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# Property Rights and the Efficiency of Bargaining\*

Ilya Segal and Michael D. Whinston<sup>†</sup>

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## Abstract

Property rights specify an initial default position from which agents may subsequently bargain to determine their ultimate allocation. The economics literature has largely focused on how property rights affect ex ante investments, under the assumption that bargaining always results in ex post efficient outcomes. In contrast, we examine how property rights affect the efficiency of bargaining and the final allocations that result. We establish a wide class of economic settings and property rights in which efficient bargaining is impossible. For these environments, we examine the optimal allocation of property rights, including simple property rights, liability rules, and dual-chooser rules.

## 1 Introduction

Property rights specify an initial default position from which agents may subsequently bargain to determine their ultimate allocation. Following the

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<sup>†</sup>Segal: Department of Economics, Stanford University, [isegal@stanford.edu](mailto:isegal@stanford.edu); Whinston: Department of Economics and Sloan School of Management, Massachusetts Institute of Technology, [whinston@mit.edu](mailto:whinston@mit.edu).

seminal article of Grossman and Hart (1986), the economics literature discussing the optimal allocation of property rights has largely focused on how they affect ex ante investments, under the assumption that bargaining always results in ex post efficient outcomes. In this paper, we instead examine how property rights affect the efficiency of bargaining and the final allocations that result.<sup>1</sup>

According to the “Coase Theorem” (Coase 1960), in the absence of “transaction costs,” parties will reach Pareto efficient agreements regardless of initial property rights. We instead examine settings in which this may not happen due to transaction costs associated with asymmetric information. That property rights may matter for the efficiency of bargaining can be seen by comparing Myerson and Satterthwaite’s (1983) conclusion that private information must generate inefficiency in bargaining between a buyer and a seller, with Cramton, Gibbons, and Klemperer’s (1987) demonstration that efficient bargaining mechanisms do exist for more evenly distributed (or randomized) property rights. Here we examine more broadly the nature of optimal property rights in such settings.

In addition to simple property rights of the sort considered by Myerson and Satterthwaite (1983) and Cramton et al. (1987), the legal literature has considered other forms of property rights. Calabresi and Melamud (1972) first highlighted the distinction between “property rules,” which correspond to the simple property rights of the economics literature, and “liability rules,” in which an agent may harm another agent (e.g., by polluting) but must make a damage payment to the victim. These liability rules may equivalently be thought of as an option-to-own.<sup>2</sup> Calabresi and Melamud (1972) considered such liability rules to be desirable only when bargaining is impractical (in which case they can make the final allocation responsive to values), but subsequent work [Ayres and Talley (1994), Kaplow and Shavell (1995-6, 1996), Ayres (2005)] has suggested the possibility that liability rules may also be desirable when bargaining is possible but imperfect. Ayres (2005) also considered in a two-agent setting “dual chooser” rules in which both agents can exercise options. In general, liability rules and dual-chooser rules may both be viewed as particular forms of *property rights mechanisms*, in which the default outcome depends on messages sent by the various agents.

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<sup>1</sup>Matouschek (2004) studies this question as well; we discuss the relation to our paper below.

<sup>2</sup>Demski and Sappington (1991) and Noldeke and Schmidt (1995) consider the use of option-to-own contracts to induce efficient ex ante investments.

This paper advances the literature in two ways. First, we establish a wide class of economic settings and property rights (including both simple property rights and liability rules) in which efficient bargaining is impossible.<sup>3</sup> Second, for these environments in which inefficiency is unavoidable, we examine the optimal allocation of property rights, including simple property rights, liability rules, and dual-chooser rules. In addition to its implications for the allocation of formal property rights, our analysis can also be applied to the allocation of decision rights within firms.

Our inefficiency result unifies a number of results in the earlier literature (Myerson and Satterthwaite (1983), Mailath and Postlewaite (1990), Williams (1999), Figueroa and Skreta (2008), Che (2006)). In contrast to the earlier literature, our approach to establishing inefficiency does not require performing any computations. Instead, it requires only the verification of two simple conditions: (i) existence of “adverse efficient-opt-out types” and (ii) non-emptiness of the core (actually, non-emptiness of a larger set that we call the “marginal core”) and its multi-valuedness with a positive probability.<sup>4</sup>

We define an “efficient-opt-out type” as a type whose non-participation is consistent with efficiency (for any types of the other agents). In addition, for settings that involve externalities, such as liability rules, we define an “adverse type” as a type who, when he does not participate and behaves noncooperatively (e.g., chooses optimally whether to damage others under a liability rule), minimizes the total expected surplus available to the other agents. (In settings with simple property rights, in which externalities are absent, any type is trivially an adverse type.) Our inefficiency result applies when each agent has a type that is simultaneously an efficient-opt-out type and an adverse type. This assumption is clearly restrictive – for example, it is not satisfied in the presence of intermediate (or randomized) property rights of the kind considered by Cramton et al. (1987) and Segal and Whinston (2011). Nevertheless, we show that this assumption is satisfied in a number of settings involving simple property rights, liability rules, and dual-chooser rules. (We also allow this assumption to hold in an asymptotic form: e.g., a type may become an “almost” adverse efficient-opt-out type as the type goes to  $+\infty$ .)

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<sup>3</sup>In Segal and Whinston (2011) we instead establish sufficient conditions for first-best efficiency to be achieved.

<sup>4</sup>Precursors to this approach can be found in Makowski and Ostroy (1989) and Segal and Whinston (2011).

In contrast, the non-emptiness and multi-valuedness of the core is a typical feature of economic settings. For example, if, under an appropriate definition of “goods,” a price equilibrium exists (e.g., a Walrasian equilibrium, or a Lindahl equilibrium), then it will be in the core, and “generically” the core will be multi-valued (except for some limiting “competitive” cases with a large number of agents, where the core may converge to a unique Walrasian equilibrium).

Having identified a class of settings in which achieving efficiency is impossible, we then turn to an analysis of the optimal allocation of property rights in those cases. In doing so, we take a mechanism design approach to bargaining, asking what property rights would be optimal if bargaining takes as efficient a form as possible given the allocation of property rights.

We use two different measures of efficiency to identify optimal property rights. In the first, we assume that there is an outside agency who will subsidize the bargaining process in order to achieve efficiency and we examine the effect of property rights on the expected subsidy that is required. One corollary of our impossibility analysis is a simple formula for this expected subsidy. The formula allows us to compare the subsidies required by the various property rights that satisfy (i) and (ii). Among such property rights, we can identify those that minimize the intermediary’s expected subsidy.

One interesting benchmark for comparison is the property rights that would maximize the expected surplus were bargaining impossible. With two agents and simple property rights that induce efficient-opt-out types, we show that the intermediary’s expected first-best subsidy equals the expected bargaining surplus, and therefore minimizing this expected subsidy is equivalent to maximizing the expected status quo surplus. For example, in the buyer-seller model of Myerson and Satterthwaite (1983), if we can choose who should initially own the object, it is optimal to give it to the agent with the higher expected value for it. We also identify the optimal option-to-own (liability rule) in this same setting, and show that it is exactly the same as the optimal option-to-own when bargaining is impossible, involving an option price (damage payment) that equals the expected value (harm) of the non-choosing agent (victim). As in the case without bargaining, the optimal option-to-own is strictly better than the best simple property right, but fails to achieve the first best when there is uncertainty about the value of the non-choosing agent.

However, the equivalence between what is best for minimizing the expected first-best bargaining subsidy and what is best absent bargaining gen-

erally breaks down when there are more than two agents: in such cases, we instead want to raise the values of coalitions including all but one agent (reducing the “hold-out power” of individual agents). We illustrate the difference in two examples, one concerning the optimal ownership of spectrum, and the other examining the optimal liability rule for pollution when there are many victims.

Evaluating property rights by their effects on the expected subsidy required for first-best bargaining may not be the right thing to do, since in most cases a benevolent intermediary willing to subsidize bargaining is not available. Our second efficiency measure is instead the maximal (“second-best”) expected surplus that can be achieved in budget-balanced bargaining. Analysis of the second-best problem is complicated by the fact that the optimal allocation rule depends on the identity of the agents least willing to participate (the “critical types”), which in turn depends on the allocation rule. Unfortunately, we are unable to solve for the second-best bargaining procedure at a comparable level of generality to our first-best subsidy calculation. For this reason, we focus on the case of just two agents.

When divided property rights are not possible, we know from Myerson and Satterthwaite (1983) that efficiency is impossible when the budget must be balanced. In such cases, however, use of a liability rule (option-to-own) may offer an improvement over (and can be no worse than) what can be achieved with the best undivided property right.

To identify the optimal liability rule, we begin by characterizing the second-best bargaining mechanism for a given liability rule. Doing so we identify the critical types for each agent, and the optimal allocation rule.<sup>5</sup> We then use this characterization to study the dependence of the maximal expected surplus on the option price.

We first consider the case in which both agents’ values are distributed uniformly. We find that the second-best expected surplus is maximized by setting the option price equal to the expected value of the non-choosing agent (the “victim”), which is  $1/2$  under our normalization. Thus, in the uniform case the optimal option price under second-best bargaining proves to be the same as the price that minimizes the expected first-best subsidy, which is in turn the same as the optimal price in the absence of bargaining.

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<sup>5</sup>Independently, Loertscher and Wasser (2014) analyze the second-best bargaining mechanism for simple intermediate property rights which do not permit first-best efficient bargaining. The two settings share a number of similar technical difficulties to be overcome.

At the same time, we find a significant difference in how the second-best expected surplus and the expected first-best subsidy vary with the option price. Namely, while in the uniform example the expected first-best subsidy is always lower the closer the price is to  $1/2$ , the second-best expected surplus does not increase monotonically with such changes. Instead, we find that setting the price close to 0 or to 1 yields a lower expected second-best surplus than setting it at exactly 0 or 1 (which corresponds to giving one of the agents a simple property right to the object). In fact, we show that the same conclusion extends to *all* distributions of the two agents' valuations (not just uniform). Thus, contrary to the intuition one might take from the results of Cramton et. al (1987), less extreme property rights (in the form of an option price that sometimes leads to exercise of the option) may be worse than extreme ones.

Finally, we explore cases in which the two agents' valuations are drawn from different distributions and show that the optimal option price is not generally equal to the victim's expected harm. Nonetheless, an intriguing fact is that in all of the cases we study the optimal option price is very close to the victim's expected harm, and the loss from instead setting the option price equal to that expected harm is small.

In addition to Myerson and Satterthwaite (1983) and Cramton, Gibbons, and Klemperer (1987), a number of other papers examine the effect of property rights on bargaining efficiency. Most, like our previous paper [Segal and Whinston (2011)] establish conditions under which the first best is achievable [for additional references, see our (2011) working paper and Segal and Whinston (2013)]. Others established impossibility of efficient bargaining in some settings [see Segal and Whinston (2013), Makowski and Ostroy (1989), and Matsushima (2012)]. Matouschek (2004) was the first paper to consider second-best optimal property rights under asymmetric information bargaining. He studied a model in which asset ownership  $x$  is set irrevocably ex ante, and bargaining over other decisions  $q$  occurs ex post after agents' types are determined. In contrast to much of our analysis, bargaining is not allowed to redistribute the initial property rights. He finds that, depending on the parameters, the optimal property rights  $x$  will either maximize the total surplus at the disagreement point (as if no renegotiation were possible) or minimize it. Mylovanov and Troger (2012) analyzes a two-agent setting like ours, but instead uses a specific bargaining protocol in which one agent has the power to make a take-it-or-leave-it offer to the other agent. Finally, in unpublished notes, Che (2006) examines the optimal option-to-own for

minimizing the expected first-best subsidy.

The paper is organized as follows: In Section 2 we describe our basic model. Section 3 derives our inefficiency result. In Section 4, we analyze the optimal property rights for minimizing the first-best subsidy. Section 5 analyzes optimal second-best property rights. Section 6 extends our analysis to consider dual-chooser rules. Finally, Section 7 concludes.

## 2 Set-Up

We consider a general model with  $N$  agents, indexed by  $i = 1, \dots, N$ , who bargain over a nonmonetary decision  $x \in X$ , as well as a vector  $t \in \mathbb{R}^N$  of monetary transfers. Each agent  $i$  privately observes a type  $\theta_i \in \Theta_i$ , and his resulting payoff is  $v_i(x, \theta_i) + t_i$ . We assume that the types  $(\tilde{\theta}_1, \dots, \tilde{\theta}_N) \in \Theta_1 \times \dots \times \Theta_N$  are independent random variables.

We will be interested in examining what is achievable given some initial property rights when the agents engage in the best possible bargaining procedure after their types are realized. To this end, we take a mechanism design approach to bargaining. Appealing to the Revelation Principle, we focus on direct revelation mechanisms  $\langle \chi, \tau \rangle$ , where  $\chi : \Theta \rightarrow X$  is the decision rule, and  $\tau : \Theta \rightarrow \mathbb{R}^N$  is the transfer rule. In particular, we will be interested in implementing an *efficient* decision rule  $\chi^*$ , which solves:

$$\chi^*(\theta) \in \arg \max_{x \in X} \sum_i v_i(x, \theta_i) \text{ for all } \theta \in \Theta.$$

We let  $V(\theta) \equiv \sum_i v_i(\chi^*(\theta), \theta_i)$  be the maximum total surplus achievable in state  $\theta$ .

When considering direct revelation mechanisms that correspond to bargaining mechanisms, we restrict them to satisfy *budget balance*:

$$\sum_i \tau_i(\theta) = 0 \text{ for all } \theta \in \Theta.$$

and (*Bayesian*) *Incentive Compatibility*:

$$\begin{aligned} & \mathbb{E}[v_i(\chi(\theta_i, \tilde{\theta}_{-i}), \theta_i) + \tau_i(\theta_i, \tilde{\theta}_{-i})] \\ & \geq \mathbb{E}[v_i(\chi(\theta'_i, \tilde{\theta}_{-i}), \theta_i) + \tau_i(\theta'_i, \tilde{\theta}_{-i})] \text{ for all } i, \theta_i, \theta'_i \in \Theta_i \end{aligned}$$

Next we consider participation constraints. For this purpose, we need to describe what outcome each agent  $i$  expects when he refuses to participate

in the bargaining mechanism. In general, this outcome will depend on the property rights mechanism in place and, in general, on the types of the other agents. For example, the other agents may make some noncooperative choices under a liability rule, and these choices may depend on their types. Alternatively, the other agents may be able to bargain with each other over some parts of the outcome without the participation of agent  $i$ , and this bargaining may have externalities on agent  $i$ . It is also possible that if agent  $i$  refuses to participate, the default will involve a noncooperative game among agents, and the outcome of this game will depend on all the agents' types.

To incorporate all these possibilities, we assume that if agent  $i$  refuses to participate and the state of the world is  $\theta$ , the nonmonetary decision is  $\hat{x}_i(\theta)$ , and agent  $i$  receives a transfer  $\hat{t}_i(\theta)$ . The resulting reservation utility of agent  $i$  is therefore

$$\widehat{V}_i(\theta) \equiv v_i(\hat{x}_i(\theta), \theta_i) + \hat{t}_i(\theta).$$

For example, in the simple special case of simple property rights that induce a fixed status quo  $(\hat{x}, \hat{t}_1, \dots, \hat{t}_N)$  that either cannot be renegotiated at all without all agents' participation or whose renegotiation by a subset of agents does not affect nonparticipating agents (e.g., because renegotiation can only involve exchange of private goods), the reservation utility would take the form  $\widehat{V}_i(\theta) = v_i(\hat{x}, \theta_i) + \hat{t}_i$ . In general, the functions  $\hat{x}_i(\theta)$  and  $\hat{t}_i(\theta)$  depend on both the property rights and assumptions about bargaining, but for much of our analysis we will take these functions as given.

Given these functions and the resulting reservation utility, the (interim) individual rationality constraints of agent  $i$  can be written as

$$\mathbb{E}[v_i(\chi(\theta_i, \tilde{\theta}_{-i}), \theta_i) + \tau_i(\theta_i, \tilde{\theta}_{-i})] \geq \mathbb{E}[\widehat{V}_i(\theta_i, \tilde{\theta}_{-i})] \text{ for all } \theta_i. \quad (1)$$

We will say that a property rights mechanism *permits efficient bargaining* if it induces functions  $\{\hat{x}_i(\cdot), \hat{t}_i(\cdot)\}_{i=1}^N$  such that there exists a budget-balanced, incentive-compatible, and individually rational mechanism implementing an efficient decision rule  $\chi^*(\cdot)$ .

### 3 An Inefficiency Theorem

In this section, we provide a set of sufficient conditions ensuring that efficient bargaining is impossible given a set of initial property rights. Our result will have Myerson and Satterthwaite's (1983) result, and several others, as special cases.



