259 ±.074	.914 ±.090	.345 ±.027	.370 ±.070	-.025 ±.085	.547 ±.088	.977 ±.049	.333 ±.035	.574 ±.138	.871 ±.054
2	0	1	0.5	0	0.1	0.05	0.5	0	Yes	0	.952 ±.427	.309 ±.092	1.434 ±.149	1.140 ±.219	.293 ±.109	.633 ±.214	-.340 ±.289	.848 ±.412	.907 ±.045	.327 ±.104	.935 ±.447	.813 ±.048
2	0	1	0.5	0	0.1	0.1	0.5	0	Yes	1	1.409 ±.927	.313 ±.058	1.687 ±.267	1.496 ±.425	.191 ±.177	.849 ±.276	-.658 ±.444	.956 ±.260	.871 ±.043	.321 ±.069	1.538 ±1.073	.745 ±.064
2	0.1	1	0.5	0	0.1	0.01	0.5	0	Yes	0	.577 ±.100	.301 ±.027	1.254 ±.083	.897 ±.089	.357 ±.021	.365 ±.091	-.008 ±.095	.560 ±.094	.961 ±.044	.323 ±.036	.586 ±.134	.884 ±.042
2	0.1	1	0.5	0	0.1	0.05	0.5	0	Yes	0	.817 ±.213	.289 ±.050	1.443 ±.119	1.134 ±.146	.309 ±.043	.600 ±.117	-.291 ±.143	.762 ±.218	.918 ±.050	.321 ±.056	.779 ±.226	.800 ±.037
2	0.1	1	0.5	0	0.1	0.1	0.5	0	Yes	0	1.445 ±.489	.363 ±.099	1.640 ±.173	1.447 ±.255	.194 ±.108	.859 ±.183	-.665 ±.278	.965 ±.230	.882 ±.056	.328 ±.058	1.582 ±.613	.746 ±.034
2	0	1	0.5	0	0.1	0.01	0.2	0	Yes	0	.311 ±.082	.309 ±.042	1.252 ±.099	.890 ±.106	.362 ±.046	.377 ±.098	-.015 ±.114	.363 ±.073	.987 ±.051	.315 ±.054	.322 ±.104	.893 ±.054
2	0	1	0.5	0	0.1	0.05	0.2	0	Yes	0	.450 ±.230	.291 ±.058	1.391 ±.077	1.044 ±.102	.347 ±.041	.576 ±.125	-.229 ±.155	.560 ±.274	.937 ±.033	.326 ±.067	.411 ±.190	.845 ±.039
2	0	1	0.5	0	0.1	0.1	0.2	0	Yes	0	.641 ±.255	.301 ±.051	1.595 ±.208	1.277 ±.227	.318 ±.036	.782 ±.168	-.464 ±.187	.657 ±.214	.906 ±.056	.322 ±.077	.645 ±.297	.784 ±.029
2	0.1	1	0.5	0	0.1	0.01	0.2	0	Yes	0	.282 ±.060	.298 ±.022	1.220 ±.060	.846 ±.071	.374 ±.023	.340 ±.062	.033 ±.072	.345 ±.098	.991 ±.046	.319 ±.042	.286 ±.057	.911 ±.050
2	0.1	1	0.5	0	0.1	0.05	0.2	0	Yes	0	.485 ±.212	.289 ±.058	1.431 ±.104	1.095 ±.127	.336 ±.034	.619 ±.114	-.283 ±.138	.501 ±.101	.923 ±.044	.308 ±.062	.509 ±.227	.837 ±.044
2	0.1	1	0.5	0	0.1	0.1	0.2	0	Yes	0	.572 ±.201	.290 ±.051	1.568 ±.146	1.250 ±.172	.318 ±.034	.744 ±.132	-.426 ±.158	.611 ±.129	.892 ±.045	.302 ±.062	.626 ±.216	.808 ±.047
2	0	1	0.5	0	0.2	0.01	0.2	0	Yes	0	.323 ±.080	.341 ±.046	1.226 ±.110	.839 ±.116	.387 ±.029	.325 ±.105	.062 ±.111	.313 ±.095	1.062 ±.045	.382 ±.040	.318 ±.083	.919 ±.063
2	0	1	0.5	0	0.2	0.05	0.2	0	Yes	0	.456 ±.157	.326 ±.053	1.390 ±.102	1.028 ±.127	.362 ±.032	.485 ±.090	-.123 ±.115	.395 ±.077	.994 ±.033	.312 ±.047	.462 ±.189	.858 ±.040
2	0	1	0.5	0	0.2	0.1	0.2	0	Yes	0	.601 ±.233	.351 ±.059	1.465 ±.093	1.118 ±.117	.348 ±.050	.576 ±.100	-.228 ±.132	.459 ±.075	.977 ±.043	.340 ±.055	.613 ±.264	.849 ±.047
2	0.1	1	0.5	0	0.2	0.01	0.2	0	Yes	0	.326 ±.108	.309 ±.047	1.224 ±.078	.838 ±.091	.386 ±.022	.338 ±.091	.047 ±.105	.304 ±.072	1.062 ±.042	.361 ±.052	.304 ±.108	.907 ±.054
2	0.1	1	0.5	0	0.2	0.05	0.2	0	Yes	0	.423 ±.115	.315 ±.043	1.369 ±.099	1.007 ±.107	.363 ±.022	.464 ±.076	-.102 ±.086	.394 ±.095	.989 ±.038	.319 ±.051	.428 ±.144	.867 ±.046
2	0.1	1	0.5	0	0.2	0.1	0.2	0	Yes	0	.511 ±.190	.340 ±.092	1.453 ±.067	1.101 ±.072	.351 ±.031	.550 ±.072	-.199 ±.088	.467 ±.080	.964 ±.031	.338 ±.046	.517 ±.258	.847 ±.044

Note: In all simulations, σ_X and a_2 are zero; β is one; s_C is 3; $d=\sigma_P$.

