Supplementary Appendix for "Are CDS Auctions Biased and Inefficient?"

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Du and Zhu (2016) characterize an equilibrium of CDS auctions and pre-auction CDS trading. In that equilibrium, the open interest is to buy if more than a half of traders have the high valuation for the bonds (the high state), and the open interest is to sell if more than a half of traders have the low valuation for the bonds (the low state). For completeness, this supplementary appendix characterizes two one-sided equilibria. Specifically, in one equilibrium, the open interest is always to buy, regardless of the state; and in the other, the open interest is always to sell, regardless of the state.

Intuitively, these one-sided equilibria may arise due to coordination. For example, if low-value traders only participate in the second stage (using limit orders) and high-value traders participate in both stages, the open interest is always to buy. And conditional on always having a buy open interest, it is self-fulfilling that:

(i) high-value traders always submit buy market orders in the first stage, anticipating the low-state price (their second-stage limit orders are executed in the high state, selling back some of their first-stage orders);

(ii) low-value traders always submit limit sell orders in the second stage (there is no benefit for them to use the first stage).

Clearly, high-value CDS sellers and high-value traders with zero CDS positions are prevented from participating in the first stage (because of their CDS position) and the second stage

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(because of an open interest to buy). Thus, this equilibrium cannot yield the competitive price or competitive allocation.

A symmetric self-fulfilling logic applies to the equilibrium that always has an open interest to sell. Clearly, both one-sided equilibria are counterfactual, since in practice some CDS auctions have buy open interest and some have sell open interest.

We note that the two-sided equilibrium in the main text and the two one-sided equilibria here fully characterize all of the equilibria in pure-strategy. Under pure strategies, there are only four possible combinations of (state, open interest):

- (high state, open interest to buy) + (low state, open interest to sell)
- (high state, open interest to buy) + (low state, open interest to buy)
- (high state, open interest to sell) + (low state, open interest to sell)
- (high state, open interest to sell) + (low state, open interest to buy)

The equilibrium characterized in the paper is the first item. The one-sided equilibria in this supplementary appendix are the second and third items. The fourth item cannot happen in equilibrium because the buy-minus-sell interest in the high state is larger than that in the low state, so if the low state generates a buy open interest then so must the high state.

Characterizing One-sided Equilibria

We now proceed with the formal analysis. The model are notations are identical to those in Du and Zhu (2016).

In the second stage, each trader in equilibrium submits the demand/supply schedule:

$$x_i(p) = \begin{cases} \max\left(-r_i + \frac{v_i - p}{\lambda}, 0\right) & \text{if } R < 0 \\ \min\left(-r_i + \frac{v_i - p}{\lambda}, 0\right) & \text{if } R > 0 \end{cases}. \tag{1}$$

Proposition 1. *Let p_H^* and p_L^* be uniquely defined by:*

$$(1-m)\frac{v_L - p_H^*}{\lambda} + \frac{m}{2}\int_{Q_i \geq 0} \min\left(Q_i, \frac{v_H - p_H^*}{\lambda}\right) dG(Q_i) = 0, \tag{2}$$

$$m\frac{v_L - p_L^*}{\lambda} + \frac{1-m}{2}\int_{Q_i \geq 0} \min\left(Q_i, \frac{v_H - p_L^*}{\lambda}\right) dG(Q_i) = 0. \tag{3}$$

We have the following equilibrium:

- *In the first stage, a high-value trader submits:*

$$r_i = \begin{cases} \min\left(-Q_i, \frac{v_H - p_L^*}{\lambda}\right) & \text{if } v_i = v_H, Q_i < 0 \\ 0 & \text{if } v_i = v_H, Q_i \geq 0 \end{cases}, \tag{4}$$

  and a low-value trader submits $r_i = 0$.

- *In the second stage, every trader i submits the demand schedule in Equation (1). The open interest is to buy in both high and low states. The final prices in the high and low states are, respectively, p_H^* and p_L^*. We have $p_H^* > p_L^*$.*

- *We have $p_H^* < p_H^c$ and $p_L^* < p_L^c$.*

Proof. We conjecture that in the high state $R > 0$ with final price p_H^*, and in the low state $R > 0$ with final price p_L^*; moreover $p_H^* > p_L^*$. We derive the equilibrium based on this conjecture, and then verify this conjecture.

We first show that the first-stage strategy in (7) is optimal, which has four cases.

1. If trader i has a high value for the bond and is a CDS seller, then he wants to buy bonds but is constrained to buy at most $-Q_i$ unit in the first stage. Suppose trader i submits a physical request of $r_i \geq 0$ in the first stage. If $r_i \in [(v_H - p_H^*)/\lambda, (v_H - p_L^*)/\lambda]$, then in the second stage trader i sells $x_i = -r_i + (v_H - p_H^*)/\lambda < 0$ units in the high state and sells $x_i = \min(0, -r_i + (v_H - p_L^*)/\lambda) = 0$ in the low state, thus getting a total allocation of $(v_H - p_H^*)/\lambda$ in the high state and r_i in the low state. Thus, the optimal physical request is $r_i = \min((v_H - p_L^*)/\lambda, -Q_i)$ under the constraint that $r_i \leq -Q_i$.