### 3.1 Characterization of Intermediary Profits

It will prove convenient to focus on mechanisms in which, for some vector of types  $(\hat{\theta}_1, \dots, \hat{\theta}_N)$ , payments take the following form:

$$\tau_i(\theta|\hat{\theta}_i) = \sum_{j \neq i} v_j(\chi^*(\theta), \theta_j) - K_i(\hat{\theta}_i) \quad (2)$$

$$\text{where } K_i(\hat{\theta}_i) = \mathbb{E}[V(\hat{\theta}_i, \tilde{\theta}_{-i}) - \widehat{V}_i(\hat{\theta}_i, \tilde{\theta}_{-i})]. \quad (3)$$

Note that these payments describe a Vickrey-Clarke-Groves (“VCG”) mechanism [see Mas-Colell, Whinston, and Green (1995), Chapter 23]. The portion of the payment that depends the agents’ announcements,  $\sum_{j \neq i} v_j(\chi^*(\theta), \theta_j)$ , causes each agent  $i$  to fully internalize his effect on aggregate surplus, thereby inducing him to announce his true type and implementing the efficient allocation rule  $\chi^*(\cdot)$ . The fixed participation fee  $K_i(\hat{\theta}_i)$ , on the other hand, equals type  $\hat{\theta}_i$ ’s expected gain from participating in the mechanism absent the fixed charge, so it causes that type’s IR constraint to hold with equality. If we imagine that there is an intermediary in charge of this trading process, its expected profit with this mechanism, assuming all agents participate, is given by

$$\begin{aligned} \pi(\hat{\theta}) &= \mathbb{E} \left[ \sum_i \tau_i(\tilde{\theta}|\hat{\theta}_i) \right] \\ &= \left( \sum_i \mathbb{E}[V(\hat{\theta}_i, \tilde{\theta}_{-i}) - \widehat{V}_i(\hat{\theta}_i, \tilde{\theta}_{-i})] \right) - (N-1) \mathbb{E}[V(\tilde{\theta})]. \end{aligned} \quad (4)$$

To ensure that all types participate, the participation fee for each agent  $i$  can be at most  $\inf_{\hat{\theta}_i \in \Theta} K_i(\hat{\theta}_i)$ , resulting in an expected profit for the intermediary of

$$\bar{\pi} \equiv \inf_{\hat{\theta} \in \Theta} \pi(\hat{\theta}). \quad (5)$$

If there exists a type  $\hat{\theta}_i$  achieving the infimum, i.e.,

$$\hat{\theta}_i \in \arg \min_{\theta_i \in \Theta_i} \mathbb{E} \left[ V(\theta_i, \tilde{\theta}_{-i}) - \widehat{V}_i(\theta_i, \tilde{\theta}_{-i}) \right],$$

it will be called agent  $i$ ’s *critical type*. This is a type that has the lowest net expected participation surplus in the mechanism.

The sign of the expected profit (5) determines whether property rights permit efficient bargaining:<sup>6</sup>

**Lemma 1** (i) Any property rights mechanism at which  $\bar{\pi} \geq 0$  permits efficient bargaining. (ii) If, moreover, for each agent  $i$ ,  $\Theta_i$  is a smoothly connected subset of a Euclidean space, and  $v_i(x, \theta_i)$  is differentiable in  $\theta_i$  with a bounded gradient on  $X \times \Theta$ , then a property rights mechanism permits efficient bargaining only if  $\bar{\pi} \geq 0$ .

### 3.1.1 Adverse efficient-opt-out Types

For each agent  $i$ , let

$$\widehat{V}_{-i}(\theta) \equiv \sum_{j \neq i} v_j(\hat{x}_i(\theta), \theta_j) - \hat{t}_i(\theta)$$

denote the joint payoff of agents other than  $i$  when agent  $i$  chooses not to participate in the bargaining mechanism. (Observe that we assume there is budget balance in the event of nonparticipation, so that the collective transfer to agents other than  $i$  when agent  $i$  opts out is  $-\hat{t}_i(\theta)$ .) Since  $V(\theta)$  is the maximal achievable surplus in state  $\theta$ , we have:

$$\widehat{V}_i(\theta) + \widehat{V}_{-i}(\theta) = \sum_j v_j(\hat{x}_i(\theta), \theta_j) \leq \sum_j v_j(\chi^*(\theta), \theta_j) = V(\theta) \text{ for all } \theta \in \Theta. \quad (6)$$

**Definition 2** Given a property rights mechanism, type  $\theta_i$  of agent  $i$  is an **efficient-opt-out type** if  $\hat{x}_i(\theta_i, \theta_{-i}) = \chi^*(\theta_i, \theta_{-i})$  for all  $\theta_{-i}$ .

Note that if  $\theta_i$  is an efficient-opt-out type, then  $V(\theta_i, \theta_{-i}) = \widehat{V}_i(\theta_i, \theta_{-i}) + \widehat{V}_{-i}(\theta_i, \theta_{-i})$  for all  $\theta_{-i}$ . That is, there are never any gains from trade between type  $\theta_i$  and the other agents, regardless of their types.

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<sup>6</sup>Versions of this result appear, for example, in Makowski and Mezzetti (1994), Krishna and Perry (1998), Neeman (1999), Williams (1999), Che (2006), Schweizer (2006), Figueroa and Skreta (2008), Segal and Whinston (2011), and Segal and Whinston (2013). Part (i) of the Lemma can be proven by building a budget-balanced mechanism as suggested by Arrow (1979) and d'Aspremont and Gérard-Varet (1979), and satisfying all agents' participation constraints with appropriate lump-sum transfers. Part (ii) follows from the classical Revenue Equivalence Theorem.

**Definition 3** Given a property rights mechanism, type  $\theta_i$  of agent  $i$  is an *adverse type* if it minimizes  $\mathbb{E}[\widehat{V}_{-i}(\theta_i, \tilde{\theta}_{-i})]$ .

Type  $\theta_i$  is an adverse type if, conditional on agent  $i$  opting out, agents other than  $i$  are worst off collectively when agent  $i$ 's type is  $\theta_i$ . Note in particular that any type  $\theta_i$  is trivially an adverse type when agent  $i$  imposes no externalities on the other agents, so  $\widehat{V}_{-i}(\theta)$  does not depend on  $\theta_i$ . This is the case, for example, with simple property rights.

**Example 4** Suppose that each of two agents  $i = 1, 2$  has a value  $\theta_i$  for an object, where  $\theta_i$  is drawn from distribution  $F_i$ . Agent 1 faces a liability rule and pays agent 2 price  $p \in [0, 1]$  if he chooses to take the object. In this case,  $\widehat{V}_1(\theta_1, \theta_2) = \max\{\theta_1 - p, 0\}$  and  $\widehat{V}_2(\theta_1, \theta_2) = \theta_2 F_1(p) + (1 - F_1(p))p$ . Then agent 2's type  $\theta_2 = p$  is an efficient-opt-out type, since when he has this value the outcome of agent 1's exercise decision is efficient regardless of agent 1's type. That type of agent 2 is trivially an adverse type because agent 1's payoff when exercising the option does not depend on agent 2's type.

On the other hand, both  $\theta_1 = 1$  and  $\theta_1 = 0$  are efficient-opt-out types for agent 1: when  $\theta_1 = 1$ , agent 1 will always exercise under the liability rule, and it is always efficient for him to do so regardless of the other agents' types; when instead,  $\theta_1 = 0$ , agent 1 will never exercise under the liability rule, which is also always efficient. Of these two types,  $\theta_1 = 0$  is an adverse type for agent 1 when  $\mathbb{E}[\theta_2] < p$ , since then agent 2 prefers for agent 1 to exercise the option and agent 1 never does, while  $\theta_1 = 1$  is an adverse type for agent 1 when  $\mathbb{E}[\theta_2] > p$ .

The significance of these definitions for our results stems from the following observation:

**Lemma 5** When agent  $i$  has a type  $\theta_i^\circ$  that is both an adverse type and an efficient-opt-out type, it is a critical type.

**Proof.** We can then write for all  $\theta_i \in \Theta_i$ ,

$$\begin{aligned} \mathbb{E} \left[ V(\theta_i^\circ, \tilde{\theta}_{-i}) - \widehat{V}_i(\theta_i^\circ, \tilde{\theta}_{-i}) \right] &= \mathbb{E} \left[ \widehat{V}_{-i}(\theta_i^\circ, \tilde{\theta}_{-i}) \right] \\ &\leq \mathbb{E} \left[ \widehat{V}_{-i}(\theta_i, \tilde{\theta}_{-i}) \right] \\ &\leq \mathbb{E} \left[ V(\theta_i, \tilde{\theta}_{-i}) - \widehat{V}_i(\theta_i, \tilde{\theta}_{-i}) \right] \end{aligned}$$

where the equality is because  $\theta_i^\circ$  is an efficient-opt-out type, the first inequality is because  $\theta_i^\circ$  is an adverse type, and the second inequality is by (6). ■

Our results will apply not only to settings in which adverse efficient-opt-out types exist, but also to settings in which their existence is only of the following asymptotic form:

**Definition 6** *The **adverse efficient-opt-out property** holds for agent  $i$  if there exists a sequence  $\{\theta_i^k\}_{k=1}^\infty$  in  $\Theta_i$  such that as  $k \rightarrow \infty$ ,*

$$\begin{aligned} \mathbb{E} \left[ V(\theta_i^k, \tilde{\theta}_{-i}) - \widehat{V}_i(\theta_i^k, \tilde{\theta}_{-i}) - \widehat{V}_{-i}(\theta_i^k, \tilde{\theta}_{-i}) \right] &\rightarrow 0, \text{ and} \\ \mathbb{E} \left[ \widehat{V}_{-i}(\theta_i^k, \tilde{\theta}_{-i}) \right] &\rightarrow \inf_{\hat{\theta}_i \in \Theta_i} \mathbb{E} \left[ \widehat{V}_{-i}(\hat{\theta}_i, \tilde{\theta}_{-i}) \right]. \end{aligned}$$

Note that this property holds whenever agent  $i$  has an adverse efficient-opt-out type  $\theta_i^\circ$  (in which case we can let  $\theta_i^k = \theta_i^\circ$  for all  $k$ ), but it may also hold in other cases – e.g., sometimes we may need to take a sequence with  $\theta_i^k \rightarrow +\infty$  (in which case we may say informally that  $\theta_i = +\infty$  is an adverse efficient-opt-out type). This property allows us to express the intermediary’s expected profits as follows:

**Lemma 7** *If the adverse efficient-opt-out property holds for each agent, the intermediary’s profits (5) can be written as follows:*

$$\bar{\pi} = \sum_i \inf_{\hat{\theta}_i \in \Theta_i} \mathbb{E}[\widehat{V}_{-i}(\hat{\theta}_i, \tilde{\theta}_{-i})] - (N-1) \mathbb{E}[V(\tilde{\theta})]. \quad (7)$$

**Proof.**

$$\begin{aligned} \bar{\pi} &= \sum_i \inf_{\hat{\theta}_i \in \Theta_i} \mathbb{E}[V(\hat{\theta}_i, \tilde{\theta}_{-i}) - \widehat{V}_i(\hat{\theta}_i, \tilde{\theta}_{-i})] - (N-1) \mathbb{E}[V(\tilde{\theta})] \\ &= \sum_i \inf_{\hat{\theta}_i \in \Theta_i} \left\{ \mathbb{E}[V(\hat{\theta}_i, \tilde{\theta}_{-i}) - \widehat{V}_i(\hat{\theta}_i, \tilde{\theta}_{-i}) - \widehat{V}_{-i}(\hat{\theta}_i, \tilde{\theta}_{-i})] + \mathbb{E}[\widehat{V}_{-i}(\hat{\theta}_i, \tilde{\theta}_{-i})] \right\} - (N-1) \mathbb{E}[V(\tilde{\theta})]. \end{aligned}$$

On the one hand, (6) guarantees that this expression is greater or equal to the right-hand side of (7). On the other hand, the adverse efficient-opt-out property for agent  $i$  ensures that

$$\inf_{\hat{\theta}_i \in \Theta_i} \left\{ \mathbb{E}[V(\hat{\theta}_i, \tilde{\theta}_{-i}) - \widehat{V}_i(\hat{\theta}_i, \tilde{\theta}_{-i}) - \widehat{V}_{-i}(\hat{\theta}_i, \tilde{\theta}_{-i})] + \mathbb{E}[\widehat{V}_{-i}(\hat{\theta}_i, \tilde{\theta}_{-i})] \right\} \leq \inf_{\hat{\theta}_i \in \Theta_i} \mathbb{E}[\widehat{V}_{-i}(\hat{\theta}_i, \tilde{\theta}_{-i})].$$

Hence, if this holds for all agents, we obtain (7). ■

## 3.2 Inefficiency Result

The adverse efficient-opt-out property is a restrictive property, but it will hold in a number of settings of interest. On the other hand, the second property we require in Proposition 9 below is usually satisfied. It makes use of the following notion:

**Definition 8**  $w \in \mathbb{R}^N$  is a *marginal core payoff vector* in state  $\theta$  if

- (i)  $\sum_{j \neq i} w_j \geq \widehat{V}_{-i}(\theta)$  for all  $i$ , and
- (ii)  $\sum_i w_i = V(\theta)$ .

Compared to the usual notion of the core, the marginal core considers only coalitions that include  $N-1$  agents. Condition (i) simply says that the coalition consisting of all agents except agent  $i$  does not block (assuming “blocking” yields the coalition the same collective payoff it receives when agent  $i$  opts out), while condition (ii) says that the maximal total surplus is achieved. Using (ii), condition (i) can be rewritten as  $w_i \leq V(\theta) - \widehat{V}_{-i}(\theta)$ , i.e., no agent  $i$  can receive more than his marginal contribution to the total surplus.

**Proposition 9** *Suppose that the assumptions of Lemma 1(ii) hold, the adverse efficient-opt-out property holds for each agent, and the set of marginal core payoff vectors is non-empty in all states and multi-valued with a positive probability. Then efficient bargaining is impossible.*

**Proof.** (7) implies that

$$\begin{aligned} \bar{\pi} &\leq \sum_i \mathbb{E}[\widehat{V}_{-i}(\tilde{\theta})] - (N-1) \mathbb{E}[V(\tilde{\theta})] \\ &= \mathbb{E} \left[ V(\tilde{\theta}) - \sum_i [V(\tilde{\theta}) - \widehat{V}_{-i}(\tilde{\theta})] \right] \end{aligned} \quad (8)$$

Now, for a marginal core payoff vector  $w$ , for each  $i$  we have

$$w_i \leq V(\theta) - \widehat{V}_{-i}(\theta).$$

If the marginal core is multi-valued, then there exists such a  $w$  with at least one inequality strict, and so

$$V(\theta) = \sum_i w_i < \sum_i \left( V(\theta) - \widehat{V}_{-i}(\theta) \right).$$

If this inequality holds with positive probability, (8) implies that  $\bar{\pi} < 0$ , and so the impossibility of efficient bargaining is implied by Lemma 1(ii). ■

### 3.3 Some Applications

The assumptions of Proposition 9 cover many classical economic settings. For one example, consider the double-auction setting of Williams (1999), in which there are  $N_s$  sellers with values drawn from a distribution on  $[\underline{\theta}_s, \bar{\theta}_s]$  and  $N_b$  buyers with values drawn from a distribution on  $[\underline{\theta}_b, \bar{\theta}_b]$  with  $(\underline{\theta}_b, \bar{\theta}_b) \cap (\underline{\theta}_s, \bar{\theta}_s) \neq \emptyset$ . Since this is a setting without externalities, all types are trivially adverse types. Moreover, since the IR constraints (1) imply that the functions  $\widehat{V}_{-i}(\cdot)$  have no effect on whether efficient bargaining can be achieved, in this setting we can without loss assume that agents  $-i$  trade efficiently among themselves in the event that agent  $i$  opts out. If so, then (i) a buyer of type  $\underline{\theta}_b$  is an efficient-opt-out type if either  $\underline{\theta}_b \leq \underline{\theta}_s$  or  $N_b > N_s$ , and (ii) a seller of type  $\bar{\theta}_s$  is an efficient-opt-out type if either  $\bar{\theta}_s \geq \bar{\theta}_b$  or  $N_s > N_b$ . Moreover, a competitive equilibrium exists in every state and is not unique with a positive probability. Since a competitive equilibrium is always in the core (and, hence, in the marginal core), Proposition 9 applies whenever both (i) and (ii) hold.<sup>7</sup>

Proposition 9 also applies to the public good setting of Mailath and Postlewaite (1990), in which each of  $N$  consumers' values is drawn from a distribution on  $[0, \bar{\theta}]$ , and the cost of provision is  $c > 0$  (whose allocation could be assumed to be equal split among participating agents in the default outcome). Assume (without loss of generality) that if an agent  $i$  opts out,

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<sup>7</sup>The argument can also be extended to show impossibility whenever  $N_b = N_s$ . In this case, note that in an efficient allocation any agent of type below  $\underline{\theta} \equiv \max\{\underline{\theta}_s, \underline{\theta}_b\}$  receives an object with probability zero, so is therefore indistinguishable from type  $\underline{\theta}$ , and any agent of type above  $\bar{\theta} \equiv \min\{\bar{\theta}_s, \bar{\theta}_b\}$  receives an object with probability one, so is therefore indistinguishable from type  $\bar{\theta}$ . Therefore, the profit in the mechanism must be the same as if all agents' types were instead distributed on the same interval  $[\underline{\theta}, \bar{\theta}]$  (with possible atoms at its endpoints), in which case efficient bargaining is impossible by the argument in the text.

the other agents choose the level of the public good to maximize their joint payoff, without any payment to or from agent  $i$ . In this case, each agent's type 0 is both an efficient-opt-out type and an adverse type. Since a Lindahl equilibrium exists in every state and is not unique with a positive probability, and a Lindahl equilibrium is in the core (and the marginal core), Proposition 9 applies.