Analysis of variance of gross profits

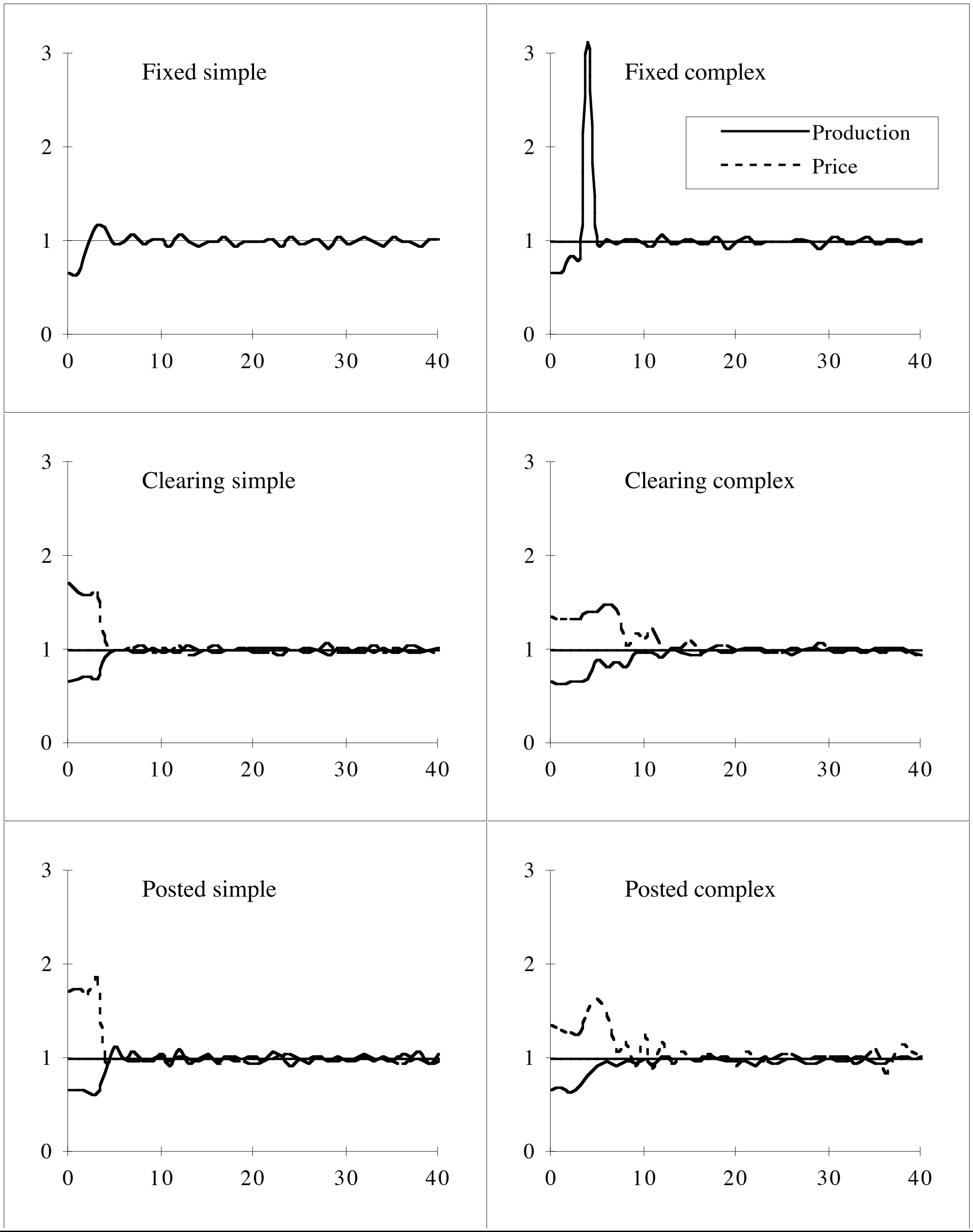
Source	Sum-of-squares	D.f.	Mean square	F-ratio	P
C	0.074	1	0.074	5.957	0.031
P	0.021	1	0.021	1.700	0.217
C*P	0.005	1	0.005	0.373	0.553
ERROR	0.149	12	0.012		

Two-way analysis of variance of normalized average profits (equation (11)) before inventory costs in each market, excluding the first 10 time periods, using price condition (P) and complexity (C) as factors, excluding the fixed-price conditions, where profits before inventory costs do not vary in the long run under fixed prices. N = 16; R^2 = .401.

Analysis of variance of inventory costs

Source	Sum-of-squares	D.f.	Mean square	F-ratio	P
C	81.308	1	81.308	30.144	0.000
P	13.070	1	13.070	4.846	0.048
C*P	33.654	1	33.654	12.477	0.004
ERROR	32.368	12	2.697		

Two-way analysis of variance of the logarithm of the average inventory costs in each market, excluding the first 10 time periods, using price condition (P) and complexity (C) as factors, excluding the clearing-price conditions, where inventories are identically zero. N = 16; R^2 = .798.



Simulated behavior of rational agents

Typical realizations of simulations of market average output and price (in a market with 4 firms) under the assumption that firms correctly estimate the structural parameters in the system and act to maximize their expected profits, given their expectation of non-cooperative rational-expectations equilibrium, with small i.i.d. random errors (standard deviation 5%) introduced in the decision rules for both price and output.

Appendix C: Questionnaires and instructions

Pre-game questionnaire

1. Date ___________

2. Firm ___________ (name indicated on your computer)

3. Your age ___________ (years)

4. What is your current educational status (check one)?

______ Faculty member
______ Doctoral student, ABD (all but dissertation)
______ Doctoral student taking courses
______ Master's student
______ Sloan Fellow
______ Undergraduate
______ Other (please specify) ______________________________

5. With what department at M.I.T. are you mostly affiliated (check one)?

______ Management (15), Behavioral and Policy Sciences
______ Management (15), Economics and Finance
______ Managmenet (15), Management Science
______ Management (15), MBA/MOT/Sloan Fellow
______ Economics (14)
______ Politicial Science (17)
______ Other (please specify) ______________________________

In the following, a "graduate course equivalent" is either one graduate-level course or two undergraduate courses.

6. What is your educational background in economics (don't count statistics or econometrics courses)?

______ Advanced (more than 3 graduate course equivalents)
______ Intermediate (2-3 graduate course equivalents)
______ Introductory (1/2 - 1 graduate course equivalent)
______ None

7. What is your educational background in relevant quantitative disciplines such as statistics, operations research, management science, engineering control theory (check one)?

