From Click to Purchase to Return: Website Browsing and Product Returns*

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Abstract

Frequent product returns from online purchases approach a staggering annual value of nearly $1 trillion in the US alone. Existing research focuses on understanding and managing returns using a purchase/return framework. However, customers' pre-purchase clicks might provide an early warning of potential product returns. We explore whether prepurchase clicks on retailers' websites provide insight into product returns. Using data from a large European apparel retailer, we propose and estimate a joint model of customer click, purchase, and return. The empirical stylized facts and our click-to-purchase-to-return model of the customer journey consistently show how customer browsing patterns foreshadow product returns. More specifically, we find that purchasing the last clicked product and browsing fewer products predicts a lower return probability. Using deep learning product embeddings, we show that customers who click on a wide variety of products are more likely to return the purchased product. Standard models of click-to-purchase or purchase-to-return cannot explain these empirical relationships. Further, standard models incorrectly estimate customer preferences for products in the presence of product returns.

Keywords: product returns, customer journey, retailing, clickstream data

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1 Introduction

Understanding and managing product returns is an important challenge for online retailers (The New Yorker, 2023). They significantly decrease profits by reducing revenue (refunds) and increasing costs (backward logistics, dry cleaning, etc.). For example, L.L. Bean spent $50 million annually on return costs, amounting to about 30% of the retailer's annual profits (Abbey et al. 2018). Return costs are often so high that major online retailers, such as Amazon and Walmart, have begun to allow customers to keep the product because extracting benefits from the returned product is less than the return costs (The Wall Street Journal, 2022). Zara started recently charging online shoppers for returns unless the products are returned to the physical store (BBC, 2022).

Product returns are typically studied in a purchase-to-return framework, where researchers assume the product purchase event is the starting point of the customer journey. In this framework, research has established that product characteristics jointly affect the probability of purchase and return because the option to return a product has value to the customer and impacts the purchase decision. From a managerial perspective, research suggests that changes in policies aimed at reducing returns (for example, towards a stricter policy) must also be evaluated based on potential negative effects on customers' purchase behavior.

The purchase-to-return framework answers many important questions. However, it overlooks an essential component of the customer journey – prepurchase clicks on the retailer's website. Before making a purchase, customers spend significant time browsing the retailer's website. They review different products, compare alternatives, and click on products they like. Only after gathering sufficient information can they make purchase decisions. This potentially valuable information is ignored in the purchase-to-return framework.

Similarly, the existing click-to-purchase models overlook customers' post-purchase decisions, namely, whether the customer decides to keep or return the product. These models extensively study how the search environment impacts customer behavior and/or managerial actions to optimize the customer experience, for example, better ranking of the options on the website. These models can be improved to provide additional insight by explicitly modeling returns. Ignoring returns can lead to incomplete analysis in categories, such as fashion retail, where return rates could be as high as 50%.

In this paper, we demonstrate empirically that customer actions during search inform retailers about potential customer returns. We base our analysis on data from a major European apparel retailer. The data capture all customer actions from when they opened the retailer's website to when they decided to return the product. We document that specific customer search patterns foreshadow the probability of returning the product. The correlation may be driven by unobserved (but modeled) consumer preferences and product characteristics that jointly influence clicks, purchases, and returns. Even if the signal is only correlative, it is still important scientifically to form hypotheses or to understand customer/purchase-occasions that might lead to returns. These relationships can be explained with a click-to-purchase-to-return framework but not with either purchase-to-return or click-to-purchase frameworks alone.

We build on purchase-to-return or click-to-purchase frameworks to propose a unified click-to-purchase-to-return framework. Our rational model is consistent with empirically based stylized facts and allows us to explore the mechanism behind the relationship between customer clicks and returns. We demonstrate how existing models may lead to incorrect estimation of parameters that describe customers and the products they purchase. Finally, we demonstrate the practicality of our model by showing that it could be estimated using real retailer data.

The remainder of the paper is organized as follows. Section 2 reviews relevant literature on product returns and customer prepurchase clicks (or, more generally, search). Section 3 describes the data used in the analysis and documents empirically-based stylized facts. Section 4 develops a model consistent with the empirical facts, and Section 5 shows that the proposed model could be estimated. Synthetic data demonstrate that a click-to-purchase-to-return model better recovers true parameters than existing models of either click-to-purchase or purchase-to-return. Section 6 demonstrates that the proposed click-to-purchase-to-return model is consistent with the stylized facts. Section 7 provides a summary, limitations, and suggested future research.