In the double-auction setting with  $KN_b$  buyers and  $KN_s$  sellers, the per-agent expected subsidy in an ex ante optimal mechanism converges to zero as the  $K \rightarrow \infty$ . Intuitively, this relates to the fact that the core converges (in probability) to the unique competitive equilibrium of the continuous limit economy, hence in the limit the agents can fully appropriate their marginal contributions [as in Makowski and Ostroy (1989, 1995, 2001)]. Specifically, a buyer's marginal contribution, in the limit, equals his value minus the equilibrium price, while a seller's marginal contribution equals the equilibrium price minus his cost. So the intermediary's expected profit converges to zero. This relates to the finding of Gresik and Satterthwaite (1989) that the inefficiency in an ex ante optimal budget-balanced mechanism also shrinks to zero as the number of agents grows. In contrast, in the public good setting of Mailaith and Postlewaite (1990), the core grows in relative size as  $N \rightarrow \infty$ , and inefficiency is exacerbated: each agent's marginal contribution when the good is provided and he is non-pivotal equals his value. As the number of agents grows large, each agent becomes non-pivotal with probability one, and the intermediary's deficit approaches the probability of provision times the project's cost. In fact, as shown by Mailaith and Postlewaite (1990), the probability of providing the public good in any budget-balanced mechanism goes to zero.

## 4 Optimal Property Rights for Minimizing the Expected First-Best Subsidy

Recall from Lemma 7 the expected subsidy needed to achieve the first best when the adverse efficient-opt-out property holds for all agents, which can be written as:

$$\bar{\pi} = \inf_{\hat{\theta}_1, \dots, \hat{\theta}_N} \mathbb{E} \left[ V(\tilde{\theta}) - \sum_i [V(\tilde{\theta}) - \hat{V}_{-i}(\hat{\theta}_i, \tilde{\theta}_{-i})] \right]. \quad (9)$$

Using this formula we compare property rights possessing this property in terms of this criterion, which in general amounts to maximizing the sum  $\sum_i \inf_{\tilde{\theta}} \mathbb{E}[\widehat{V}_{-i}(\hat{\theta}_i, \tilde{\theta}_{-i})]$ , or  $\sum_i \mathbb{E}[\widehat{V}_{-i}(\hat{\theta}_i^\circ, \tilde{\theta}_{-i})]$  when adverse efficient-opt-out types  $\theta_1^\circ, \dots, \theta_N^\circ$  exist for all agents. In the remainder of this section we explore the implications of this prescription.

## 4.1 Two Agents

We first consider situations with two agents and analyze optimal property rights for an indivisible good. Specifically, as in Myerson and Satterthwaite (1983), each agent  $i$ 's value  $\theta_i$  for the good is drawn from a full support distribution  $F_i$  on  $[0, 1]$ . We first consider which of the two agents should own the good if the goal is to minimize the expected first-best bargaining subsidy. We then investigate whether options to own can improve on simple ownership. This corresponds to the legal literature's question of whether property rules or liability rules are better.<sup>8</sup>

### 4.1.1 Who Should Own?

Consider any situation with two agents in which the property rights induce a fixed status quo  $(\hat{x}, \hat{t}_1, \dots, \hat{t}_N)$ , and the two agents both have efficient-opt-out types.<sup>9</sup> We then have

$$\begin{aligned}\widehat{V}_{-1}(\theta_1^\circ, \theta_2) &= v_2(\hat{x}, \theta_2) + \hat{t}_2, \\ \widehat{V}_{-2}(\theta_1, \theta_2^\circ) &= v_1(\hat{x}, \theta_1) + \hat{t}_1,\end{aligned}$$

and, using (9), the expected first-best subsidy is

$$\begin{aligned}\bar{\pi}(\hat{x}) &= \mathbb{E}\{V(\tilde{\theta}) - [V(\tilde{\theta}) - \widehat{V}_{-1}(\theta_1^\circ, \tilde{\theta}_2)] - [V(\tilde{\theta}) - \widehat{V}_{-2}(\tilde{\theta}_1, \theta_2^\circ)]\} \\ &= \mathbb{E}\{v_2(\hat{x}, \tilde{\theta}_2) + v_1(\hat{x}, \tilde{\theta}_1) + (\hat{t}_1 + \hat{t}_2) - V(\tilde{\theta})\} \\ &= \mathbb{E}\{v_2(\hat{x}, \tilde{\theta}_2) + v_1(\hat{x}, \tilde{\theta}_1) - V(\tilde{\theta})\} < 0.\end{aligned}$$

In words, a mediator who implements the first best must subsidize the entire renegotiation surplus. Thus, the status quo  $\hat{x}$  that minimizes the

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<sup>8</sup>In Section 6 we also discuss “dual chooser” rules in settings with two agents.

<sup>9</sup>Recall that with a fixed status quo, any efficient-opt-out type is trivially also an adverse type.



expected subsidy (within a class of those that have efficient-opt-out types) must maximize the expected status quo surplus  $\mathbb{E}[v_1(\hat{x}, \tilde{\theta}_1) + v_2(\hat{x}, \tilde{\theta}_1)]$ . Thus, we have:

**Proposition 10** *Suppose that the assumptions of Lemma 1(ii) hold and there are two agents. Then, among the property rights mechanisms that induce a fixed status quo and cause both agents to have efficient-opt-out types, the one that minimizes the first-best subsidy is the one that maximizes the two agents' joint payoff in the absence of bargaining.*

Since, as we saw in Section 3, both agents have efficient-opt-out types in the setting of Myerson and Satterthwaite, we have the following corollary:

**Corollary 11** *Consider the Myerson-Satterthwaite setting in which each of two agents  $i = 1, 2$  has value  $\theta_i \in [0, 1]$  drawn from a full support distribution  $F_i$ . Then assigning ownership to the agent with the higher expected value minimizes the first-best subsidy.*

Thus, to minimize the first-best bargaining subsidy, ownership is best assigned exactly as if bargaining were impossible.

#### 4.1.2 Property Rules vs. Liability Rules

We now consider the possibility that instead of a simple property right, one agent may be given an option to own. Specifically, imagine that agent 1 can choose to acquire the good from agent 2 at a price  $p$ . This arrangement may be thought of as a liability rule in which agent 1 can take the good from agent 2, but must then make damage payment  $p$  to agent 2.

As we saw in Example 4, both agents will have adverse efficient-opt-out types in this case. For agent 2 it is his type  $\hat{\theta}_2 = p$ , while for agent 1 it is his type  $\hat{\theta}_1 = 1$  if  $\mathbb{E}[\theta_2] < p$ , and type  $\hat{\theta}_1 = 0$  if  $\mathbb{E}[\theta_2] > p$ . The marginal core in this case is nonempty and is multivalued in any state  $\theta$  in which  $\widehat{V}_1(\theta_1) + \widehat{V}_2(\theta_1, \theta_2) < V(\theta)$ ; that is, whenever the exercise decision by type  $\theta_1$  is not efficient. Hence, by Proposition 9, efficiency cannot be achieved with a liability rule.

To identify the subsidy-minimizing liability rule, we write the intermediary's profit as

$$\begin{aligned}\bar{\pi} &= \mathbb{E} \left[ \widehat{V}_1(\tilde{\theta}_1) + \widehat{V}_2(\hat{\theta}_1, \tilde{\theta}_2) - V(\tilde{\theta}) \right] \\ &= \underbrace{\mathbb{E} \left[ \widehat{V}_1(\tilde{\theta}_1) + \widehat{V}_2(\tilde{\theta}_1, \tilde{\theta}_2) - V(\tilde{\theta}) \right]}_{<0 \text{ when option exercise is not first-best}} + \underbrace{\mathbb{E} \left[ \widehat{V}_2(\hat{\theta}_1, \tilde{\theta}_2) - \widehat{V}_2(\tilde{\theta}_1, \tilde{\theta}_2) \right]}_{\leq 0 \text{ since } \hat{\theta}_1 \text{ is an adverse type}}\end{aligned}$$

Observe that the first term is the expected welfare loss from agent 1's optimal exercise of the option. It is negative provided that agent 1's exercise decision is not always ex post optimal. However, the inefficiency is minimized when  $p = \mathbb{E}[\theta_2]$ , which is the optimal exercise price in the absence of bargaining. The second term, on the other hand, is non-positive, and is strictly negative unless agent 2 is indifferent, on average, about agent 1's exercise decision. That occurs when  $p = \mathbb{E}[\theta_2]$ . Thus, we see that when agent 1 has the option, the option price that minimizes the first-best subsidy has  $p = \mathbb{E}[\theta_2]$  and it results in a positive expected subsidy (confirming that achieving the first best is impossible, as implied by Proposition 9).<sup>10</sup> This option price corresponds exactly to the traditional legal liability rule in which the damage payment equals the victim's expected damage.

Next, consider which agent should have the option. When agent  $i$  gets the option and  $p = \mathbb{E}[\theta_{-i}]$ , the first-best subsidy exactly equals the welfare loss from agent  $i$ 's optimal exercise of the option in the absence of bargaining. Hence, the agent  $i$  should be given the option if and only if he is best assigned the option when bargaining is impossible.

Finally, since the case of a simple property right corresponds to setting  $p = 0$  or  $p = 1$ , the optimal liability rule is strictly better than the best simple property right, exactly as in the case without bargaining.

In summary:

**Proposition 12** *In the Myerson and Satterthwaite setting, the option-to-own (i.e., liability rule) that minimizes the expected first-best bargaining subsidy sets the option price equal to the non-chooser's ("victim's") expected value ("harm") and assigns the option to the agent whose optimal exercise in the absence of bargaining results in the greatest expected surplus. The resulting expected subsidy is lower than it is with any simple property right.*

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<sup>10</sup>Che (2006) also derives this result and notes the impossibility of two agents achieving efficiency under a liability rule.

## 4.2 More than Two Agents

When there are more than two agents, choosing the subsidy-minimizing property rights requires that we consider the coalitional, rather than individual, values. For example, shifting the property right to a private good (generating no externalities) from one agent to another is efficiency enhancing in the absence of bargaining if it increases the joint payoff of the two agents. In contrast, this change increases efficiency when bargaining is possible (in the sense of reducing the expected first-best subsidy) if it reduces the sum of these agents' marginal contributions to the total surplus.<sup>11</sup>

We illustrate the new effects through two examples.

### 4.2.1 Application: Spectrum Licenses

Consider the following example: Simple property rights to two spectrum licenses,  $L_1$  and  $L_2$ , are to be allocated among three firms. Firms 1 and 2 are specialists and each firm  $i = 1, 2$  has a value  $\theta_i \in \mathbb{R}_+$  for license  $L_i$ , drawn from distribution  $F$  with mean  $\mu$ , and no value for license  $L_{-i}$ . Firm G is a generalist firm, and has a value  $\theta_G \in \mathbb{R}_+$  for both licenses, and value  $\lambda\theta_G$  for just one of the licenses, where  $\theta_G$  is drawn from distribution  $H$ , with mean  $\mu_G$ , and  $\lambda \in (0, 1)$ . The values  $\theta_1$ ,  $\theta_2$ , and  $\theta_G$  are independent random variables. When  $\lambda < 1/2$ , the licenses are complements for G; when  $\lambda > 1/2$ , they are substitutes. For example, the licenses might be in two different regions, with firms 1 and 2 being regional firms and firm G being a national firm. In that case, G is likely to find the two licenses complements ( $\lambda > 1/2$ ). Alternatively, the licenses might be to different frequencies, with firms 1 and 2 each having a product that can use either of the frequencies effectively, while firm G may have two products, each of which can use one of the two frequencies effectively. In that case, the frequencies are likely to be substitutes ( $\lambda > 1/2$ ).

We will compare an allocation of **BOTH** licenses to G with an allocation of **NONE** of the licenses to G (so that each license  $L_i$  is allocated to the specialist firm  $i$ ). Absent bargaining, the expected surplus is larger at **BOTH**

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<sup>11</sup>This effect can therefore be interpreted as the effect on the two agents' joint payoff in bargaining, if this bargaining permits each agent to extract his marginal contribution to the grand coalition. This relates to the analysis of Segal (2003); however, the latter considers the Shapley value, in which agents receive weighted combinations of their marginal contributions to different coalitions, hence the results are not directly comparable.

than at ONE if

$$\mu_G - 2\mu > 0 \tag{10}$$

Note that the best choice between these two allocations of property rights in the absence of bargaining is independent of  $\lambda$ .

Now consider the subsidy-minimizing property rights when there is bargaining. Under both of these property rights allocations, the adverse efficient-opt-out property holds. The following table summarizes the coalitional values, under the assumption that each two-agent coalition maximizes its joint payoff:

<b>Property Rights Allocation:</b>	$\widehat{V}_{-G}$	$\widehat{V}_{-1}$	$\widehat{V}_{-2}$
BOTH	0	$\max\{\theta_G, \lambda\theta_G + \theta_2\}$	$\max\{\theta_G, \lambda\theta_G + \theta_1\}$
NONE	$\theta_1 + \theta_2$	$\max\{\lambda\theta_G, \theta_2\}$	$\max\{\lambda\theta_G, \theta_1\}$

So, BOTH is better than NONE if

$$\sum_{i=1,2} \mathbb{E}[\max\{\theta_G, \lambda\theta_G + \theta_i\}] > 2\mu + \sum_{i=1,2} \mathbb{E}[\max\{\lambda\theta_G, \theta_i\}],$$

which can be rewritten as

$$\mu_G - 2\mu > (2\lambda - 1)\mu_G + \sum_{i=1,2} (\mathbb{E}[\max\{0, \theta_i - \lambda\theta_G\}] - \mathbb{E}[\max\{0, \theta_i - (1 - \lambda)\theta_G\}]) \tag{11}$$

Thus, bargaining changes the optimal property rights according to the sign of the term on the right-hand side of (11). This term equals zero when  $\lambda = 1/2$ , so that the licenses are neither substitutes nor complements for firm G. In that case, the best property rights allocation is the same as in the absence of bargaining. The derivative of the right-hand side with respect to  $\lambda$  is

$$\mu_G \left( 2 - \sum_{i=1,2} [\Pr(\theta_i - \lambda\theta_G \geq 0) + \Pr(\theta_i - (1 - \lambda)\theta_G \geq 0)] \right) > 0$$

Thus, when  $\lambda > 1/2$  (substitutes), bargaining pushes the optimal property rights toward NONE, and when  $\lambda < 1/2$  (complements) bargaining pushes the optimal property rights toward BOTH.

### 4.2.2 Application: Liability Rule for Pollution

Consider a setting in which agent 0 (the “firm”) chooses whether to pollute, labeled by  $x \in \{0, 1\}$ . The firm’s utility is  $v_0(x, \theta_0) = \theta_0 x$ , where  $\theta_0 \in \mathbb{R}_+$  denotes its value for polluting. Agents  $i = 1, \dots, N$  are consumers, whose utilities are given by  $v_i(x, \theta_i) = (1 - x)\theta_i$  with  $\theta_i \in \mathbb{R}_+$ . Efficient pollution is therefore given by

$$\chi^*(\theta) = 1 \text{ if and only if } \theta_0 \geq \sum_{i \geq 1} \theta_i.$$

We assume that, for all  $i$ ,  $\tilde{\theta}_i$  has a full-support absolutely continuous distribution on  $\mathbb{R}_+$ .