______ Advanced (more than 3 graduate course equivalents)
______ Intermediate (2-3 graduate course equivalents)
______ Introductory (1/2 - 1 graduate course equivalent)
______ None

8. What is your educational background in "system dynamics" (check one)?

______ Several courses (e.g. 15.872/15.874, 15.873, individual study)
______ One course (e.g. 15.872/15.874)
______ None

9. Has anyone who has played this game before ever told you any datails about the game? (Answer yes or no) ________

10. Have you ever played any of the following simulation games (check any that apply)?

______ The market game during IAP, January 1990
______ The "People Express" Management Flight Simulator
______ The "Beer Distribution Game"
______ Bent Bakken's Real Estate Investment Game
______ Bent Bakken's Oil Tanker Game
______ Ernst Diehl's Control Game
______ The "Long Wave" Capital Investment Game.

Game instructions, fixed-price, simple condition

Goal

As in life, the purpose of the game is for you to maximize your profits. For the experimenters, the purpose is to investigate how markets with different structures and price regimes behave.

Your dollar reward is based on your cumulative net profits in the game. Your cumulative profits are converted to real money using a fixed exchange rate. On average, you should earn about $30 in three hours of playing.

In addition, you will receive a bonus for your forecasting performance (see below). On average, the bonus will amount to a couple of dollars.

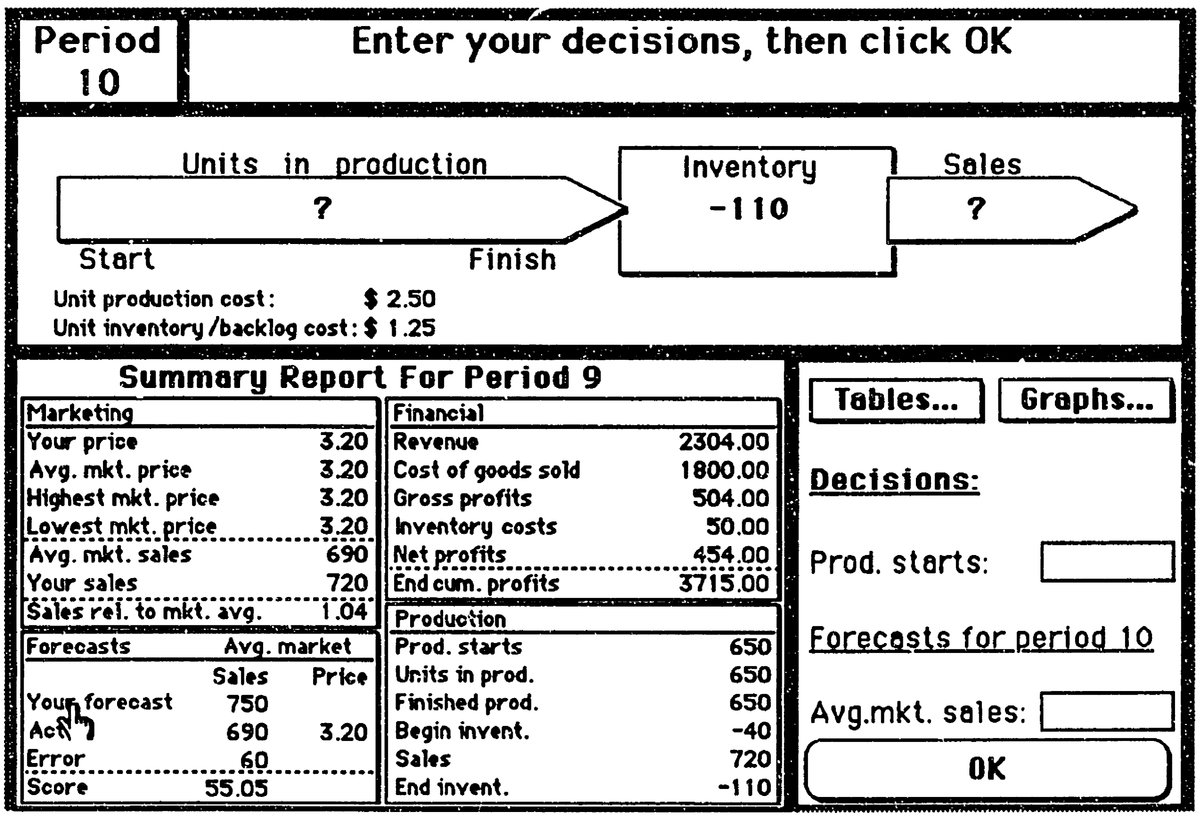


Figure 1

Your firm and the market

Figure 1 shows the screen that you will be seeing in the game. (The numbers in the figure are for illustration only--the numbers in your game will be different.)

Your firm competes with other firms in the market. All firms are identical with respect to production cost, inventory cost, etc. Each firm sells only one product, which for experimental reasons has been kept abstract. The products of different firms are quite alike but not identical. You can think of the product as a tire, a softdrink, or a men's shirt.

Production, inventory, and backlog

Each period, you have only one decision to make, namely how much to produce.

If you sell less than the finished production, the rest piles up in your inventory. On the other hand, if you sell more than you have available, the excess sales accumulate in an order backlog (i.e. a negative inventory).

Theoretically, there is no limit on how much you produce, but you cannot cancel initiated production or destroy your inventory, i.e. you cannot produce negative amounts.

Profits and costs

Your profits each period is your revenue (sales x price) less your costs. Costs consist of production cost, incurred at the time of sale (called "cost of goods sold"), and inventory/backlog costs.

The "cost of goods sold" is simply proportional to sales, i.e. the unit production cost is constant.

Inventory/backlog costs are proportional to your inventory or backlog that you have at the beginning of each period. You might also say that your inventory/backlog cost is proportional to the absolute value of your inventory.

In the case where you have positive inventory, you can interpret the cost as a storage and financing charge. When you have a backlog (negative inventory), you can think of the cost as a rebate that you have to offer your customers to compensate for delayed delivery, or a penalty imposed on you for late delivery by a central planning ministry.

Your gross profits are your revenues less cost of goods sold. Your net profits are gross profits less inventory/backlog costs.

Note that both the unit production cost and the unit inventory/backlog cost are constant (and will not change at any time during the game).

Price

In this version of the game, prices are fixed by government decree.

Sales

Your sales are influenced by only one factor (in other versions of the market where prices are variable, sales would also be affected by price):

- External factors Demand for your product, as well as demand for the products of your competitors, can be influenced by exogenous factors which are independent of anything you or your competitors do in the game.

  The pattern of these "factors" will be known only to the experimenter, but you can be sure that there will not be a consistent trend of growth and decline. Moreover, these "factors" will remain in a "reasonable" range.