2 Related Research

This paper contributes mainly to the literature on product returns. Research on product returns has been both theoretical and empirical. Theoretically, researchers have focused on return policies to demonstrate that the option to return products serves as a risk-reducing mechanism that encourages the customer to experience the product (Che 1996); also studied empirically by Petersen and Kumar (2015); or as a signal of product quality (Moorthy and Srinivasan 1995).

Empirical research has focused on firms' optimization of return policies. In an attempt to identify the optimal return policy, researchers recognize the trade-off between higher demand and higher return rates when firms use lenient policies and suggest that the optimal return policy must be balanced (Davis et al. 1998, Bower and Maxham III 2012, Abbey et al. 2018) because overly strict return policies lead to a decrease in purchases (Bechwati and Siegal 2005). Janakiraman et al. (2016) extensively review the effect of return policy leniency on purchases and returns. Anderson et al. (2009) propose a structural model where the option to return is embedded in a customer purchase decision – the customer learns private information only after purchasing the product. Other empirical studies demonstrate that a variety of policy factors affect the probability of product returns, including price, discounts, marketing instruments (e.g., free shipping), or the truthfulness of product reviews (Petersen and Kumar 2009, 2010, Sahoo et al. 2018, Shehu et al. 2020, El Kihal and Shehu 2022). Empirical studies suggest prescriptive instruments, such as visualization systems, to decrease return rates. These instruments decrease return rates by decreasing uncertainty in the product match to the customer (Hong and Pavlou 2014). Other researchers use machine learning to accurately predict returns and identify product-related features that enable the firm to better select and design fashion products for the retailer's website (Cui et al. 2020, Dzyabura et al. 2021).

We contribute to the product returns literature by including prepurchase clicks that precede purchase to better understand the customer journey. Clicks are an aspect of customer search – the latter is an established and mature field of research. The literature typically follows sequential (Weitzman 1979) or simultaneous (Stigler 1961) approaches. Both approaches assume the customer knows the distribution of the rewards and searches to resolve uncertainty. For example, Weitzman examines a stylized problem of sequentially opening boxes to learn their value and then deciding when to stop searching and collect the value of the best box (but paying the search cost for every box opened). If the value distributions are known for all boxes, Weitzman proves that the optimal (dynamic programming) search strategy is an index strategy – choose next the box with the highest index and stop searching when the value of the best box already opened exceeds the indices of all remaining boxes. Many papers expand this simple framework to study various aspects of real-world search. Most of the literature focuses on sequential search buttressed by Bronnenberg et al. (2016), who report strong evidence to support sequential search.

Recent papers allow for flexible preference heterogeneity (Morozov et al. 2021), add learning (Ke et al. 2016, Branco et al. 2012, Dzyabura and Hauser 2019), multiple attributes (Kim et al. 2010), intermediaries (Dukes and Liu 2016), search duration (Ursu et al. 2020), and search fatigue (Ursu et al. 2023). The availability of click-stream data has enabled researchers to empirically study customer search behavior (Bronnenberg et al. 2016, Chen and Yao 2017, Ursu et al. 2020) and provide detailed insights on click-to-purchase customer behavior. For example, Bronnenberg et al. (2016) examine customer search behavior for cameras and show that early search is highly predictive of customer purchase and that the first-time discovery of the purchased alternative happens towards the end of the search. Chen and Yao (2017) show that refinement tools significantly impact customer behavior and the market structure. Ursu et al. (2020) study search duration, quantify customer preferences and search costs, and develop insights on how much information to provide on a platform.

To date, researchers have focused primarily on the purchase-to-return sub-journey (returns literature) or the click-to-purchase sub-journey (search literature). Research on the click-to-return is scarce and uses a theoretical lens (Jerath and Ren 2024, Janssen and Williams 2024). We expand these research streams to focus on the entire click-to-purchase-to-return journey in the empirical setting. Our research provides complementary insights to the returns literature (search predicts returns) and to the search literature (the possibility of returning a product changes a customer's optimal sequential search strategy). We demonstrate that by focusing on the entire customer journey, we gain additional insight into customer behavior and explore when the existing models may fail.

3 Data and Empirically-based Stylized Facts

3.1 Data from a Fashion Retailer

We sought and obtained online-channel individual-level data from a large apparel retailer in Western Europe. We focus on the online channel because (1) most returns are through the online channel (in total, 53% of sold products are being returned – typical for the European apparel industry), and (2) the online channel is an ideal situation in which to observe clicks, purchases, and returns for each customer. We preprocessed the data by removing noise and outliers (for example, extremely short/long sessions). Appendix Appendix A provides a detailed description of data pre-processing.