The property rights are given by a liability rule: the firm can choose to pollute, in which case it must pay pre-specified “damages”  $p_i \geq 0$  to each consumer  $i \geq 1$ . Thus, if the firm does not participate in bargaining, it optimally chooses  $\hat{x}_0(\theta) = \chi^*(\theta_0, \sum_i p_i)$ , and its transfer is given by  $\hat{\tau}_0(\theta) = -(\sum_i p_i) \hat{x}_0(\theta)$ .

We must also specify what happens if agent  $i \geq 1$  does not participate. To obtain the results in the simplest possible way, we assume for now that all the other agents then bargain efficiently among each other, given that agent  $i$  must be paid compensation  $p_i$  if pollution is chosen. Thus, they optimally choose pollution level  $\hat{x}_i(\theta) = \chi^*(p_i, \theta_{-i})$ , and agent  $i$ ’s compensation is  $\hat{\tau}_i(\theta) = p_i \hat{x}_i(\theta)$ . (We will discuss the role of this assumption later.)

Given these assumptions, each agent  $i \geq 1$  has an efficient-opt-out type  $\theta_i^\circ = p_i$ . This type is also trivially adverse, since the agent imposes no externalities on the others. Hence, by Lemma 5, it is agent  $i$ ’s critical type.

The firm, on the other hand, has two efficient-opt-out types:  $\theta_0 = 0$  (which never pollutes in the first best and does not pollute when it does not participate) and  $\theta_0 = +\infty$  (which always pollutes in the first best and pollutes when it does not participate). Furthermore,  $\theta_0 = 0$  is an adverse type if  $\sum_{i \geq 1} p_i \geq \mathbb{E}[\sum_{i \geq 1} \tilde{\theta}_i]$  while  $\theta_0 = +\infty$  is an adverse type if the inequality is reversed. (Of course, formally speaking  $\theta_0 = +\infty$  is not a “type,” but taking a sequence  $\theta_0^k \rightarrow +\infty$  shows that the firm does satisfy the adverse efficient-opt-out property.)

**Remark 13** *How would this conclusion be affected if agent  $i \geq 1$  expected a different outcome  $\hat{x}_i(\theta)$  from non-participation, while still expecting compensation  $\hat{\tau}_i(\theta) = p_i \hat{x}_i(\theta)$  according to the liability rule? Observe that the*

reservation utility of type  $\theta_i = p_i$  is independent of  $\hat{x}_i(\theta)$ . Since type  $\theta_i = p_i$  was agent  $i$ 's critical type above, where efficient bargaining was impossible, the participation constraints of this type continue to imply that efficient bargaining is impossible.

Furthermore, we can argue that if the intermediary can choose  $\hat{x}_i(\theta)$  following nonparticipation of agent  $i \geq 1$  to minimize the expected first-best subsidy, then it can do no better than setting  $\hat{x}_i(\theta) = \chi^*(p_i, \theta_{-i})$ , as we assumed above. Indeed, since the intermediary has to satisfy the participation constraint of type  $\theta_i = p_i$  regardless of  $\hat{x}_i(\theta)$ , formula (7) bounds below the intermediary's expected subsidy. On the other hand, by choosing  $\hat{x}_i(\theta) = \chi^*(p_i, \theta_{-i})$  the intermediary ensures that type  $\theta_i = p_i$  is a critical type, and therefore its participation constraints imply all the other types' participation constraints, so the lower bound on the expected subsidy is actually achieved. Therefore, the following analysis of optimal damages  $p$  applies to the situation where the intermediary can choose  $\hat{x}_i(\theta)$  optimally following nonparticipation by individual agents.

Now we identify the vector of damages  $p = (p_1, \dots, p_N)$  that minimizes the expected first-best subsidy. Using (7), the maximization problem can be written as

$$\max_{p_1, \dots, p_N \geq 0} \sum_{i=0}^N \mathbb{E} \left[ \widehat{V}_{-i}(\theta_i^\circ, \tilde{\theta}_{-i}) \right]$$

where

$$\mathbb{E} \left[ \widehat{V}_{-0}(\theta_0^\circ, \tilde{\theta}_{-0}) \right] = \min \left\{ \sum_{i \geq 1} \mathbb{E}[\tilde{\theta}_i], \sum_{i \geq 1} p_i \right\}$$

and

$$\mathbb{E} \left[ \widehat{V}_{-i}(\theta_i^\circ, \tilde{\theta}_{-i}) \right] = \mathbb{E} \left[ \max \left\{ \tilde{\theta}_0 - p_i, \sum_{j \neq i, j \geq 1} \tilde{\theta}_j \right\} \right] \text{ for } i \geq 1.$$

Note that using the Envelope Theorem, for  $i \geq 1$ ,

$$\frac{\partial \mathbb{E} \left[ \widehat{V}_{-i}(\theta_i^\circ, \tilde{\theta}_{-i}) \right]}{\partial p_i} = - \Pr \left\{ \tilde{\theta}_0 - p_i > \sum_{j \neq i, j \geq 1} \tilde{\theta}_j \right\} \in (-1, 0), \quad (12)$$

while

$$\frac{\partial \mathbb{E} \left[ \widehat{V}_{-0}(\theta_0^\circ, \tilde{\theta}_{-0}) \right]}{\partial p_i} = \left\{ \begin{array}{l} 1 \text{ if } \sum_{i \geq 1} \mathbb{E}[\tilde{\theta}_i] > \sum_{i \geq 1} p_i \\ \text{otherwise.} \end{array} \right\}.$$

Therefore, at the optimum we must have  $\sum_{i \geq 1} p_i = \sum_{i \geq 1} \mathbb{E}[\tilde{\theta}_i]$ , i.e., the total damages paid by the firm should equal the total expectation damages for the affected parties. This would also be optimal in a setting where bargaining is impossible.

However, in contrast to the setting without bargaining, it now matters how the damages are allocated among consumers. The problem of optimal allocation of damages can be formulated as

$$\max_{p_1, \dots, p_N \geq 0} \sum_{i \geq 1} \mathbb{E}[\widehat{V}_{-i}(p_i, \tilde{\theta}_{-i})] \text{ s.t. } \sum_{i \geq 1} p_i = \sum_{i \geq 1} \mathbb{E}[\tilde{\theta}_i].$$

Note by (12),  $\partial \mathbb{E}[\widehat{V}_{-i}(p_i, \tilde{\theta}_{-i})] / \partial p_i$  is nondecreasing in  $p_i$ , so the objective function is convex, and is therefore maximized at a vertex of the feasible set, i.e., a point  $p$  such that  $p_{i^*} = \sum_{i \geq 1} \mathbb{E}[\tilde{\theta}_i]$  for

$$i^* \in \arg \max_{j \geq 1} \left\{ \mathbb{E} \left[ \widehat{V}_{-j} \left( \sum_{i \geq 1} \mathbb{E}[\tilde{\theta}_i], \tilde{\theta}_{-j} \right) \right] + \sum_{i \geq 1, i \neq j} \mathbb{E} \left[ \widehat{V}_{-i}(0, \tilde{\theta}_{-i}) \right] \right\}$$

and  $p_i = 0$  for all  $i \neq i^*$ . Thus, all of the damages should be paid to a single consumer, with the consumer selected to maximize the total expected surplus of the  $N$  coalitions consisting of the firm and  $N - 1$  affected parties.

## 5 Optimal Property Rights with Second-Best Bargaining

In many circumstances, there isn't an intermediary available to subsidize trade. In that case, a more appropriate approach to determining optimal property rights involves looking at second-best mechanisms that maximize expected surplus subject to a budget balance constraint. Analyzing that problem, however, is complicated by the interplay between the mechanism chosen and the agents' critical types: those critical types depend on the mechanism being employed, but the best mechanism depends on the agent's critical types (because they determine which IR constraints bind).<sup>12</sup> In this section, we analyze this problem. As this is a much harder problem than the first-best problem studied earlier, we restrict attention to the case of

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<sup>12</sup>For any mechanism, not just VCG mechanisms, we now refer to an agent's "critical type" as a type that has the smallest participation surplus among all of the agent's types.

two agents trading a single indivisible good, where those agents' types are independently distributed on  $[0, 1]$ .<sup>13</sup>

Myerson and Satterthwaite (1983) characterized the optimal second-best mechanism for the case of simple property rights, where one agent is a seller (the initial owner) and the other agent is the buyer. The optimal mechanism when agent 2 is the seller and agents' types are uniformly distributed is shown in Figure 1 [the shaded region shows the type profiles  $(\theta_1, \theta_2)$  at which agent 1 ends up getting the good], which leads to a surplus loss of  $7/64$  (from the first-best surplus of  $3/4$ ). It involves a trading “gap”  $l = 1/4$ , which represents the amount that the buyer's value must exceed the seller's value for trade to occur. Cramton, Gibbons, and Klemperer (1987) showed that the first best is achievable for a convex set of intermediate property rights if dividing or randomizing property rights is possible [see also Segal and Whinston (2011)]. However, when divided or randomized property rights are not possible, it may be possible to improve upon simple undivided property rights with more complex property rights mechanisms. In the remainder of this section, we examine this possibility for liability rules.

## 5.1 Characterization of the Second-best Bargaining Mechanism for a Given Liability Rule

Without loss of generality, we will take the agent who possesses the option to have the good to be agent 1 (the “active agent”); agent 2 is the “passive agent” or “victim.” Note that if  $p = 0$  then agent 1 will always exercise his option in the default, so this liability rule is equivalent to agent 1 being the owner with a simple property right. If, instead,  $p = 1$ , then agent 1 will never exercise his option, so the liability rule is then equivalent to agent 2 being the owner with a simple property right. Hence, the *optimal* liability rule cannot be worse than the optimal simple property right. However, we will see that there are always some liability rules that are worse than the best simple property right.

Our analysis hinges on identifying critical types. For the passive agent 2, we can observe that any type  $\theta_2^p$  whose probability of trade in the mechanism is  $p$  is a critical type. Indeed, any other type  $\theta_2'$  can guarantee the same participation surplus by pretending to be type  $\theta_2^p$  since the probability of trade in the mechanism then exactly equals the probability of trade if agent

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<sup>13</sup>The restriction to  $[0, 1]$  is just a normalization.



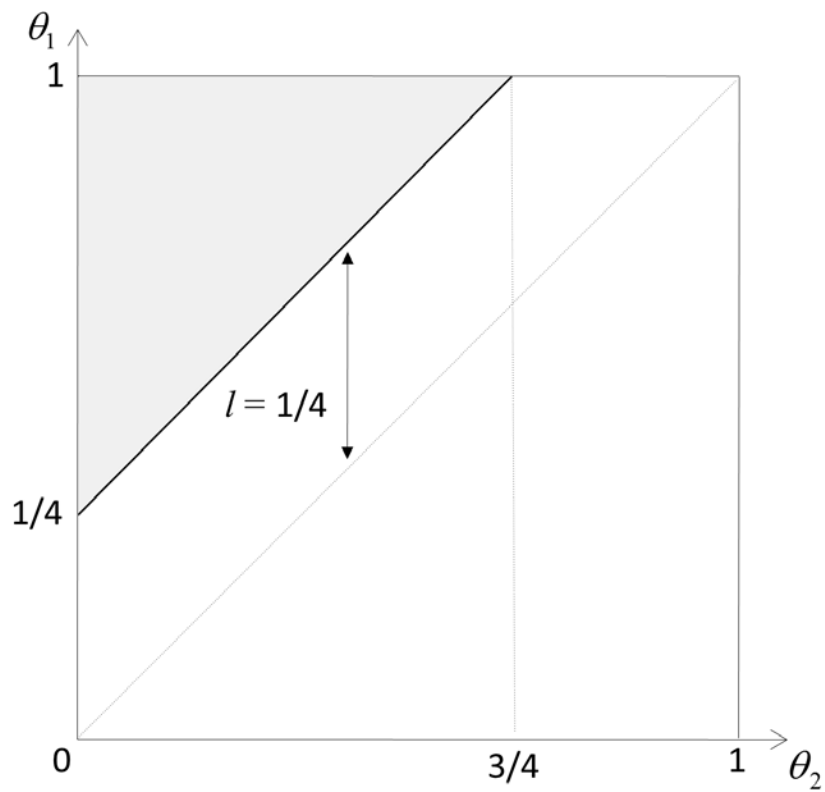


Figure 1: The second-best optimal (Myerson-Satterthwaite) mechanism when agent 1 (the buyer) and agent 2 (the seller) both have values that are uniformly distributed on the interval  $[0, 1]$ .

2 refuses to participate. As a result, if type  $\theta_2^p$  is willing to participate, then so is *every* type. In general, however, there will not be a single critical type for agent 2 but an interval  $[\underline{\theta}_2, \bar{\theta}_2]$  of them.

As for the active agent 1, we observe that this agent's critical types always include either  $\hat{\theta}_1 = 0$ , or  $\hat{\theta}_1 = 1$ , or both. To see this, observe that in the default outcome this agent's payoff is  $V_1(\theta_1) = \max\{\theta_1 - p, 0\}$ , which is a convex function whose derivative is 0 below  $p$  and 1 above  $p$ . This agent's expected payoff  $U_1(\theta_1)$  in any mechanism, on the other hand, has a derivative  $U_1'(\theta_1)$  at each  $\theta_1$  that equals that type's expected probability of receiving the good in the mechanism, so  $U_1'(\theta_1) \in [0, 1]$  for all  $\theta_1$ . Thus, if  $V_1(\theta_1) = U_1(\theta_1)$  at some  $\theta_1$ , then this must be true at either  $\theta_1 = 0$  or at  $\theta_1 = 1$  (or at both).

In what follows, we consider the general case in which the c.d.f.'s of the two agents' types are  $F_1, F_2$  respectively, with strictly positive densities  $f_1, f_2$ . For  $i = 1, 2$ , and for  $\lambda \in [0, 1]$  let

$$\underline{\omega}_i(\theta_i|\lambda) \equiv \theta_i - \lambda \frac{1 - F_i(\theta_i)}{f_i(\theta_i)} \quad \text{and} \quad \bar{\omega}_i(\theta_i|\lambda) \equiv \theta_i + \lambda \frac{F_i(\theta_i)}{f_i(\theta_i)}$$

denote agent  $i$ 's virtual values when his downward/upward ICs bind. We assume that both  $\underline{\omega}_i(\cdot|1)$  and  $\bar{\omega}_i(\cdot|1)$  are strictly increasing and continuous functions in  $\theta_i$  (this implies the same properties for any  $\lambda \in [0, 1]$ ). Note that

$$\underline{\omega}_i(\theta_i|\lambda) \leq \theta_i \leq \bar{\omega}_i(\theta_i|\lambda),$$

and that the inequalities are strict for  $\theta_i \in (0, 1)$ , provided that  $\lambda > 0$ .

Also, for  $\gamma \in [0, 1]$ , let agent 1's weighted virtual value be

$$\omega_1(\theta_1|\lambda, \gamma) \equiv (1 - \gamma)\underline{\omega}_1(\theta_1|\lambda) + \gamma\bar{\omega}_1(\theta_1|\lambda).$$

In our characterization, the weight  $\gamma$  will equal 0 when  $\theta_1 = 0$  is a critical type for agent 1 (so that his downward IC constraints bind), and will equal 1 when  $\theta_1 = 1$  is a critical type (so that agent 1's upward incentive constraints bind). We can have  $\gamma \in (0, 1)$  when both of these types are critical for agent 1.