Forecasts

In addition to your production decision, we ask you to make a forecast of the average sales per firm in the market (i.e. not the total market sales, and not your own sales). We ask you to forecast both of these figures for the current period.

On top of your profit-based reward, you will get a "bonus" of a couple of dollars, depending how you rank in your forecasting performance. Your performance is measured by a "score" which is the average standard deviation of your forecast (i.e. the root mean squared error). Thus, the lower your score, the better your performance. The fact that errors are squared before they're summed means that it is better to have many small errors than a few large ones.

Length of game

We cannot tell you exactly when the game will end. There is no time limit on your decisions, but, since rewards are based on cumulative profits, the more periods you play, the more money you will accumulate in the game, so there's an incentive to not take too long to make your decision. Moreover, since all the other players in the market must wait for the last player to make up his or her mind, we suggest that you do not keep your co-players waiting for too long.

Practice rounds

In order to familiarize you with the game and the mechanics of using the machine, you will have a practice session of 3 time periods.

At the end of period 4, your cumulative profits are then "reset" so that they are equal to the profits you made during that period. Your forecasting score is reset in the same manner. However, your inventory is not reset!!!

Making your decisions

When you are ready to make your decisions, click the mouse on the text fields in the lower right-hand corner of your screen to bring the cursor into the appropriate field. (Hitting the <tab> or <enter> or <return> key does the same thing.) Type in your decision and, when you're finished, click on the "OK" button to execute them. You will then be asked to confirm each decision.

After all participants have entered their decisions, the computers calculate the results for that period and advance to the next period.

Previous-period report

On the lower left part of the screen, you will see a summary report for the previous period. The report lists both prices for last period, your forecasting performance, your production, inventory and sales, and your profits, costs and cumulative profits.

Graphs and Tables

Just above your decision area in the lower left part of the screen, you will find two "pop-up menus" which allow you to look at data for previous periods in both graphical and tabular form. To use the menus, move the mouse to the appropriate menu and depress and hold down the mouse button, and the menu will "pop" up. Move the mouse (still holding down the button) to the desired item in the menu (items will turn black as you move over them) and then release the mouse button to select that graph or table.

Some graphs show variables over time while others plot one variables as a function of the other.

Tables list historical data in numerical form. Unlike the graphs, which cannot be changed, you can re-define the tables. Click on the "Define table..." button. Then use the "pop-up menus" that now appear above each column in the table to select the variable you want to list in that column. (The column-definition menus are hierarchical, i.e. each item in the main menu opens a sub-menu from which you choose your variable. It takes a bit of practice to use these.)

We encourage you to use the practice round to gain familiarity with the mechanics of the game software such as using the mouse, looking at graphs and tables, defining tables, etc.

Speaking with others

You are not allowed to communicate directly with the other players. Moreover, we request that you do not tell other people about the game until the end of the semester. We appreciate your cooperation in this matter.

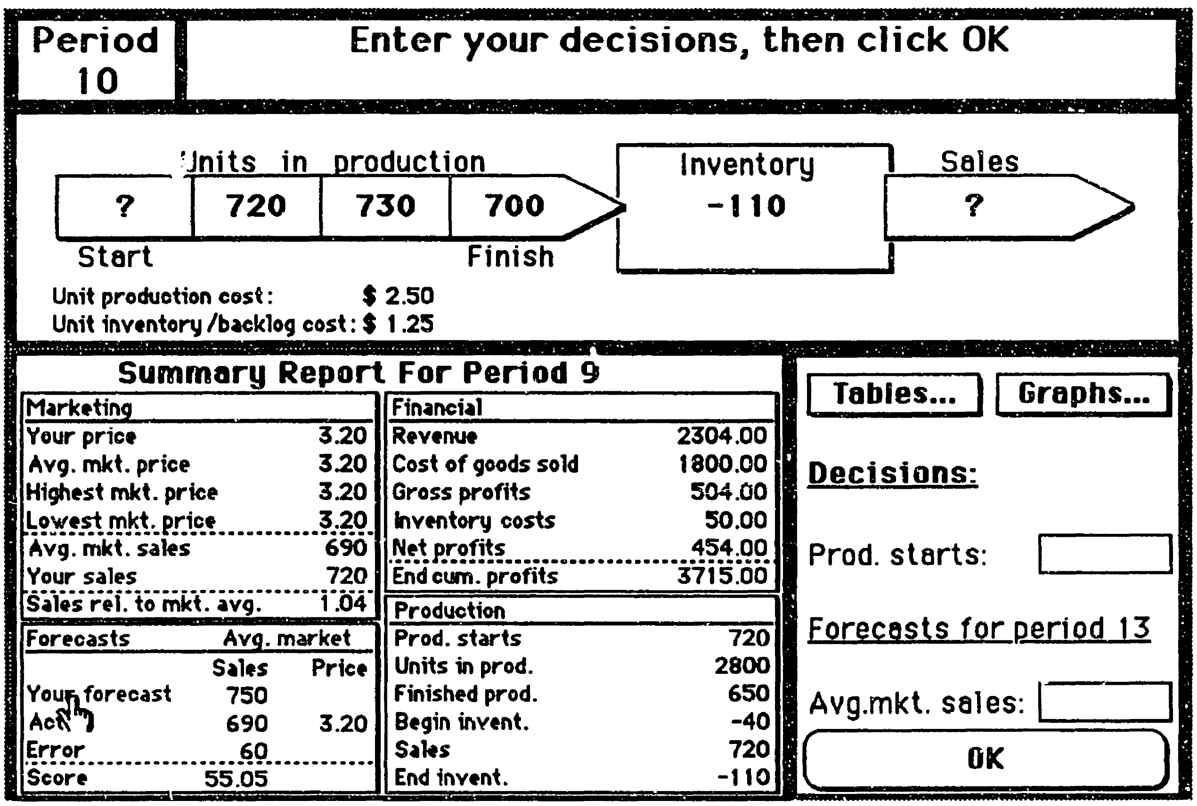
Game instructions, fixed-price, complex condition

Goal

As in life, the purpose of the game is for you to maximize your profits. For the experimenters, the purpose is to investigate how markets with different structures and price regimes behave.

Your dollar reward is based on your cumulative net profits in the game. Your cumulative profits are converted to real money using a fixed exchange rate. On average, you should earn about $30 in three hours of playing.

In addition, you will receive a bonus for your forecasting performance (see below). On average, the bonus will amount to a couple of dollars.