In this paper, we focus on orders that had at most one product purchased. This focus illustrates an important situation where the full purchase journey matters. The focus is insightful because it excludes bracketing situations when the customer purchases several colors or variations of a product, intending to keep only one. While such situations are important and realistic, they require model augmentation, obscuring basic incremental insights about the advantages of modeling the more complete customer journey. We leave model augmentation and empirical analysis of multiple-product orders for future research.

The retailer sells medium-priced fashion products for women, men, and children. Its main product is adult apparel, which accounts for 95% of purchases. As is typical for Europe, the retailer has a generous return policy. Products can be returned for free or a full refund within 60 days after the purchase, with or without providing a reason. Prior to our analysis, the retailer did not use customer clicks to understand or manage returns. The retailer did not attempt any interventions to discourage (or encourage) returns based on a customers' clicks.

Data include both mobile and desktop usage and consist of three main components:

- Prepurchase (clicks): Website browsing records the sequence of clicks made by the customer during the browsing session at the retailer's website. We observe products listed on the website for the customer, the set of products considered (clicked to view the detailed product page), and the sequence of these clicks. We also observe all actions (e.g., clicking on a product, sorting by price) and the timing between the different actions, allowing us to observe how much time a customer spends on a specific product page.
- Purchase: Purchases include the product purchased (if any) by the customer during browsing sessions. These data include product characteristics, such as price, category, fabric, size, brand, color, and product image.
- Post-purchase (returns): Returns contain information on whether the customer kept or returned the purchased product and when the return occurred.

A unique identifier matches all three data components. For each session, we observe the customer journey from opening the retailer's website to deciding whether to keep or return a fashion product. If and when appropriate data become available, our analyses might be extended to examine the impact of clicks before visiting the retailer's website or clicks from a previous visit. However, even if such data were available, the retailer may not be able to use the data due to the increasing concern for privacy in the European Union. Padilla et al. (2023) document the increased emphasis on "first-party data" by large online retailers.

The observation period is between October 1, 2019, and February 28, 2020[1]. After the pre-processing (Appendix Appendix A), we observe 837,404 single-item browsing sessions, of which 51,858 (6.2%) resulted in a purchase. In 40.9% of these purchases, customers return the purchased product. As anticipated and consistent with multiple-product bracketing, the return rate for the single-item subsample is lower (40.9%) than that for the multiple-item subsample (53%).

Customers can access the retailer's website through a desktop or mobile device (54.8% accessed through a mobile device in our data). On the website, the customer observes a product list, which displays a small image of the product, its price, and its category. When the customer clicks on a specific product, further information is revealed on the product page, such as more (and higher quality) product images and detailed product descriptions. To illustrate the information available to the customer, we provide the retailer's website screenshot in Appendix Appendix A. During these 837,404 browsing sessions, the customers

[1] We have access to data until May 15, 2020. However, we excluded the months when the COVID-related restrictions took place in the country where our retailer primarily operates.

review, on average, 3.5 products (median 2). In 26.3% of sessions, the customer used at least one filtering tool (for example, by color), and in 26.9% of cases, reviewed more than one color variety of the product. Figure 1 provides the marginal distributions of product clicks, product-color clicks, number of filters, and product-category clicks.

The retailer's website displays 16 high-level product categories predefined by the retailer (e.g., jeans, blouses, dresses, coats, shoes). The most popular purchased product categories are "jackets and coats" (30.4%) and "jeans" (16.2%). "Dresses" and "jumpsuits" have the highest return rate (56.8% and 57.6% respectively), and "T-shirts" have the lowest return rate (10.2%). Figure 2 displays return rates by category and the sales share of each category.

3.2 Empirically-based Stylized Facts

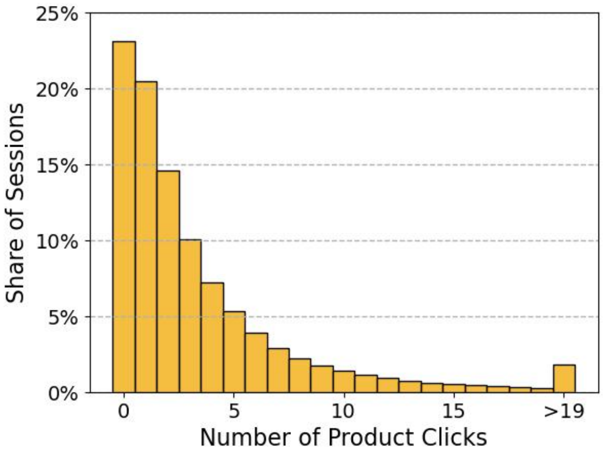
We begin with illustrative stylized facts. Browsing data are high dimensional because of the number of options available to the customer and because the order of customer actions matters. To gain intuition, we summarize customer search with aggregate statistics of browsing that relate to product returns. While these relationships are not necessarily causal, a minimum criterion for any click-to-purchase-to-return model is that the model is consistent with the stylized facts. Indeed, in Section 4, we introduce a formal model in which clicks, purchases, and returns are driven by customer preferences and product characteristics ("shocks"). In some ways, we can think of the observations as an "early warning system." If the retailer observes a customer's behavior in certain patterns, then that customer is more likely to return a purchased product.