Finally, for  $\lambda \in [0, 1]$  and  $\gamma \in [0, 1]$ , define  $\bar{\theta}_1(\theta_2|\lambda, \gamma) \equiv \omega_1^{-1}(\bar{\omega}_2(\theta_2|\lambda)|\lambda, \gamma)$  and  $\underline{\theta}_1(\theta_2|\lambda, \gamma) \equiv \omega_1^{-1}(\underline{\omega}_2(\theta_2|\lambda)|\lambda, \gamma)$ . Given an agent 2 type  $\theta_2$ , and values  $\lambda$  and  $\gamma$ , these are the types for agent 1 at which agent 1's weighted virtual valuation equals agent 1's upward and downward virtual values, respectively. They will form part of the boundary of the set of type profiles at which

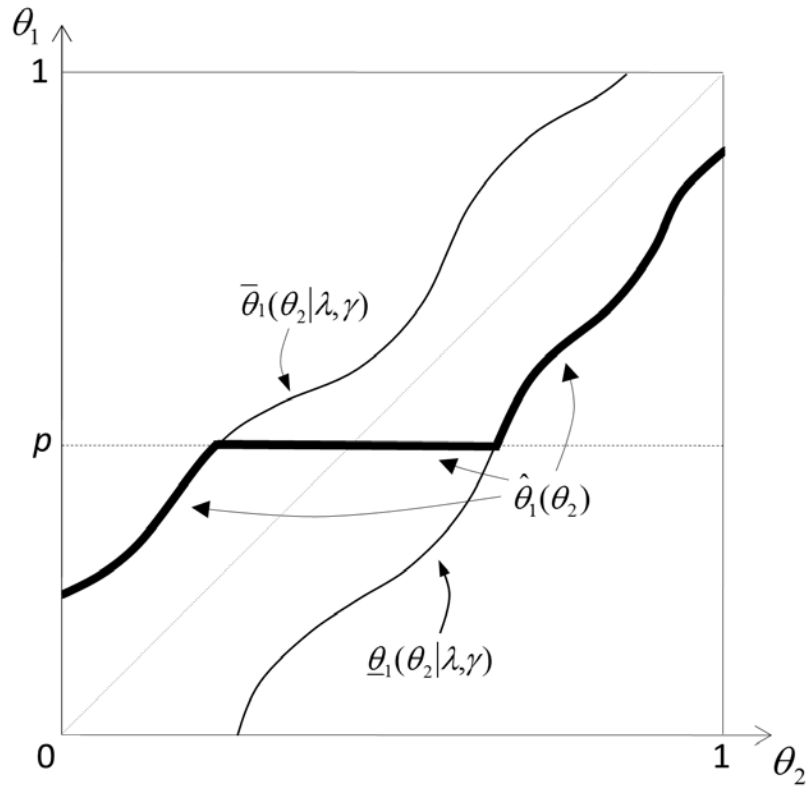


Figure 2: The types  $\bar{\theta}_1(\theta_2|\lambda, \gamma)$  and  $\underline{\theta}_1(\theta_2|\lambda, \gamma)$ , and the boundary  $\hat{\theta}_1(\theta_2)$  of Lemma 14.

agent 1 gets the good in the optimal mechanism, and are depicted in Figure 2. Under our assumptions they are both continuous increasing functions and  $\bar{\theta}_1(\theta_2|\lambda, \gamma) \geq \underline{\theta}_1(\theta_2|\lambda, \gamma)$  for all  $\theta_2$ .

We begin with the following characterization result (all proofs for results in this section are in the Appendix):

**Lemma 14** *When there is a liability rule in which agent 1 has the option to own in return for a payment of  $p \in [0, 1]$ , the second-best solution takes the following form (with probability 1): For some fixed  $\lambda$  and  $\gamma$*

$$x_1(\theta_1, \theta_2) = \begin{cases} 1 & \text{for } \theta_1 > \hat{\theta}_1(\theta_2), \\ 0 & \text{for } \theta_1 < \hat{\theta}_1(\theta_2), \end{cases} \quad (13)$$

where

$$\hat{\theta}_1(\theta_2) \equiv \max \{ \underline{\theta}_1(\theta_2|\lambda, \gamma), \min \{ p, \bar{\theta}_1(\theta_2|\lambda, \gamma) \} \}. \quad (14)$$

Furthermore,

$$\gamma \mathbb{E}[\hat{\theta}_1(\tilde{\theta}_2) - p] \geq 0 \text{ and } (1 - \gamma) \mathbb{E}[\hat{\theta}_1(\tilde{\theta}_2) - p] \leq 0. \quad (15)$$

In Figure 2, the function  $\hat{\theta}_1(\theta_2)$  defined in (14), which forms the boundary of the region in which agent 1 gets the good in the mechanism, is shown in heavy trace.

Condition (15) reflects agent 1's IR constraint. Under the liability rule, agent 1's utility is exactly  $1 - p$  larger when he is type 1 than when he is type 0. On the other hand, the difference in expected payoffs for types  $\theta_1 = 1$  and  $\theta_1 = 0$  equals  $1 - \mathbb{E}[\hat{\theta}_1(\tilde{\theta}_2)]$  in the mechanism.<sup>14</sup> Thus, type  $\theta_1 = 0$  must be a critical type for agent 1 when  $\mathbb{E}[\hat{\theta}_1(\tilde{\theta}_2)] > p$ .

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<sup>14</sup>To see this, observe that by the Envelope Theorem, this difference in expected payoffs in the mechanism equals

$$\int_0^1 \left( \int_0^{\hat{\theta}_2(\theta_1)} f_2(\theta_2) d\theta_2 \right) d\theta_1$$

where  $\hat{\theta}_2(\cdot) = \min\{\theta_2 : \hat{\theta}_1(\theta_2) = \theta_1\}$ . But, reversing the order of integration, this can be rewritten as

$$\int_0^1 \left( \int_{\hat{\theta}_1(\theta_2)}^1 d\theta_1 \right) f_2(\theta_2) d\theta_2 = 1 - \mathbb{E}[\hat{\theta}_1(\theta_2)]$$

For the specific case in which both agents' types are drawn from the uniform distribution, this characterization implies that the second-best mechanism takes the following form:

**Proposition 15** *When both agents' types are drawn from the uniform distribution and there is a liability rule in which agent 1 has the option to own in return for a payment of  $p \in [0, 1]$ , the optimal second-best allocation rule takes the following form, for some function  $l(p)$ :*

- **For  $p < 3/8$ :**  $x_1(\theta_1, \theta_2) = 1$  if and only if (i)  $\min\{\theta_1, p\} \geq \theta_2$ , (ii)  $\theta_1 \geq p$  and  $\theta_2 \in [p, p + l(p)]$ , or (iii)  $\theta_1 \geq \theta_2 - l(p)$  and  $\theta_2 > p + l(p)$ ;
- **For  $p \in [3/8, 5/8]$ :**  $x_1(\theta_1, \theta_2) = 1$  if and only if (i)  $\theta_1 \geq \theta_2 + l(p)$  and  $\theta_2 < 3/8$ , (ii)  $\theta_1 \geq p$  and  $\theta_2 \in [3/8, 5/8]$ , or (iii)  $\theta_1 \geq \theta_2 - l(p)$  and  $\theta_2 > 5/8$ ;
- **For  $p > 5/8$ :**  $x_1(\theta_1, \theta_2) = 1$  if and only if (i)  $\theta_1 \geq \theta_2 \geq p$ , (ii)  $\theta_1 \geq p$  and  $\theta_2 \in [p, p - l(p)]$ , or (iii)  $\theta_1 \geq \theta_2 + l(p)$  and  $\theta_2 < p - l(p)$ .

Figures 3-5 show the sets of types for which agent 1 receives the good for the three cases identified in Proposition 15. The three cases correspond to situations in which agent 1's critical type is  $\hat{\theta}_1 = 1$  (for  $p < 3/8$ ),  $\hat{\theta}_1 = 0$  (for  $p > 5/8$ ), and both types 0 and 1 are critical types (for  $p \in [3/8, 5/8]$ ). Note that the critical type is 1 (resp. 0) for low (resp. high)  $p$ , which are cases where the property right is relatively close to agent 1 (resp. 2) having a simple ownership right.<sup>15</sup> The "gap" function  $l(p)$  is similar to the Myerson-Satterthwaite gap seen in Figure 1, and like that gap its size is set to achieve budget balance.

## 5.2 Second-best Surplus Given Option Price $p$

The second-best expected surplus as a function of  $p$  can be derived analytically in the case in which both agents' values are uniformly distributed (see the Appendix for its derivation). Figure 6 graphs the resulting second-best

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<sup>15</sup>To compare these figures to Figure 2, note that in the special case of the uniform distribution,  $\bar{\theta}_1(\theta_2|\lambda, 0) = \theta_2$  and  $\underline{\theta}_1(\theta_2|\lambda, 1) = \theta_2$  for all  $\theta_2$ .

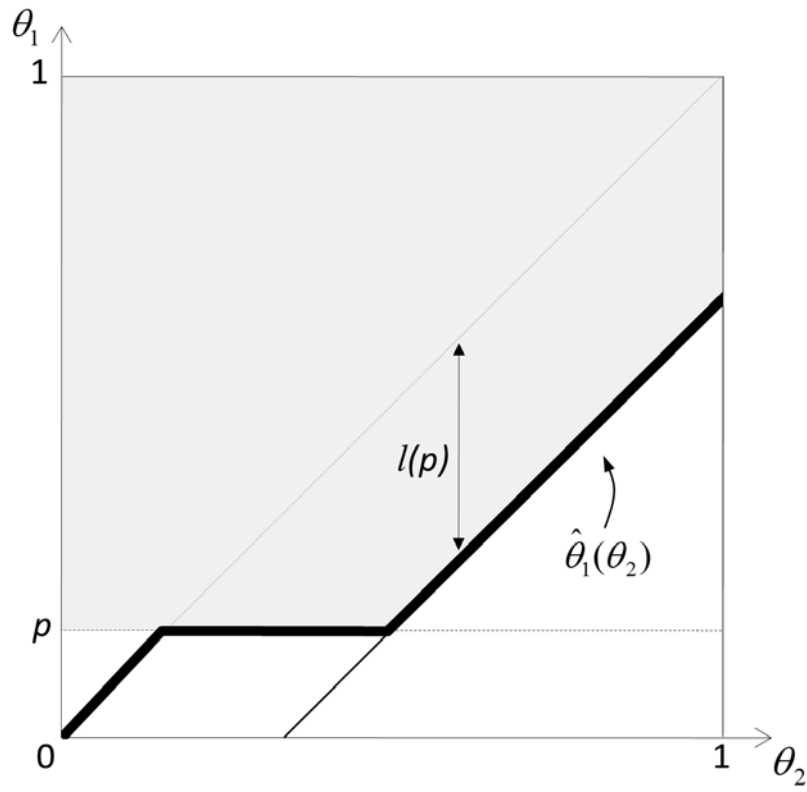


Figure 3: The second-best mechanism for  $p < 3/8$  when both agents' values are uniformly distributed on the interval  $[0, 1]$ .

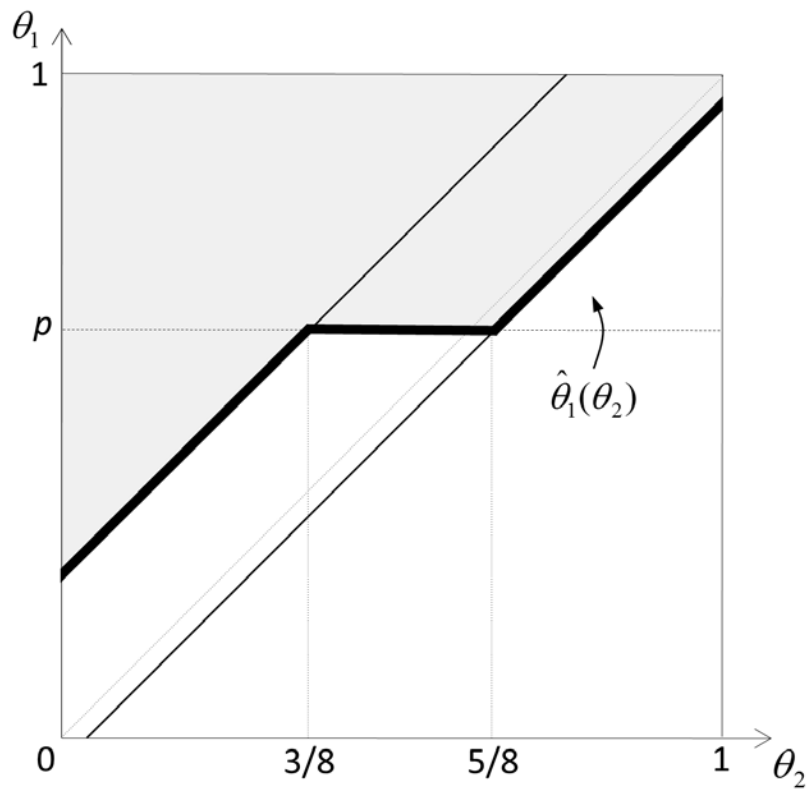


Figure 4: The second-best mechanism for  $3/8 < p < 5/8$  when both agents' values are uniformly distributed on the interval  $[0, 1]$ .

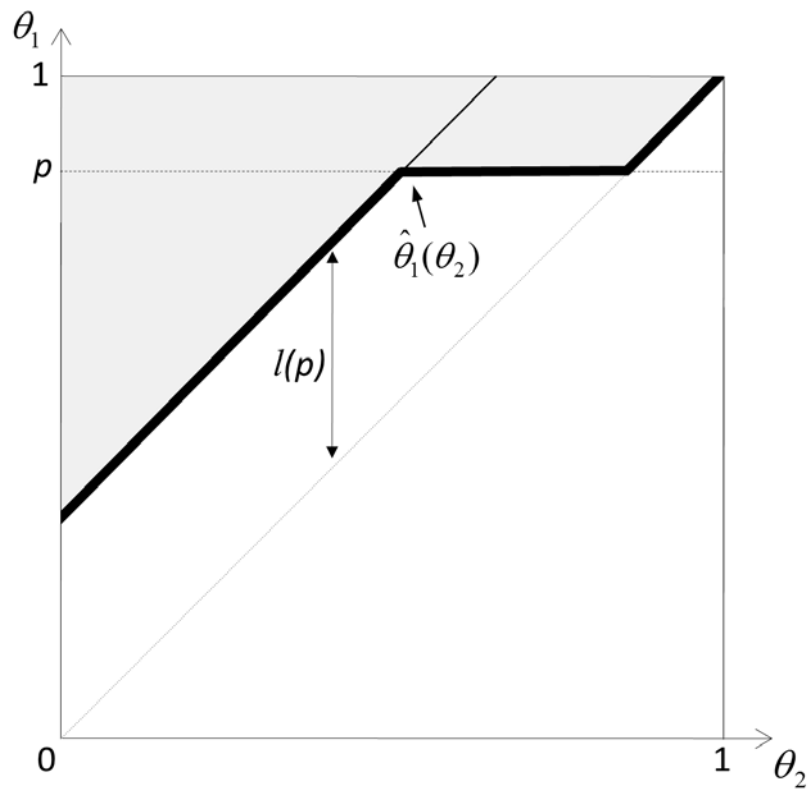
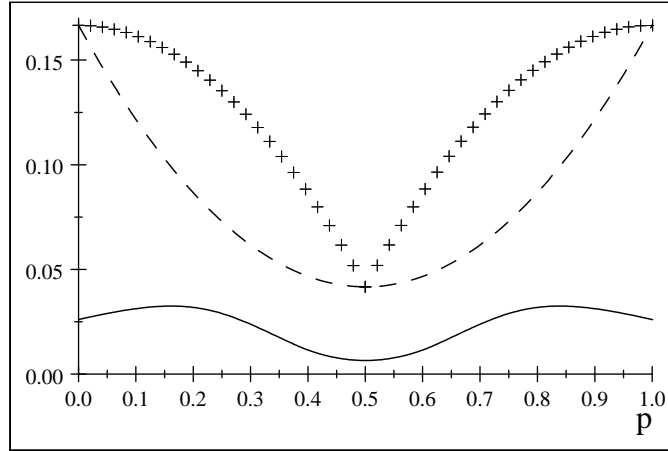


Figure 5: The second-best mechanism for  $p > 5/8$  when both agents' values are uniformly distributed on the interval  $[0, 1]$ .



inefficiency as a function of  $p$ . For comparison, the figure also shows the inefficiency with no bargaining and the deficit for a planner who would subsidize trade to achieve the first best. As can be seen in the figure, the optimal property right has  $p = 1/2$  – equal to the expected value of the passive agent – in all three cases.



Expected Subsidy Needed for First-Best (+ curve)  
 Expected Inefficiency if there were No Bargaining (dashed curve)  
 Expected Inefficiency in Second-Best Bargaining (solid curve)

While the optimal second-best  $p$  is the same as when bargaining is not possible and as when there is an intermediary willing to subsidize trade, Figure 6 shows one significant difference between the second-best case and these others: the surplus achievable with a liability rule is not monotone increasing as  $p$  moves toward  $1/2$ , and is in fact lower for  $p$  close to  $0$  (resp.  $1$ ) than at  $p = 0$  (resp.  $1$ ). That is, a slightly interior  $p$  is worse than the simple property right it is near.<sup>16</sup> The fact that liability rules which induce

<sup>16</sup>In contrast, Lemma 19 in the Appendix shows that in the case of randomized simple property rights the set of status quos for which first-best is achieved is convex [the set is nonempty by Segal and Whinston (2011)], and that a move in the direction of this set raises the second-best expected surplus. Loertscher and Wasser (2014) characterizes the second-best optimal bargaining mechanism with simple property rights when the first best is not attainable, including for cases with more than two agents.

default allocations close to but different from a simple property right are worse than that simple property right does not depend on our assumption of a uniform distribution. As the following proposition shows, it is true for *any* distributions of values for the two agents:

**Proposition 16** *There exists a  $\delta > 0$  such that any liability rule with  $p \in [1, 1 - \delta]$  (resp.  $p \in [0, \delta]$ ) has a lower second-best expected surplus than  $p = 1$  (resp.  $p = 0$ ), which is equivalent to simple ownership by agent 2 (resp. agent 1).*

To understand Proposition 16, note that starting at  $p = 1$  a small reduction in  $p$  weakly increases the default payoff of every type of the active agent 1. At the same time, it increases the expected payoff in the default to essentially all types of the passive agent 2 (whose default payoff when  $p = 1$  simply equals his type), since he then gains  $(p - \theta_2)[1 - F_1(p)]$ .<sup>17</sup> Thus, this change tightens IR constraints, reducing the achievable second-best surplus.