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Your firm competes with other firms in the market. All firms are identical with respect to production cost, inventory cost, etc. Each firm sells only one product, which for experimental reasons has been kept abstract. The products of different firms are quite alike but not identical. You can think of the product as a tire, a softdrink, or a men's shirt.

Production, inventory, and backlog

Each period, you have only one decision to make, namely how much production to initiate. There is a three-period production lag between the time production is initiated and the time it becomes "finished", available for sale (or storage in inventory).

If you sell less than the finished production, the rest piles up in your inventory. On the other hand, if you sell more than you have available, the excess sales accumulate in an order backlog (i.e. a negative inventory).

Theoretically, there is no limit on how much you produce, but you cannot cancel initiated production or destroy your inventory, i.e. you cannot produce negative amounts.

Profits and costs

Your profits each period is your revenue (sales x price) less your costs. Costs consist of production cost, incurred at the time of sale (called "cost of goods sold"), and inventory/backlog costs.

The "cost of goods sold" is simply proportional to sales, i.e. the unit production cost is constant.

Inventory/backlog costs are proportional to your inventory or backlog that you have at the beginning of each period. You might also say that your inventory/backlog cost is proportional to the absolute value of your inventory.

In the case where you have positive inventory, you can interpret the cost as a storage and financing charge. When you have a backlog (negative inventory), you can think of the cost as a rebate that you have to offer your customers to compensate for delayed delivery, or a penalty imposed on you for late delivery by a central planning ministry.

Your gross profits are your revenues less cost of goods sold. Your net profits are gross profits less inventory/backlog costs.

Note that both the unit production cost and the unit inventory/backlog cost are constant (and will not change at any time during the game).

Price

In this version of the game, prices are fixed by government decree.

Sales

Your sales are influenced by two factors (in other versions of the market where prices are variable, sales would also be affected by price):

- Multiplier effect You can think of the market you are in as part of a regional economy where, if every firm has a high level of production, more people will be employed, which in turn will influence incomes and thus demand for all goods in the region, including your product. Conversely, if all firms produce less, people are laid off, and demand for your product will fall somewhat.

  Thus, the demand for your product (and for the product of other firms) depends partly on the total amount of units in production. (Units in production includes both production started this period plus the amount in the "pipeline" started during the previous three periods).

- External factors Demand for your product, as well as demand for the products of your competitors, can be influenced by exogenous factors which are independent of anything you or your competitors do in the game.

  The pattern of these "factors" will be known only to the experimenter, but you can be sure that there will not be a consistent trend of growth and decline. Moreover, these "factors" will remain in a "reasonable" range.

Forecasts

In addition to your production decision, we ask you to make a forecast of the average sales per firm in the market (i.e. not the total market sales, and not your own sales).

We ask you to forecast this figure 3 periods into the future. For instance, at the beginning of period 5, we ask you to forecast what you think the average sales per firm in the market will be in period 8.

On top of your profit-based reward, you will get a "bonus" of a couple of dollars, depending how you rank in your forecasting performance. Your performance is measured by a "score" which is the average standard deviation of your forecast (i.e. the root mean squared error). Thus, the lower your score, the better your performance. The fact that errors are squared before they're summed means that it is better to have many small errors than a few large ones.

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Practice rounds

In order to familiarize you with the game and the mechanics of using the machine, you will have a practice session of 6 time periods .

At the end of period 7, your cumulative profits are then "reset" so that they are equal to the profits you made during that period. Your forecasting score is reset in the same manner.

However, your inventory and your production pipeline are not reset!!! Note that, due to the lag, your decisions will start to be important beginning in period 4.

Making your decisions

When you are ready to make your decisions, click the mouse on the text fields in the lower right-hand corner of your screen to bring the cursor into the appropriate field. (Hitting the <tab> or <enter> or <return> key does the same thing.) Type in your decision and, when you're finished, click on the "OK" button to execute them. You will then be asked to confirm each decision.

After all participants have entered their decisions, the computers calculate the results for that period and advance to the next period.

Previous-period report

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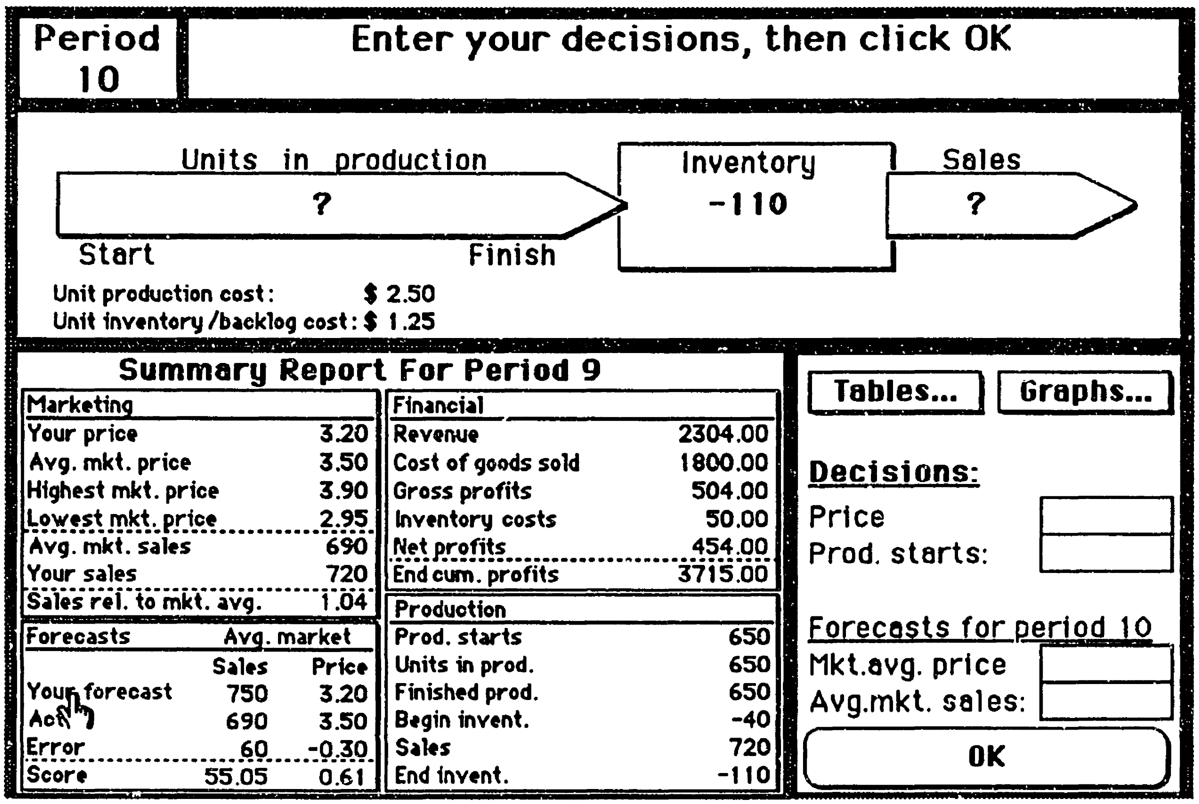

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Each period, you have two decisions to make: what price to charge for your product, and how much to produce.