2. If trader i has a high value for the bond and has a positive or zero CDS position, then he wants to buy bonds but is constrained to sell in the first stage. By our conjecture, the open interest is always to buy, so trader i can only sell in the second stage as well. Thus the optimal physical request is $r_i = 0$, and trader i does not trader in either stage.

3. If trader i has a low value for the bond and is a CDS buyer, then he wants to sell bonds but is constrained to sell at most $-Q_i$ unit the first stage. Since he can sell in the second stage, trader i is indifferent between every $r_i \in [-(v_L - p_L^*)/\lambda, 0]$, as they all lead to a total allocation of $(v_L - p_L^*)/\lambda$ in the low state and $(v_L - p_H^*)/\lambda$ in the high state, which are his optimal allocations.

4. If trader i has a low value for the bond and has a negative or zero CDS position, then he wants to sell bonds but is constrained to buy in the first stage. Thus, the optimal physical request is $r_i = 0$, and trader i sells in the second stage, getting a total allocation of $(v_L - p_L^*)/\lambda$ in the low state and $(v_L - p_H^*)/\lambda$ in the high state, similar to Case 3.

Aggregating the allocations across the two stages we have the following market-clearing conditions in the high and low states:

$$(1-m)\frac{v_L - p_H^*}{\lambda} + \frac{m}{2}\int_{Q_i \geq 0} \min\left(Q_i, \frac{v_H - p_H^*}{\lambda}\right) dG(Q_i) = 0, \tag{5}$$

$$m\frac{v_L - p_L^*}{\lambda} + \frac{1-m}{2}\int_{Q_i \geq 0} \min\left(Q_i, \frac{v_H - p_L^*}{\lambda}\right) dG(Q_i) = 0. \tag{6}$$

The left-hand side of Equation (5) is clearly decreasing in p_H^*, is positive when $p_H^* = v_L$, and is negative when $p_H^* = v_H$. Thus, there exists a unique $p_H^* \in (v_L, v_H)$ that satisfies Equation (5). The left-hand side of Equation (5), for every value of p_H^*, is strictly dominated by $(1-m)(v_L - p_H^*)/\lambda + m(v_H - p_H^*)/\lambda$, which is equal to zero when $p_H^* = p_H^c$. Thus the solution p_H^* to Equation (5) is less than p_H^c.

Likewise, Equation (6) has a unique solution p_L^* such that $p_L^* < p_L^c$. Moreover, the solutions p_H^* and p_L^* clearly satisfy $p_H^* > p_L^*$.

Finally, the first-stage strategy in (7) implies that $R > 0$ in both high and low states, as we have conjectured.

This completes the derivation and verification of the equilibrium.

Finally, we consider the alternative conjecture of $R > 0$ in the high state with final price p_H^*, $R > 0$ in the low state with final price p_L^*, and $p_H^* \leq p_L^*$. Given this conjecture, the high value traders submit the physical request anticipating the low price p_H^*,

$$r_i = \begin{cases} \min\left(-Q_i, \frac{v_H - p_H^*}{\lambda}\right) & \text{if } v_i = v_H, Q_i < 0 \\ 0 & \text{if } v_i = v_H, Q_i \geq 0 \end{cases}, \tag{7}$$

and sells some requests back through second-stage limit-orders if $r_i \geq \frac{v_H - p_L^*}{\lambda}$ and the state turns out to be low. Thus a high value CDS seller obtains a total bond allocation of $\min\left(-Q_i, \frac{v_H - p_H^*}{\lambda}\right)$ in the high state and $\min\left(-Q_i, \frac{v_H - p_L^*}{\lambda}\right)$ in the low state, which is the same allocation as before. Likewise the allocations of other traders are also the same as before. Thus Equations (5) and (6) still define the market-clearing prices p_H^* and p_L^* in the high and low states, respectively. Since Equations (5) and (6) imply that $p_H^* > p_L^*$, the alternative conjecture cannot be an equilibrium.

$\square$

Proposition 2. *Let p_H^* and p_L^* be uniquely defined by:*

$$m\frac{v_H - p_H^*}{\lambda} + \frac{1-m}{2}\int_{Q_i \geq 0} \max\left(-Q_i, \frac{v_L - p_H^*}{\lambda}\right) dG(Q_i) = 0, \tag{8}$$

$$(1-m)\frac{v_H - p_L^*}{\lambda} + \frac{m}{2}\int_{Q_i \geq 0} \min\left(-Q_i, \frac{v_L - p_L^*}{\lambda}\right) dG(Q_i) = 0. \tag{9}$$

We have the following equilibrium:

- *In the first stage, a low-value trader submits:*

$$r_i = \begin{cases} \max\left(-Q_i, \frac{v_L - p_H^*}{\lambda}\right) & \text{if } v_i = v_L, Q_i > 0 \\ 0 & \text{if } v_i = v_L, Q_i \leq 0 \end{cases}, \tag{10}$$

  and a high-value trader submits $r_i = 0$.

- *In the second stage, every trader i submits the demand schedule in Equation (1). The open interest is to sell in both high and low states. The final prices in the high and low states are, respectively, p_H^* and p_L^*. We have $p_H^* > p_L^*$.*

- *We have $p_H^* > p_H^c$ and $p_L^* > p_L^c$.*

The proof of Proposition 2 is analogous to that of Proposition 1 and is omitted.

References

Du, S., and H. Zhu, 2016, “Are CDS Auctions Biased and Inefficient?,” Working paper.