We have also explored computationally a range of cases in which the two agents' values are drawn from differing distributions. In general, it is not the case that the optimal option price  $p$  equals the victim's expected harm,  $\mathbb{E}[\theta_2]$ . Figure 7 illustrates this fact for an example in which the active agent's (agent 1) value has density  $f_1(\theta_1) = 0.2 + 1.6(\theta_1)$  while the passive agent's (agent 2) value is uniformly distributed. The figure plots the expected surplus with the second-best mechanism for each  $p \in [0.45, 0.55]$  in increments of 0.001. The optimal  $p$  is 0.516. The expected surplus at this  $p$  is 0.726996, compared to the expected surplus of 0.726881 when  $p = 0.5 = \mathbb{E}[\theta_2]$ .<sup>18</sup> Interestingly, just as in this example, in all of the cases we have computed, the optimal option price  $p$  is extremely close to the victim's expected harm and the loss from setting instead  $p = \mathbb{E}[\theta_2]$  is very small.<sup>19</sup>

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<sup>17</sup>The complication in the proof is that this is not true for type  $\theta_2 = 1$  (or types near it for  $p < 1$ ).

<sup>18</sup>As a check that this finding is not a result of computational imprecisions, we also examined the case in which we switched the two agents value distributions, as well as other cases with linear density functions for agent 2. For such cases, we know that the optimal  $p$  in fact equals  $\mathbb{E}[\theta_2]$  (see Lemma 18 in the Appendix). Our computational algorithm yields an optimal  $p$  exactly equal to  $\mathbb{E}[\theta_2]$  in all of these cases.

<sup>19</sup>Unfortunately, we have been unable to produce an analytical result showing that this is generally true.

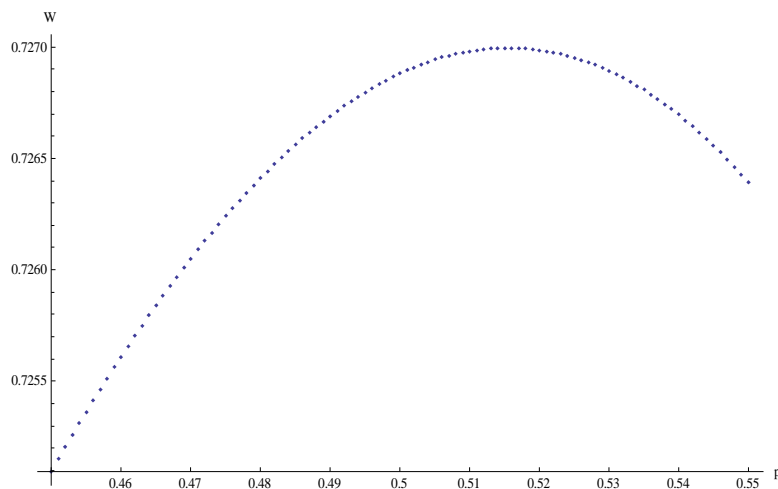


Figure 6: Welfare as a function of  $p$  in an example in which the active agent’s value is distributed with density  $f_1(\theta_1) = 0.2 + 1.6(\theta_1)$  and the passive agent’s value is uniformly distributed. The optimal  $p$  differs from  $1/2$ , the expected value of the passive agent’s value.

## 6 Dual-Chooser Rules

For another application, in this section we consider whether “dual-chooser” rules, as described by Ayres (2005), can improve upon simple property rights or liability rules when there are two agents. In a dual chooser rule, one agent (we will assume agent 2) is the initial owner of the good, but the other agent can get it if both agents agree to this at a pre-specified price  $p$ .

We assume that both agents’ values for the good are independently drawn from the same interval, which we normalize to be  $[0, 1]$ . Our first observation is that with this property rights mechanism, agent 2’s type  $\hat{\theta}_2 = 1$  is an adverse efficient-opt-out type, while agent 1’s type  $\hat{\theta}_1 = 0$  is an adverse efficient-opt-out type (these types never trade, either in the default mechanism or in the efficient mechanism). Since these types have the same reservation utilities as in the standard Myerson-Satterthwaite setting in which agent 2 is the owner, we see immediately that the expected first-best subsidy is the same as in the Myerson-Satterthwaite setting, regardless of  $p$ .

As for the second-best expected surplus, observe that for no  $p$  can it exceed that in the Myerson-Satterthwaite setting where agent 2 has a simple property right because, for any  $p$ , the participation constraints of types  $\hat{\theta}_2 = 1$

and  $\hat{\theta}_1 = 0$  must still be satisfied, and the reservation utilities of these types are the same as in the Myerson-Satterthwaite setting. On the other hand, the second-best expected surplus can be *strictly lower* than in the Myerson-Satterthwaite setting. For example,

**Proposition 17** *If  $\tilde{\theta}_1, \tilde{\theta}_2 \sim U[0, 1]$ , then the Myerson-Satterthwaite mechanism fails to satisfy IR for any dual-chooser rule with posted price  $p \in (0, \frac{1}{2}) \cup (\frac{1}{2}, 1)$ .*

**Proof.** Consider first agent 1. His expected utility in the dual-chooser rule is  $\hat{U}_1(\theta_1) = \max\{0, (\theta_1 - p)p\}$ , while his expected utility in the Myerson-Satterthwaite mechanism is  $U_1(\theta_1) = (\theta_1 - 1/4)_+^2 / 2$  (this can be calculated either from the dominant-strategy “pricing” implementation of this allocation rule, or by the integral formula using the allocation rule). So clearly for  $p < 1/4$ , the IR of  $\theta_1 = 1/4$  will fail. For  $1/4 \leq p < 1/2$ , consider instead type  $\theta_1 = p + 1/4$  (this will actually be the “critical type”). For this type the participation surplus is  $p^2/2 - p/4 = p(p - 1/2)/2 < 0$ .

Now consider agent 2. His expected utility in the dual-chooser rule is  $\hat{U}_2(\theta_2) = \theta_2 + \max\{0, (1 - p)(p - \theta_2)\}$ . while his expected utility in the M-S mechanism is  $U_2(\theta_2) = \theta_2 + (3/4 - \theta_2)_+^2 / 2$ . So clearly for  $p > 3/4$ , the IR of  $\theta_2 = 3/4$  will fail. For  $1/2 < p \leq 3/4$ , consider type  $\theta_2 = p - 1/4$  (this will actually be the “critical type”). For this type the participation surplus is  $(1 - p)^2 / 2 - (1 - p) / 4 = (1 - p)(1/2 - p) / 2 < 0$ . ■

We see then that when bargaining under asymmetric information will take place (of the form we have considered), dual chooser rules cannot improve upon simple property rights, and can be worse. (They must therefore also be weakly worse than the best liability rule.) This can be contrasted with the case in which bargaining is impossible, where for any  $p \in (0, 1)$  the surplus is *strictly higher* under a dual chooser rule than under a simple property right.

## 7 Conclusion

The critical role of property rights for economic efficiency has long been recognized. In this paper, we shed new light on this role, by examining how property rights affect efficiency when agents will bargain under conditions of asymmetric information.

Our results have implications for several literatures. Relative to the theoretical mechanism design literature, we provide a new set of sufficient conditions characterizing when efficiency through bargaining is impossible, which applies not only to the traditional case of simple property rights, but also to more general property rights mechanisms. We then show how efficiency is affected by the property rights allocation in such cases.

In organizational economics, losses due to ex post bargaining inefficiencies were a central theme of Williamson’s Transaction Cost Economics approach to the firm. One can view our analysis, in which we study how property rights can affect those losses, as taking the Grossman-Hart-Moore [Grossman and Hart (1986), Hart and Moore (1990), Hart (1995)] Property Rights Theory approach of asking how asset ownership affects efficiency, but doing so focusing instead on Williamson’s costs of haggling, rather than on inefficiencies in ex ante investments. Like the Property Rights Theory, our approach has implications not only for asset ownership, but also for allocation of decision rights within firms.

Finally, in the legal literature, ever since Calabresi and Malemud (1972), scholars have been interested in the performance of different property rights regimes. Our results shed new light on this issue when bargaining is imperfect due to the presence of asymmetric information. In particular, we have highlighted the effect that bargaining has on the choice among property rights regimes, relative to the case in which bargaining is impossible.

## 8 Appendix: Proofs

### 8.1 Proof of Lemma 14

**Proof.** Note that in any Bayesian incentive compatible mechanism, agent 2’s expected consumption  $1 - E[x_1(\tilde{\theta}_1, \theta_2)]$  must be nondecreasing in  $\theta_2$ , therefore there will be a type  $\hat{\theta}_2$  such that

$$\left\{ 1 - F_1(p) - E[x_1(\tilde{\theta}_1, \theta_2)] \right\} \text{sign}(\theta_2 - \hat{\theta}_2) \geq 0 \text{ for all } \theta_2 \quad (16)$$

Consider the designer’s “relaxed problem” in which she chooses  $\hat{\theta}_2$ , the allocation rule  $x(\cdot)$ , and interim expected utilities  $U_1(\cdot)$ ,  $U_2(\cdot)$  to maximize expected surplus subject to (16), expected budget balance, first-order incentive compatibility (ICFOC), agent 1’s participation constraints  $IR_1(0)$  and

$\text{IR}_1(1)$ , and agent 2's participation constraint  $\text{IR}_2(\hat{\theta}_2)$ .<sup>20</sup> The Lagrangian for this problem (leaving ICFOC as constraints) is

$$\begin{aligned} & E \left[ \tilde{\theta}_1 x_1(\tilde{\theta}) + \tilde{\theta}_2(1 - x_1(\tilde{\theta})) \right] + E \left[ \delta(\tilde{\theta}_2)(1 - F_1(p) - x_1(\tilde{\theta})) \text{sign}(\tilde{\theta}_2 - \hat{\theta}_2) \right] \\ & + \lambda \left\{ E[\tilde{\theta}_1 x_1(\tilde{\theta}) + \tilde{\theta}_2(1 - x_1(\tilde{\theta}))] - E[U_1(\tilde{\theta}_1)] - E[U_2(\tilde{\theta}_2)] \right\} \\ & + \mu_0 U_1(0) + \mu_1 (U_1(1) - (1 - p)) + \nu [U_2(\hat{\theta}_2) - p[1 - F_1(p)] - \hat{\theta}_2 F_1(p)] \\ & \text{s.t. ICFOC} \end{aligned}$$

It is easy to see that  $\lambda > 0$  (since the first-best is impossible), while  $\mu_0, \mu_1 \geq 0$  must satisfy the complementary slackness conditions

$$\mu_0 U_1(0) = 0 \text{ and } \mu_1 [U_1(1) - (1 - p)] = 0 \quad (17)$$

and  $\delta(\cdot) \geq 0$  must satisfy the complementary slackness conditions

$$\delta(\theta_2) \left\{ E[x_1(\tilde{\theta}_1, \theta_2)] - (1 - F_1(p)) \right\} = 0 \text{ for all } \theta_2 \neq \hat{\theta}_2. \quad (18)$$

Note that the solution must have  $\nu = \lambda$  (otherwise we could raise the Lagrangian by adding a constant to  $U_2(\cdot)$  without affecting ICFOC), and  $\mu_0 + \mu_1 = \lambda$  (otherwise we could raise the Lagrangian by adding a constant to  $U_1(\cdot)$  without affecting ICFOC). Hence, we can rewrite the Lagrangian as

$$\begin{aligned} & (1 + \lambda) E \left[ \tilde{\theta}_1 x_1(\tilde{\theta}) + \tilde{\theta}_2(1 - x_1(\tilde{\theta})) \right] + E \left[ \delta(\tilde{\theta}_2)(1 - F_1(p) - x_1(\tilde{\theta})) \text{sign}(\tilde{\theta}_2 - \hat{\theta}_2) \right] \\ & - \lambda \left( \frac{\lambda - \mu_1}{\lambda} \right) \left\{ E[U_1(\tilde{\theta}_1)] - U_1(0) \right\} - \lambda \left( \frac{\mu_1}{\lambda} \right) \left\{ E[U_1(\tilde{\theta}_1)] - U_1(1) + 1 - p \right\} \\ & - \lambda \left\{ E[U_2(\tilde{\theta}_2)] - U_2(\hat{\theta}_2) \right\} - \lambda \left\{ p[1 - F_1(p)] + \hat{\theta}_2 F_1(p) \right\} \end{aligned}$$

Note also that we can always satisfy one of the complementary slackness conditions (17) by adding a constant to  $U_1(\cdot)$ . To be able to satisfy both of them at the same time while satisfying  $\text{IR}_1(1)$  and  $\text{IR}_1(0)$ , we need to be in one of the following three cases: (i)  $\mu_0, \mu_1 > 0$  implies  $U_1(1) - U_1(0) = 1 - p$ ,

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<sup>20</sup>By standard arguments that adjust the transfer rule [e.g., Lemma 1 in Segal and Whinston (2011)], ex ante budget balance can be strengthened to ex post budget balance without affecting expected surplus or any of the other constraints. Moreover, provided that the allocation rule is monotone, all IC and IR constraints will be satisfied for the reasons stated in text.

(ii)  $\mu_1 = 0$  implies  $\mu_0 = \lambda > 0$  hence  $U_1(0) = 0$  and  $U_1(1) - U_1(0) = U_1(1) \geq 1 - p$ , or (iii)  $\mu_0 = 0$  implies  $\mu_1 = \lambda$  hence  $U_1(1) = 1 - p$  and  $U_1(1) - U_1(0) = 1 - p - U_1(0) \leq 1 - p$ .

For now, fix  $\hat{\theta}_2$  and consider maximizing with respect to the allocation rule  $x_1(\cdot)$  the simplified Lagrangian (which drops all terms not involving the allocation rule)

$$\begin{aligned} & (1 + \lambda) E \left[ \tilde{\theta}_1 x_1(\tilde{\theta}) + \tilde{\theta}_2 (1 - x_1(\tilde{\theta})) \right] - E \left[ \delta(\tilde{\theta}_2) x_1(\tilde{\theta}) \text{sign}(\tilde{\theta}_2 - \hat{\theta}_2) \right] \\ & - \lambda \left( \frac{\lambda - \mu_1}{\lambda} \right) \left\{ E[U_1(\tilde{\theta}_1)] - U_1(0) \right\} - \lambda \left( \frac{\mu_1}{\lambda} \right) \left\{ E[U_1(\tilde{\theta}_1)] - U_1(1) + 1 - p \right\} \\ & - \lambda \left\{ E[U_2(\tilde{\theta}_2)] - U_2(\hat{\theta}_2) \right\} \end{aligned}$$

Now using ICFOC and integration by parts, dividing by  $1 + \lambda$ , and letting  $\hat{\lambda} \equiv \lambda / (1 + \lambda)$ ,  $\hat{\delta}(\theta_2) \equiv \delta(\theta_2) / (1 + \lambda)$  and  $\gamma \equiv \mu_1 / \lambda$ , this simplified Lagrangian can be rewritten as

$$E \left[ \omega_1(\tilde{\theta}_1 | \hat{\lambda}, \gamma) x_1(\tilde{\theta}) + \left\{ 1_{\{\tilde{\theta}_2 \geq \hat{\theta}_2\}} \underline{\omega}_2(\tilde{\theta}_2 | \hat{\lambda}) + 1_{\{\tilde{\theta}_2 < \hat{\theta}_2\}} \bar{\omega}_2(\tilde{\theta}_2 | \hat{\lambda}) \right\} (1 - x_1(\tilde{\theta})) \right. \\ \left. - \text{sign}(\theta_2 - \hat{\theta}_2) \delta(\tilde{\theta}_2) x_1(\tilde{\theta}) \right].$$

It is maximized pointwise by a solution of the form (13), where

$$\hat{\theta}_1(\theta_2) = \begin{cases} \omega_1^{-1}(\bar{\omega}_2(\theta_2 | \hat{\lambda}) - \hat{\delta}(\theta_2) | \hat{\lambda}, \gamma) & \text{for } \theta_2 < \hat{\theta}_2, \\ \omega_1^{-1}(\underline{\omega}_2(\theta_2 | \hat{\lambda}) + \hat{\delta}(\theta_2) | \hat{\lambda}, \gamma) & \text{for } \theta_2 > \hat{\theta}_2. \end{cases} \quad (19)$$

Let  $\bar{\theta}_2(\theta_1 | \lambda, \gamma) \equiv \{\theta_2 : \underline{\theta}_1(\theta_2 | \lambda, \gamma) = \theta_1\}$  and  $\underline{\theta}_2(\theta_1 | \lambda, \gamma) \equiv \{\theta_2 : \bar{\theta}_1(\theta_2 | \lambda, \gamma) = \theta_1\}$ , and define  $\bar{\theta}_2 \equiv \bar{\theta}_2(p | \hat{\lambda}, \gamma)$  and  $\underline{\theta}_2 \equiv \underline{\theta}_2(p | \hat{\lambda}, \gamma)$ . Note that under our assumptions we have  $\underline{\theta}_2 \leq \bar{\theta}_2$ . Observe that by (16) and the definition of  $\underline{\theta}_2, \bar{\theta}_2$ , we have  $\hat{\theta}_1(\theta_2) \leq p < \bar{\theta}_1(\theta_2 | \lambda, \gamma)$  for all  $\theta_2 \in (\underline{\theta}_2, \hat{\theta}_2)$  and  $\hat{\theta}_1(\theta_2) \geq p > \underline{\theta}_1(\theta_2 | \lambda, \gamma)$  for all  $\theta_2 \in (\hat{\theta}_2, \bar{\theta}_2)$ . This in turn implies, using (19), that for all  $\theta_2 \in (\min\{\underline{\theta}_2, \hat{\theta}_2\}, \max\{\hat{\theta}_2, \bar{\theta}_2\})$  we have  $\hat{\delta}(\theta_2) > 0$  and therefore for all such  $\theta_2$ , by (18),  $E[x_1(\hat{\theta}_1, \theta_2)] = 1 - F_1(p)$ , which implies  $\hat{\theta}_1(\theta_2) = p$ .