Production, inventory, and backlog

If you sell less than what you produce, the rest piles up in your inventory. On the other hand, if you sell more than you have available, the excess sales accumulate in an order backlog (i.e. a negative inventory).

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Your gross profits are your revenues less cost of goods sold. Your net profits are gross profits less inventory/backlog costs.

Note that both the unit production cost and the unit inventory/backlog cost are constant (and will not change at any time during the game).

Price

You are free to choose any price you want for your product. The only limitation is that price must be greater than zero. If you charge a higher price, you will make more profits per unit but also sell fewer units, as described below.

Sales

Your sales are influenced by three factors:

- Your price, relative to the average market price that period will have a strong effect on how much you sell. Since the products of your competitors are fairly similar to your product, customers are quite sensitive to price. Thus, if you charge a higher price than the market average, you will probably sell quite a bit less than the average firm does. Conversely, if you charge a lower price, you will sell more than the average.

- The average market price charged by all firms in the market will affect how much the average firm sells, but this effect is weaker than the effect of how much you charge, relative to the other firms. Thus, if all firms increase their price, they will all sell somewhat less, but if only one firm raises its price, that firm will sell a lot less.

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In order to familiarize you with the game and the mechanics of using the machine, you will have a <u>practice session of 3 time periods</u> .

At the end of period 4, your <u>cumulative profits are then "reset"</u> so that they are equal to the profits you made during that period. Your <u>forecasting score is reset</u> in the same manner. However, your <u>inventory is not reset!!!</u>

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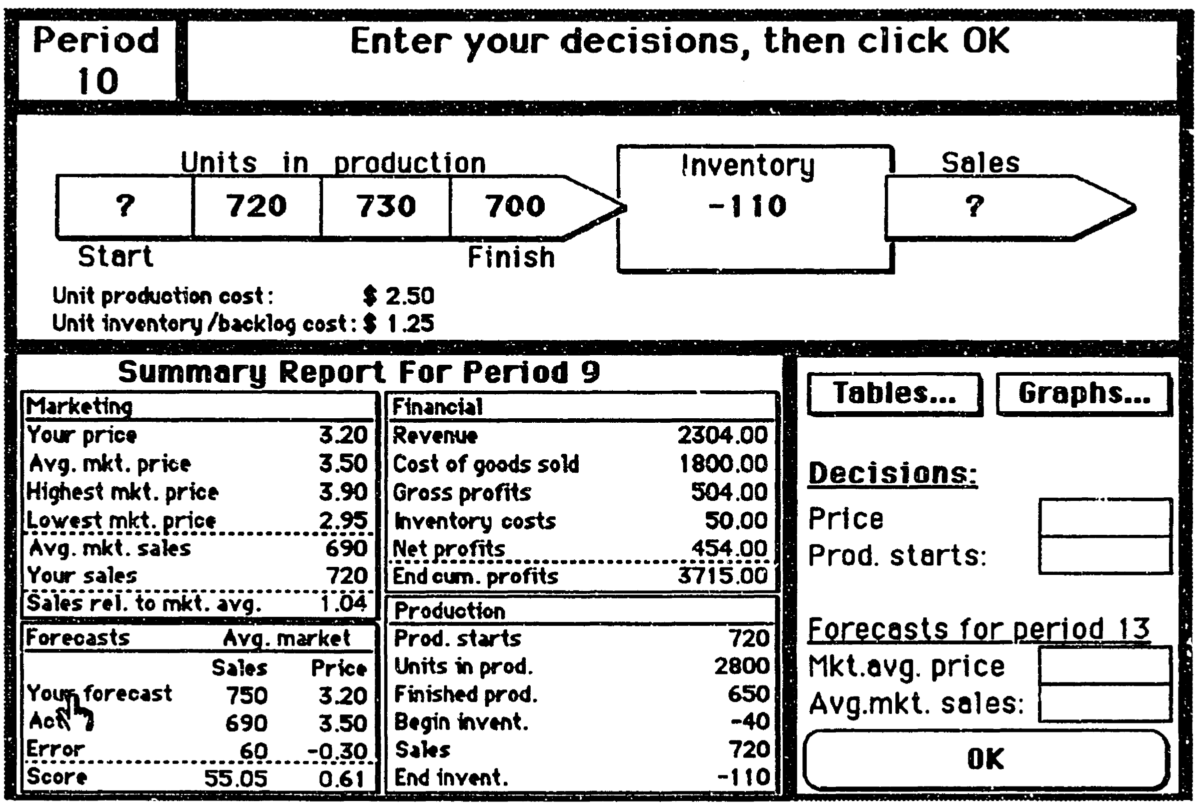

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product, which for experimental reasons has been kept abstract. The products of different firms are quite alike but not identical. You can think of the product as a tire, a softdrink, or a men's shirt.

Each period, you have two decisions to make: what price to charge for your product, and how much production to initiate.

Production, inventory, and backlog

There is a three-period production lag between the time production is initiated and the time it becomes "finished", available for sale (or storage in inventory).

If you sell less than the finished production, the rest piles up in your inventory. On the other hand, if you sell more than you have available, the excess sales accumulate in an order backlog (i.e. a negative inventory).

Theoretically, there is no limit on how much you produce, but you cannot cancel initiated production or destroy your inventory, i.e. you cannot produce negative amounts.

Profits and costs

Your profits each period is your revenue (sales x price) less your costs. Costs consist of production cost, incurred at the time of sale (called "cost of goods sold"), and inventory/backlog costs.

The "cost of goods sold" is simply proportional to sales, i.e. the unit production cost is constant.

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- Multiplier effect You can think of the market you are in as part of a regional economy where, if every firm has a high level of production, more people will be employed, which in turn will influence incomes and thus demand for all goods in the region, including your product. Conversely, if all firms produce less, people are laid off, and demand for your product will fall somewhat.

  Thus, the demand for your product (and for the product of other firms) depends partly on the total amount of units in production. (Units in production includes both production started this period plus the amount in the "pipeline" started during the previous three periods).

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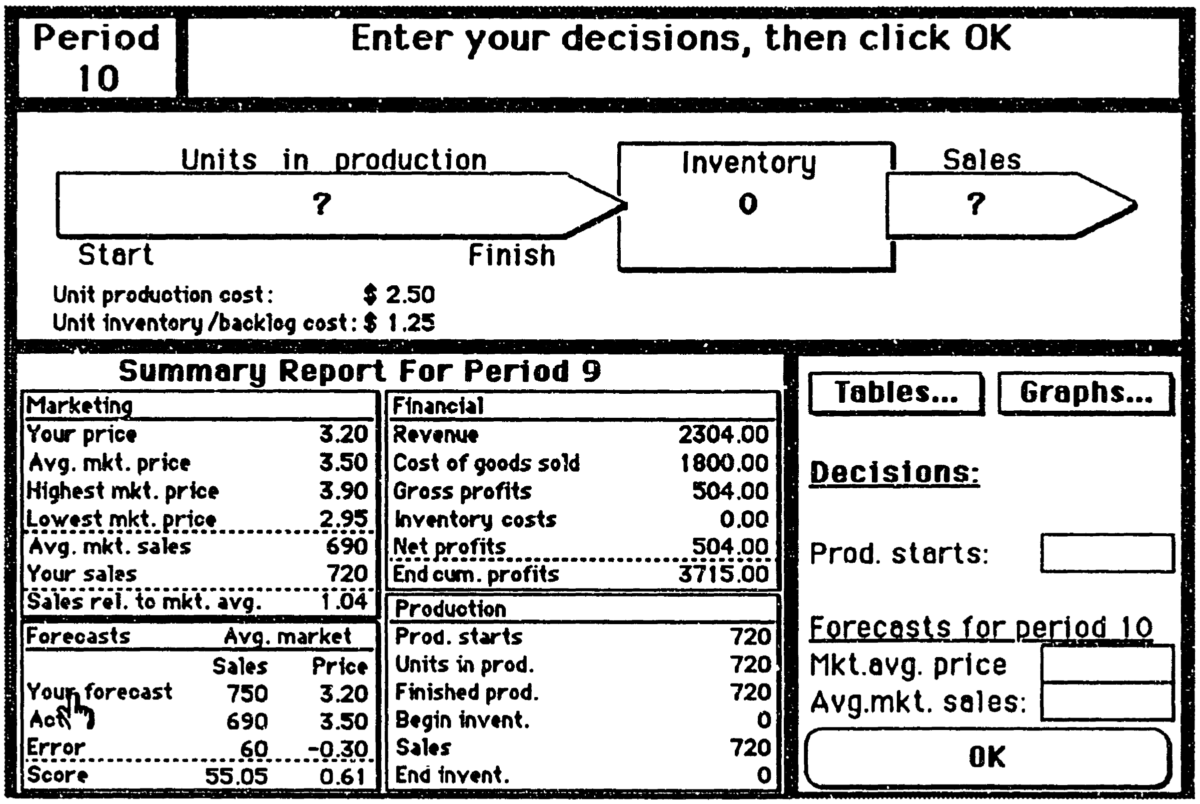

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Production, inventory, and backlog

Each period, you must decide how much production to initiate.

All that you produce is brought to market, i.e. inventories or backlogs are not allowed in this version of the game.

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Profits and costs

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The price you receive for your product depends on three factors:

- How much you are trying to sell, relative to the amounts supplied by other firms in the market. Since the products of your competitors are fairly similar to your product, this effect is relatively small. Thus, if you supply more than the market average, you will probably get a somewhat lower price, but not a dramatically lower price. Conversely, if you supply less than the market average, your price will be higher, but probably not a lot higher.

- The average amount supplied by all firms in the market will have a stronger effect on the average price received. Thus, if all firms increase their output, they will all receive a significantly lower price. Conversely, if everyone lowers their output, the average price will increase.

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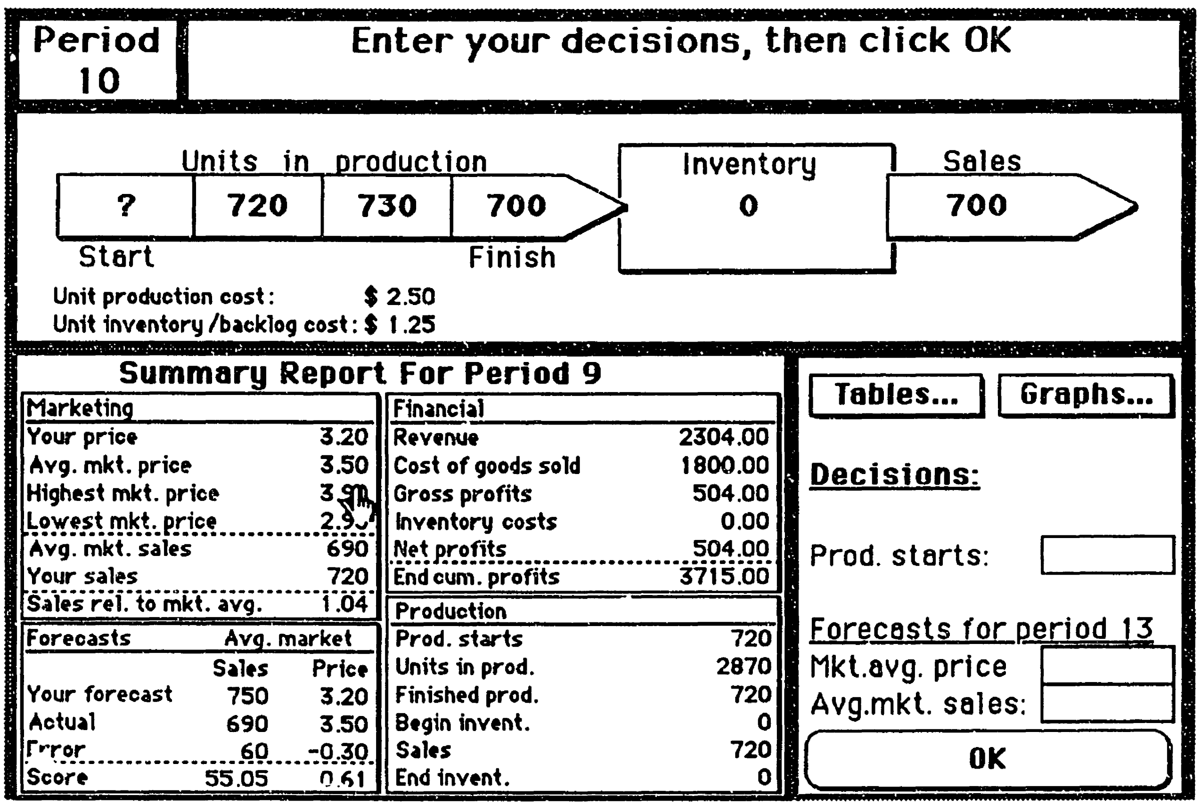