Next, for  $\theta_2 < \min\{\underline{\theta}_2, \hat{\theta}_2\}$ ,  $\hat{\delta}(\theta_2) \geq 0$  implies that  $\hat{\theta}_1(\theta_2) \leq \bar{\theta}_1(\theta_2 | \lambda, \gamma) < p$ , therefore  $E[x_1(\hat{\theta}_1, \theta_2)] > 1 - F_1(p)$ , and so by (18)  $\hat{\delta}(\theta_2) = 0$ , implying  $\hat{\theta}_1(\theta_2) = \bar{\theta}_1(\theta_2 | \lambda, \gamma)$ . Similarly for  $\theta_2 > \max\{\hat{\theta}_2, \bar{\theta}_2\}$ ,  $\hat{\delta}(\theta_2) \geq 0$  implies that  $\hat{\theta}_1(\theta_2) \geq \underline{\theta}_1(\theta_2 | \lambda, \gamma) > p$ , therefore  $E[x_1(\hat{\theta}_1, \theta_2)] < 1 - F_1(p)$ , and so by (18)

$\hat{\delta}(\theta_2) = 0$ , implying  $\hat{\theta}_1(\theta_2) = \underline{\theta}_1(\theta_2|\lambda, \gamma)$ . Thus, the solution takes the form

$$\hat{\theta}_1(\theta_2) = \begin{cases} \bar{\theta}_1(\theta_2|\lambda, \gamma) & \text{for } \theta_2 < \min\{\underline{\theta}_2, \hat{\theta}_2\}, \\ p & \text{for } \theta_2 \in (\min\{\underline{\theta}_2, \hat{\theta}_2\}, \max\{\hat{\theta}_2, \bar{\theta}_2\}), \\ \underline{\theta}_1(\theta_2|\lambda, \gamma) & \text{for } \theta_2 > \max\{\hat{\theta}_2, \bar{\theta}_2\}. \end{cases} \quad (20)$$

Now consider optimal choice of  $\hat{\theta}_2$ : since the solution for  $\hat{\theta}_2 < \underline{\theta}_2$  or  $\hat{\theta}_2 > \bar{\theta}_2$  is feasible in the problem for  $\hat{\theta}_2 \in [\underline{\theta}_2, \bar{\theta}_2]$  (while the converse is not true), it is optimal to choose  $\hat{\theta}_2 \in [\underline{\theta}_2, \bar{\theta}_2]$ , in which case the function  $\hat{\theta}_1(\theta_2)$  is given by (14).

The complementary slackness conditions (17) are given by (15), since by ICFOC and Fubini's Theorem

$$U_1(1) - U_1(0) = \int_0^1 E[x_1(\theta_1, \tilde{\theta}_2)] d\theta_1 = E \left[ \int_0^1 x_1(\theta_1, \tilde{\theta}_2) d\theta_1 \right] = 1 - E[\hat{\theta}_1(\tilde{\theta}_2)]. \quad (21)$$

Finally, note that the constructed solution actually satisfies all the incentive constraints (since for each  $i$ ,  $E_{\tilde{\theta}_i} x_i[(\theta_i, \tilde{\theta}_i)]$  is nondecreasing in  $\theta_i$ ) and all of the participation constraints (by the argument in the text before the proposition). ■

We now describe a transfer rule that implements the allocation rule above in a dominant strategy IC mechanism that has the right participation constraints binding. When  $\gamma < 1$ , i.e.,  $\text{IR}_1(0)$  binds, we let  $t_1(\theta_1, \theta_2) = -\hat{\theta}_1(\theta_2) x_1(\theta_1, \theta_2)$  – i.e., agent 1 pays  $\hat{\theta}_1(\theta_2)$  when he consumes the object.<sup>21</sup> When  $\gamma > 0$ , i.e.,  $\text{IR}_1(1)$  binds, we let  $t_1(\theta_1, \theta_2) = -p + (1 - x_1(\theta_1, \theta_2)) \hat{\theta}_1(\theta_2)$  – i.e., agent 1 first takes the object at  $p$  and then is paid  $\hat{\theta}_1(\theta_2)$  when he gives it up.<sup>22</sup> For  $\gamma \in (0, 1)$ , by (15) the two payments have the same expectation over  $\theta_2$  for every  $\theta_1$ . In particular, in that case we can elect the first option for  $t_1$  when  $\theta_1 < p$  and the second option when  $\theta_1 > p$ , yielding transfer rule

$$t_2(\theta_1, \theta_2) = \begin{cases} -\hat{\theta}_1(\theta_2) x_1(\theta_1, \theta_2) & \text{if } \theta_1 < p, \\ -p + (1 - x_1(\theta_1, \theta_2)) \hat{\theta}_1(\theta_2) & \text{if } \theta_1 > p, \end{cases} \quad (22)$$

<sup>21</sup>Since  $x_1(0, \theta_2) = 1$  for all  $\theta_2$  when  $\gamma < 1$ , type 0 of agent 1 has an expected payoff of 0.

<sup>22</sup>Since  $x_1(0, \theta_2) = 0$  for all  $\theta_2$  when  $\gamma > 1$ , type 1 of agent 1 has an expected payoff of  $1 - p$ .



As for agent 2, we let

$$t_2(\theta_1, \theta_2) = \begin{cases} \underline{\theta}_2(\theta_1|\hat{\lambda}, \gamma) x_1(\theta_1, \theta_2) & \text{if } \theta_1 < p, \\ p - \bar{\theta}_2(\theta_1|\hat{\lambda}, \gamma)(1 - x_1(\theta_1, \theta_2)) & \text{if } \theta_1 > p, \end{cases} \quad (23)$$

That is, if agent 1 would not exercise his option at the default, then agent 2 receives  $\underline{\theta}_2(\theta_1|\hat{\lambda}, \gamma)$  whenever he sells the object, while if agent 1 would exercise his option at the default, then agent 2 receives  $p$  but pays back  $\bar{\theta}_2(\theta_1|\hat{\lambda}, \gamma)$  whenever he ends up keeping the object.<sup>23</sup>

Adding the two transfer rules (22) and (23) yields a budget deficit of

$$\begin{aligned} & \left[ \underline{\theta}_2(\theta_1|\hat{\lambda}, \gamma) - \hat{\theta}_1(\theta_2) \right] x_1(\theta_1, \theta_2) \quad \text{when } \theta_1 < p, \\ & \left[ \hat{\theta}_1(\theta_2) - \bar{\theta}_2(\theta_1|\hat{\lambda}, \gamma) \right] (1 - x_1(\theta_1, \theta_2)) \quad \text{when } \theta_1 > p. \end{aligned} \quad (24)$$

## 8.2 Lemma 18

**Lemma 18** *Suppose that agent 1's type  $\theta_1$  is uniformly distributed and let  $\hat{\theta}_1(\theta_2|p)$  describe the second-best allocation rule given  $p$  [as specified in (14)]. Then, if for any  $p$  and  $p'$  both  $IR_1(0)$  and  $IR_1(1)$  bind in the optimal second-best mechanism, then  $\hat{\theta}_1(\theta_2|p') = \hat{\theta}_1(\theta_2|p) + (p' - p)$ . Among such  $p$ , expected surplus is maximized by setting  $p = E(\theta_2)$ .*

**Proof.** Define the two regions  $A_p \equiv \{\theta : \theta_1 \in (\hat{\theta}_1(\theta_2|p), p)\}$  and  $B_p \equiv \{\theta : (p, \hat{\theta}_1(\theta_2|p))\}$ . By (15), the probabilities of these two regions must be equal (these are the two regions in which the final allocation differs from what would happen if agent 1 simply exercised his option optimally). Observe that with a constant shift in  $\hat{\theta}_1(\theta_2)$  budget balance is preserved: For every state  $\theta$  in region  $A_p$ , the deficit is now exactly  $\delta$  smaller, while for every state  $\theta$  in region  $B_p$ , the deficit is now exactly  $\delta$  larger. Given the uniform distribution of  $\theta_1$  and the fact that we integrate in each case over the same sets of  $\theta_2$ , this change has no effect on the expected deficit. Thus, if  $\hat{\theta}_1(\theta_2|p)$  maximizes expected

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<sup>23</sup>Observe that with this payment rule, type  $\hat{\theta}_2$  of agent 2 has expected payoff  $\hat{\theta}_2 F_1(p) + p[1 - F_1(p)]$ , so  $IR_2(\hat{\theta}_2)$  holds with equality.

surplus with budget balance under default  $p$ , then  $\hat{\theta}_1(\theta_2|p') = \hat{\theta}_1(\theta_2|p) + (p' - p)$  must do so under default  $p'$ .

Next, observe that the optimal mechanism in the region in which  $\text{IR}_1(0)$  and  $\text{IR}_1(1)$  both bind therefore [again, given the uniform distribution of  $\theta_1$ ] has a constant improvement in expected surplus over the expected surplus arising when agent 1 simply exercises his option optimally. Since the latter expected surplus is maximized at  $p = 1/2$ , so is the former. ■

### 8.3 Proof of Proposition 15

Consider the special case of  $F_1, F_2$  being uniform on  $[0, 1]$ , in which

$$\underline{\omega}_i(\theta_i|\lambda) = (1 + \lambda)\theta_i - \lambda \text{ and } \bar{\omega}_i(\theta_i|\lambda) = (1 + \lambda)\theta_i. \quad (25)$$

Then we have

$$\begin{aligned} \underline{\theta}_2(\theta_1) &= \theta_1 - \underline{l} \\ \bar{\theta}_2(\theta_1) &= \theta_1 + \bar{l} \\ \bar{\theta}_1(\theta_2) &= \theta_2 + \underline{l} \\ \underline{\theta}_1(\theta_2) &= \theta_2 - \bar{l} \end{aligned}$$

and

$$\hat{\theta}_1(\theta_2) = \min \left\{ \max \left\{ \theta_2 - \bar{l}, p \right\}, \theta_2 + \underline{l} \right\},$$

where  $\underline{l} = \lambda(1 - \gamma)/(1 + \lambda)$  and  $\bar{l} = \lambda\gamma/(1 + \lambda)$ .

#### 8.3.1 Only $\text{IR}_1(0)$ binds

Now consider the relaxed problem in which we ignore  $\text{IR}_1(1)$ . The solution to that problem corresponds to the case in which  $\gamma = 0$ , hence  $\bar{l} = 0$ . Let  $\underline{l} = l = \lambda/(1 + \lambda)$ . We can use the following transfer for agent 1:

$$\begin{aligned} t_1(\theta_1, \theta_2) &= -\hat{\theta}_1(\theta_2)x_1(\theta_1, \theta_2) \text{ when } \theta_1 < p, \\ t_1(\theta_1, \theta_2) &= -E[\hat{\theta}_1(\tilde{\theta}_2)] + (1 - x_1(\theta_1, \theta_2))\hat{\theta}_1(\theta_2) \text{ when } \theta_1 > p. \end{aligned}$$

since in both cases  $E[t_1(\theta_1, \tilde{\theta}_2)] = E[\hat{\theta}_1(\tilde{\theta}_2)x_1(\theta_1, \tilde{\theta}_2)]$ . Given these transfers, the budget deficit is then

$$\begin{aligned} &\left[ \underline{\theta}_2(\theta_1) - \hat{\theta}_1(\theta_2) \right] x_1(\theta_1, \theta_2) \text{ when } \theta_1 < p, \\ &\left[ \hat{\theta}_1(\theta_2) - \bar{\theta}_2(\theta_1) \right] (1 - x_1(\theta_1, \theta_2)) + p - E[\hat{\theta}_1(\tilde{\theta}_2)] \text{ when } \theta_1 > p. \end{aligned}$$

Focusing first on the region where  $p < \theta_1 < \theta_2$ , the subsidy there is the whole gains from trade  $\theta_2 - \theta_1$ . Since the efficient expected gains from trade in the M-S model with  $U [0, 1]^2$  distribution is  $1/6$ , the expected gains from trade on the region  $p < \theta_1 < \theta_2$  is  $(1 - p)^3 / 6$  (the probabilities and the gains themselves are scaled by  $1 - p$ ).<sup>24</sup> Now, in the region  $\theta_2 + l < \theta_1 < p$ , the subsidy  $\theta_1 - \theta_2 - 2l$  can be interpreted as (a) paying the gains from trade as if agent 1's value were  $\theta_1 - l$  and trade were efficient for that value, and then (b) getting back  $l$  on every trade that happened. In expectation, (a) yields  $(p - l)^3 / 6$ , and (b) yields  $l(p - l)^2 / 2$ .<sup>25</sup> Finally, we have the term  $p - E[\theta_1(\tilde{\theta}_2)]$ , which has to be paid when  $\theta_1 > p$ , which in expectation costs  $(1 - p) [(p - l)^2 / 2 - (1 - p)^2 / 2]$ . Adding all the terms yields

$$\begin{aligned} & (1 - p)^3 / 6 + (p - l)^3 / 6 - l(p - l)^2 / 2 + ((p - l)^2 / 2 - (1 - p)^2 / 2) (1 - p) \\ = & p - lp + \frac{1}{2}l^2 - \frac{2}{3}l^3 - \frac{1}{2}p^2 + l^2p - \frac{1}{3} \end{aligned}$$

Requiring ex ante budget balance sets this expression to 0. We want to express  $l$  through  $p$  but it's easier to do the reverse. The solutions are:

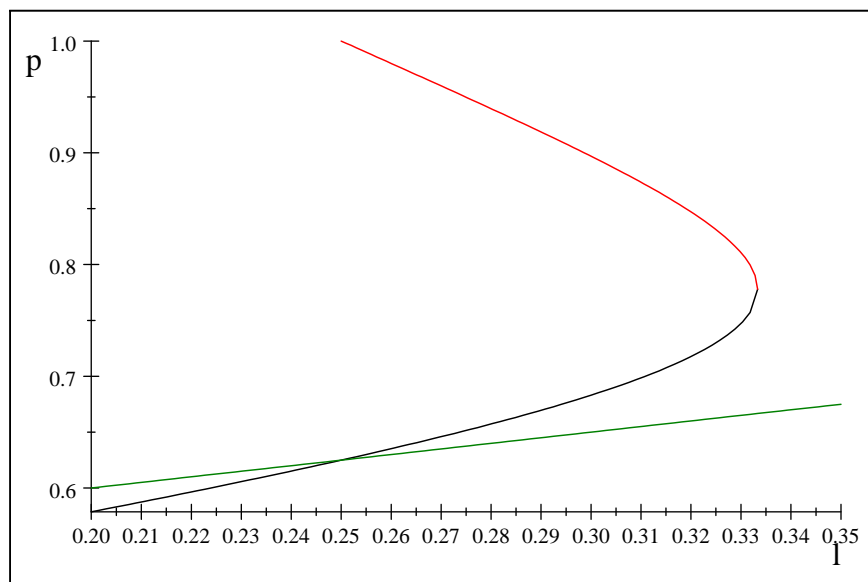
$$p = 1 - l(1 - l) \pm \frac{1}{3}\sqrt{3(1 - l)^3(1 - 3l)}$$

Putting both solutions on the same graph as red and black curves we get

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<sup>24</sup>In general, the Myerson-Satterthwaite deficit with uniform distributions on  $[0, 1]$  and a "gap" equal to  $l$  is  $(1 - 4l)(1 - l)^2 / 6$  [see Myerson and Satterthwaite (1983, p. 277)]. So when  $l = 0$ , the deficit is  $1/6$ . We get  $(1 - p)^3 / 6$  because the probability of being in the region  $p < \theta_1 < \theta_2$  is  $(1 - p)^2$  and the region is  $[0, 1]^2$  scaled down by  $(1 - p)$ .

<sup>25</sup>Alternatively, the region  $\max\{\theta_1, \theta_2\} < p$  is a scaled-down version of a Myerson-Satterthwaite  $[0, 1]^2$  trading box with a "gap" equal to  $l/p$ . Using the formula in footnote XX, the Myerson-Satterthwaite deficit would be  $(1 - 4\frac{l}{p})(1 - \frac{l}{p}) / 6$ . The probability of an outcome in this region is  $p^2$  and the deficit is scaled down by  $p$ , so this region contributes  $(p - 4l)(p - l) / 6$  to the expected deficit.



Combining the red and black curves yields  $l(p)$ , which we see is inverse U-shaped.

The green line is the boundary of when  $IR_1(1)$  is satisfied, which is when  $E[\hat{\theta}_1(\theta_2)] \leq p$ ; i.e.,  $p - l \geq 1 - p$ , or  $p \geq (1 + l)/2$ . Intersecting the line with the black curve we solve

$$1 - l(1 - l) - \frac{1}{3}\sqrt{3(1 - l)^3(1 - 3l)} = (1 + l)/2$$

The solution is:  $l = \frac{1}{4}$ , and the corresponding level of  $p$  is  $p = 1 - l(1 - l) - \frac{1}{3}\sqrt{3(1 - l)^3(1 - 3l)} = 5/8 = 0.625$

Thus, since the solution to the relaxed problem satisfies  $IR_1(1)$  when  $p \geq 5/8$ , it is the actual solution in those cases. The case where  $p \leq 3/8$  and only  $IR_1(1)$  binds is symmetric.

**Expected Welfare Loss Calculation** Now, calculate the expected welfare loss when  $p \geq 5/8$ . It is

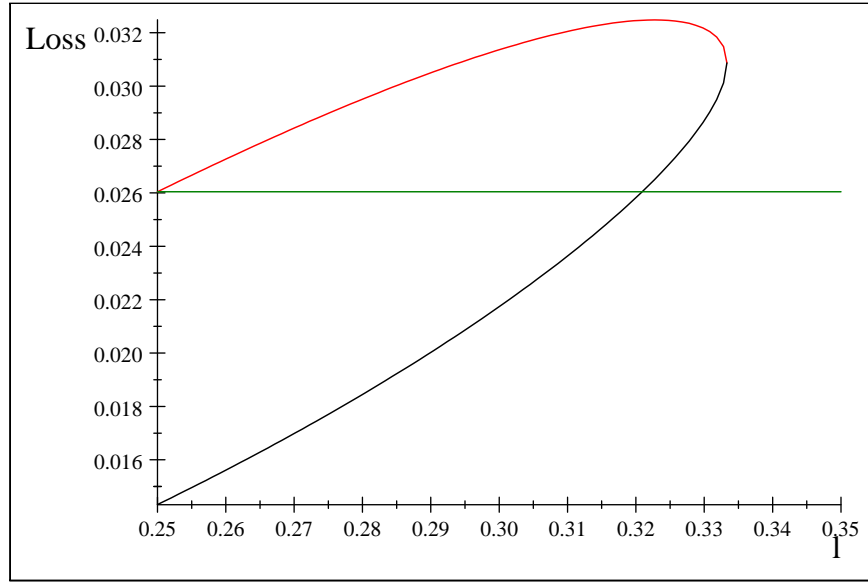
$$p^3/6 - (p - l)^3/6 - l(p - l)^2/2 = \frac{1}{6}l^2(3p - 2l).$$

(The first term is if there were no trade at all for values below  $p$ , the second term is expected gains from trade on the triangle below  $p$  assuming that  $l$  is wasted each time, and the third term accounts for  $l$  not being wasted.)

Substituting  $p$  from the red and black curves into this expression yields the welfare loss

$$\frac{1}{6}l^2 \left( 3l^2 - 5l + 3 \pm \sqrt{3(1-l)^3(1-3l)} \right),$$

which can be plotted as follows:



Reducing  $p$  from 1 corresponds to moving along the curve clockwise, starting on the red curve and then shifting to the black curve. Note that the loss is increasing  $p$  as we reduce  $p$  from 1 for most of the red curve. The green line here is the M-S welfare loss (i.e., when  $p = 1$  and  $l = 1/4$ ) which is  $\frac{5}{192}$ . The loss exceeds the M-S loss for  $l > \hat{l} \approx 0.321$ , which corresponds to  $\hat{p} \geq p \approx 0.720$ .

The point where welfare loss is maximized is given by solving

$$0 = \frac{d}{dl} \left( l^2 \left( 3l^2 - 5l + 3 + \sqrt{3(1-l)^3(1-3l)} \right) \right)$$

$$l \in [1/4, 1/2]$$

The solution is  $l \approx 0.323$ , which corresponds to  $p \approx 0.839$ .

### 8.3.2 Only $\text{IR}_1(1)$ binds

By symmetry, the solution has only  $\text{IR}_1(1)$  bind when  $p \leq 3/8$  and results, for each such  $p$ , in identical welfare losses to the case with  $p + 1/2$ .

### 8.3.3 Both $\text{IR}_1(0)$ and $\text{IR}_1(1)$ bind

When  $p \in (3/8, 5/8)$  both  $\text{IR}_1(0)$  and  $\text{IR}_1(1)$  must bind. Applying (24), the budget deficit in this case is

$$\begin{aligned} \theta_1 - \theta_2 - 2\underline{l} & \text{ when } \theta_2 + \underline{l} < \theta_1 < p, \\ \theta_2 - \theta_1 - 2\bar{l} & \text{ when } p < \theta_1 < \theta_2 - \bar{l}, \\ 0 & \text{ otherwise.} \end{aligned}$$

By (15),  $E[\hat{\theta}_1(\tilde{\theta}_2)] = p$ . This implies that the probabilities of the two regions  $A \equiv \{\theta : \theta_2 + \underline{l} < \theta_1 < p\}$  and  $B \equiv \{\theta : p < \theta_1 < \theta_2 - \bar{l}\}$  are equal (these are the two regions in which the final allocation differs from what would happen if agent 1 simply exercised his option optimally). This involves having  $\bar{l} = \underline{l} + (1 - 2p)$ .

Consider first the case of  $p = 1/2$ . In this case, regions  $A$  and  $B$  have equal probability when  $\underline{l} = \bar{l}$ . The optimal allocation in this case can be interpreted as separate Myerson-Satterthwaite mechanisms for the cases  $\theta_1, \theta_2 < 1/2$  (in which agent 1 is the buyer) and  $\theta_1, \theta_2 > 1/2$  (in which agent 1 is the seller), with no cross-subsidization. The unique gap that achieves budget balance and maximizes expected surplus is half of the Myerson-Satterthwaite gap:  $l_{1/2} = 1/2 \cdot 1/4 = 1/8$ . For other  $p \in (3/8, 5/8)$ , we apply Lemma BOTH IRs BIND.

## 8.4 Lemma 19

**Lemma 19** *When random or divisible property rights are feasible, resulting in payoffs that are linear in  $x$ , the maximal second-best welfare is convex in the status quo level of  $x$ ,  $\hat{x}$ .*

**Proof.** Let  $\langle \chi, \tau \rangle$  be a second-best optimal mechanism for status quo  $\hat{x}$  and  $\langle \chi', \tau' \rangle$  be a second-best optimal mechanism for property rights  $\hat{x}'$ . For  $\alpha \in [0, 1]$ , consider the mechanism  $\langle \chi'', \tau'' \rangle = \alpha \langle \chi, \tau \rangle + (1 - \alpha) \langle \chi', \tau' \rangle$ . (Given linearity of payoffs, it is equivalent to the randomized mechanism that implements  $\langle \chi, \tau \rangle$  with probability  $\alpha$  and  $\langle \chi', \tau' \rangle$  with probability  $1 - \alpha$ .)

Mechanism  $\langle \chi'', \tau'' \rangle$  inherits IC and BB from mechanisms  $\langle \chi, \tau \rangle$  and  $\langle \chi', \tau' \rangle$ , and satisfies IR for status quo  $\hat{x}'' = \alpha \hat{x} + (1 - \alpha) \hat{x}'$ . ■

Lemma 19 implies, in particular, that the set of status quos for which first-best is achieved is convex (the set is nonempty by Segal-Whinston 2011), and that a move in the direction of this set raises the second-best expected surplus.

## 8.5 Proof of Proposition 16

Consider the Myerson-Satterthwaite solution, which corresponds to the case of  $p = 1$  and  $\gamma = 0$  of the above Lemma. Let  $\lambda_1$  denote the Lagrange multiplier on expected budget balance in this solution, and let  $\hat{\lambda}_1 \equiv \lambda_1 / (1 + \lambda_1)$ .

Now, fix the option price  $\bar{p}$  and a type  $\hat{\theta}_2$  and consider the program  $R(\bar{p}, \hat{\theta}_2)$  of choosing the allocation rule, the utility mappings  $U_1(\cdot)$  and  $U_2(\cdot)$ , and the “ironing point  $p$ ” [which affects the solution through constraint (16)] to maximize expected surplus plus  $\lambda_1$  times expected revenue subject to only  $\text{IR}_2(\hat{\theta}_2)$ ,  $\text{IR}_1(0)$ , ICFOC, and constraint (16), with optimization being over the allocation rule  $x_1(\cdot)$  and  $p \in [\hat{\theta}_2, 1]$ . We will first show that there is a type  $\hat{\theta}_2 \in (\underline{\theta}_2(1), 1)$  such that the solution has  $p = 1$  and the same allocation rule as in the Myerson-Satterthwaite solution. Thus, this program achieves the Myerson-Satterthwaite expected surplus, and, if  $\bar{p} = 1$ , satisfies expected budget balance. Moreover, it also satisfies  $\text{IR}_2(\theta_2)$  for all  $\theta_2$  as well as monotonicity (and thus, global IC). Thus, the value of program  $R(1, \hat{\theta}_2)$  is exactly the second-best (Myerson-Satterthwaite) expected surplus.

To see this, observe that by arguments in the proof of Lemma 14), the solution takes the form described by (13) and (20) [note that we are in the case  $\underline{\theta}_2(p) \leq \underline{\theta}_2(1) < \hat{\theta}_2$ ]. Now maximization of the Lagrangian over  $p$  takes the form

$$p \in \arg \max_{p' \in [\hat{\theta}_2, 1]} (1 - F_1(p')) E \left[ \hat{\delta}(\tilde{\theta}_2) \text{sign}(\tilde{\theta}_2 - \hat{\theta}_2) \right]. \quad (26)$$

By (19) and the fact that  $\underline{\omega}_i(\theta_i | \hat{\lambda}) \leq \theta_i \leq 1$  for each  $i = 1, 2$ ,  $\theta_i \in [0, 1]$ , we must have

$$\begin{aligned} \hat{\delta}(\theta_2) &= \max\{\bar{\omega}_2(\theta_2 | \hat{\lambda}_1) - \underline{\omega}_1(p | \hat{\lambda}_1, 0)\} \geq \max\{\bar{\omega}_2(\theta_2 | \hat{\lambda}_1) - 1, 0\} \text{ for } \theta_2 < \hat{\theta}_2, \\ \hat{\delta}(\theta_2) &= \max\{\underline{\omega}_1(p | \hat{\lambda}_1) - \bar{\omega}_2(\theta_2 | \hat{\lambda}_1, 0)\} \leq 1 - \bar{\omega}_2(\theta_2 | \hat{\lambda}_1) \text{ for } \theta_2 > \hat{\theta}_2, \end{aligned}$$

and therefore

$$\begin{aligned}
& E[\hat{\delta}(\hat{\theta}_2) \text{sign}(\tilde{\theta}_2 - \hat{\theta}_2)] \\
\leq & \int_{\underline{\theta}_2(p|\hat{\lambda}_1,0)}^{\underline{\theta}_2(1|\hat{\lambda}_1,0)} [1 - \bar{\omega}_2(\theta_2|\hat{\lambda})] dF_2(\theta_2) + \int_{\underline{\theta}_2(1|\hat{\lambda}_1,0)}^{\hat{\theta}_2} [1 - \bar{\omega}_2(\theta_2|\hat{\lambda})] dF_2(\theta_2) \\
& + \int_{\hat{\theta}_2}^1 [1 - \underline{\omega}_2(\theta_2|\hat{\lambda})] dF_2(\theta_2) \\
= & \int_{\underline{\theta}_2(p|\hat{\lambda}_1,0)}^{\underline{\theta}_2(1|\hat{\lambda}_1,0)} [1 - \bar{\omega}_2(\theta_2|\hat{\lambda})] dF_2(\theta_2) + \int_{\underline{\theta}_2(1|\hat{\lambda}_1,0)}^1 [1 - \bar{\omega}_2(\theta_2|\hat{\lambda})] dF_2(\theta_2) \\
& + \int_{\hat{\theta}_2}^1 [\bar{\omega}_2(\theta_2|\hat{\lambda}) - \underline{\omega}_2(\theta_2|\hat{\lambda})] dF_2(\theta_2).
\end{aligned}$$

The second integral is strictly negative [since  $\bar{\omega}_2(\theta_2|\hat{\lambda}) > \bar{\omega}_2(\underline{\theta}_2(1)|\hat{\lambda}) = 1$  for all  $\theta_2 > \underline{\theta}_2(1)$ ], while the first and third approach zero as  $\hat{\theta}_2 \rightarrow 1$  [the third integral equals  $\hat{\lambda}(1 - \hat{\theta}_2)$ , while  $\underline{\theta}_2(p|\hat{\lambda}_1, 0) \rightarrow \underline{\theta}_2(1|\hat{\lambda}_1, 0)$  as  $\hat{\theta}_2$ , and hence  $p$ , approaches 1]. Hence, their sum is negative for  $\hat{\theta}_2 \in (\underline{\theta}_2(1), 1)$  close enough to 1. Then (26) implies that the program for such values of  $\hat{\theta}_2$  is solved by setting  $p = 1$ .

Now fix  $\hat{\theta}_2^* \in (\underline{\theta}_2(1), 1)$  observe the following:

- The second-best expected surplus for  $p = 1$  equals the value of program  $R(1, \hat{\theta}_2^*)$ .
- The value of program  $R(1, \hat{\theta}_2^*)$  exceeds the value of program  $R(\bar{p}', \hat{\theta}_2^*)$  for any  $\bar{p}' \in (\hat{\theta}_2^*, 1)$ . This follows because a change from  $\bar{p} = 1$  to  $\bar{p}' \in (\hat{\theta}_2^*, 1)$  tightens the constraint  $\text{IR}_2(\hat{\theta}_2)$  in the relaxed program by  $[1 - F_1(p)](p - \hat{\theta}_2)$  and does not affect any other constraints.
- The value of program  $R(\bar{p}', \hat{\theta}_2^*)$  for any  $\bar{p}' \in (\hat{\theta}_2^*, 1)$  exceeds the value achieved if instead the ironing point  $p$  is set equal to  $\bar{p}'$  (by the argument above), which in turn exceeds the value that is achieved when we maximize expected surplus plus  $\hat{\lambda}_1$  times expected revenue subject to all of the IR and IC constraints.
- Since the Myerson-Satterthwaite surplus exceeds the value that is achieved when we maximize expected surplus plus  $\hat{\lambda}_1$  times expected revenue



subject to all of the IR and IC constraints, it must exceed the second-best surplus that is achievable with option price  $\bar{p}'$  (otherwise the second-best solution would have at least as large a value of expected surplus plus  $\hat{\lambda}_1$  times expected revenue as the Myerson-Satterthwaite expected surplus).

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