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Post-game questionnaire, simple conditions

Question 1

It was mentioned in the introduction to the game that overall market "demand" was influenced by two factors:

- Prices (both your own prices and the aggregate average price)
- "External factors" such as weather, macro-economic trends, etc.

If prices were constant, or if you set your own price, a higher/lower "demand" was expressed as more/less sales. If you did not set prices but prices were found by the computer, a higher/lower "demand" was expressed in higher/lower prices.

Although you were not told what the pattern of the "external factors" was, we would like you to sketch, on the graph on the next page, a curve over time of your best guess at what you think the pattern of "outside factors" was. Remember that the factors work to raise or lower demand, all other things equal. Remember that demand is also influenced by prices.

The value 0 (zero), indicates a neutral influence or, if you will, the "average" influence of the external factors. Points where the curve goes above 0 would indicate times that the factors worked to raise demand, all other things equal, above what it would otherwise have been. Conversely, points below 0 indicate that demand, again all other things equal would fall below "normal".

Don't worry too much about the exact numerical values or timing--just give a rough picture of the pattern of the factors. If you can, try to indicate an approximate magnitude of the factors by scaling the graph. The scale is expressed as a percentage of normal. For instance, in case prices were fixed or set by you, the factors might have worked to raise sales approximately 10% above normal. In the price-clearing case (where prices were found by the computer), a value of 10% would indicate that prices were 10% above where they would have been if no such factors were present.

___ Approximate maximum (in %)

___ Approximate minimum (in %) Time period

Question 2

Tell us, in your own words, how ' your production decision. Explain what happened in the co game. You might try to address the following questions in your explanation:

- What were you "trying to do"--i.e. what was your major strategy with regard to production?
- Did you have a specific procedure for arriving at a number? What were the factors of variables you primarily looked at or thought were the most relevant?
- How long did it take you to "settle" on your strategy?
- Were you surprised by the results? How?
- Were your consistent in your strategy, or did it change in the course of the game. How? When?

Question 3

Ignore this question if you did not set your own price

Now tell us how you made your price decision. Explain what happened in the course of the game. You might try to address the following questions in your explanation:

- What were you "trying to do"--i.e. what was your major strategy with regard to price?

- Did you have a specific procedure for arriving at a number? What were the factors of variables you primarily looked at or thought were the most relevant?
- How long did it take you to "settle" on your strategy?
- Were you surprised by the results? How?
- Were your consistent in your strategy, or did it change in the course of the game. How? When?

Question 4

Ignore this question if you did not set your own price

Which do you feel was the "most important" decision--price or production? Explain.

Question 5

Although you do not know specifically how well other participants did in the game, you do know whether your profits were above or below the market average. What do you think you did "right" or "wrong" which led to your relative performance?

Post-game questionnaire, complex conditions

Question 1

It was mentioned in the introduction to the game that overall market "demand" was influenced by three factors:

- Prices (both your own prices and the aggregate average price)
- Overall level of production ("units in production") in the market (the multiplier effect).]
- "External factors" such as weather, macro-economic trends, etc.

If prices were constant, or if you set your own price, a higher/lower "demand" was expressed as more/less sales. If you did not set prices but prices were found by the computer, a higher/lower "demand" was expressed in higher/lower prices.

Although you were not told what the pattern of the "external factors" was, we would like you to sketch, on the graph on the next page, a curve over time of your best guess at what you think the pattern of "outside factors" was. Remember that the factors work to raise or lower demand, all other things equal. Remember that demand is also influenced by prices and by the multiplier effect.

The value 0 (zero), indicates a neutral influence or, if you will, the "average" influence of the external factors. Points where the curve goes above 0 would indicate times that the factors worked to raise demand, all other things equal, above what it would otherwise have been. Conversely, points below 0 indicate that demand, again all other things equal would fall below "normal".

Don't worry too much about the exact numerical values or timing--just give a rough picture of the pattern of the factors. If you can, try to indicate an approximate magnitude of the factors by scaling the graph. The scale is expressed as a percentage of normal. For instance, in case prices were fixed or set by you, the factors might have worked to raise sales approximately 10% above normal. In the price-clearing case (where prices were found by the computer), a value of 10% would indicate that prices were 10% above where they would have been if no such factors were present.

___ Approximate maximum (in %)

___ Approximate minimum (in %) Time period

<u>Question 2</u>

Tell us, in your own words, how you made your production decision. Explain what happened in the course of the game. You might try to address the following questions in your explanation:

- What were you "trying to do"--i.e. what was your major strategy with regard to production?
- Did you have a specific procedure for arriving at a number? What were the factors of variables you primarily looked at or thought were the most relevant?
- How long did it take you to "settle" on your strategy?
- Were you surprised by the results? How?
- Were your consistent in your strategy, or did it change in the course of the game. How? When?

<u>Question 3</u>

<u>Ignore this question if you did not set your own price</u>

Now tell us how you made your price decision. Explain what happened in the course of the game. You might try to address the following questions in your explanation:

- What were you "trying to do"--i.e. what was your major strategy with regard to price?

- Did you have a specific procedure for arriving at a number? What were the factors of variables you primarily looked at or thought were the most relevant?
- How long did it take you to "settle" on your strategy?
- Were you surprised by the results? How?
- Were your consistent in your strategy, or did it change in the course of the game. How? When?

Question 4

Ignore this question if you did not set your own price

Which do you feel was the "most important" decision--price or production? Explain.

Question 5

Although you do not know specifically how well other participants did in the game, you do know whether your profits were above or below the market average. What do you think you did "right" or "wrong" which led to your relative performance?