Because product characteristics are potentially correlated with product purchases and returns, we illustrate the stylized facts using fixed-effects controls for product characteristics on the probability of returns. After controlling for product characteristics, we relate the residual effects to customer behavior. Intuitively, the dependencies that we explore in this section imply that if we observe two customers who purchased exactly the same product but had different search sessions, we study how the different search-session characteristics relate to customer behavior.

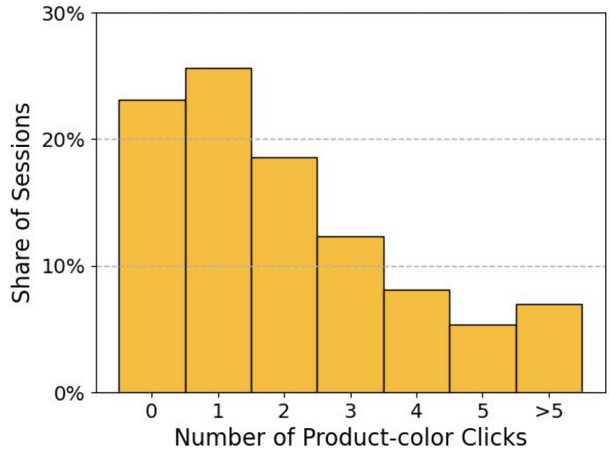
Figures 3, 5 and 6 illustrate the stylized facts. The graphs' vertical axes represent the return probability net product-specific effects as a function of the control variable (for example, number of product clicks) net of product-specific effects[2]. The gold lines indicate the mean and 95% confidence interval, while the blue line indicates the regression line plus 95% confidence intervals for the estimated curve. We used a natural log transformation for continuous independent variables to account for potential non-linearity and reduce the

[2]Formally, removing the product fixed-effects would center the graph around zero. To improve the interpretability, we transformed all variables by adding corresponding average values.

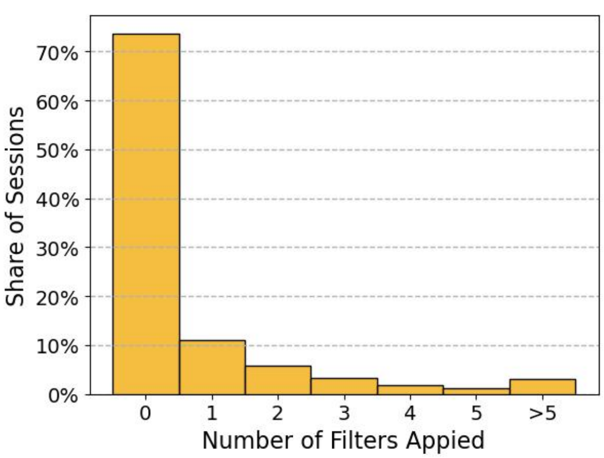
Figure 1: Descriptive Statistics of Customer Browsing Behavior at the Retailer's Website



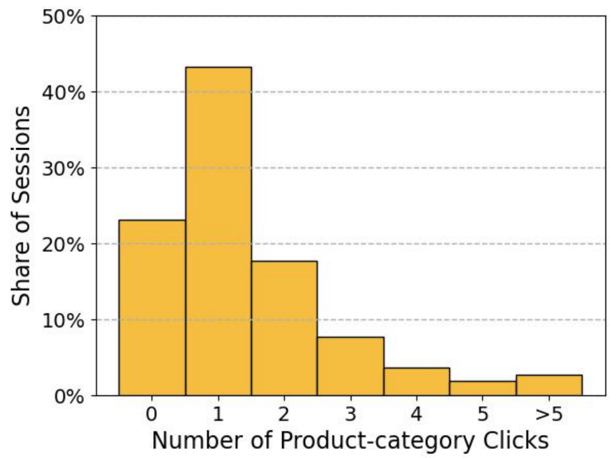
(a) Number of product clicks



(b) Number of product-color clicks

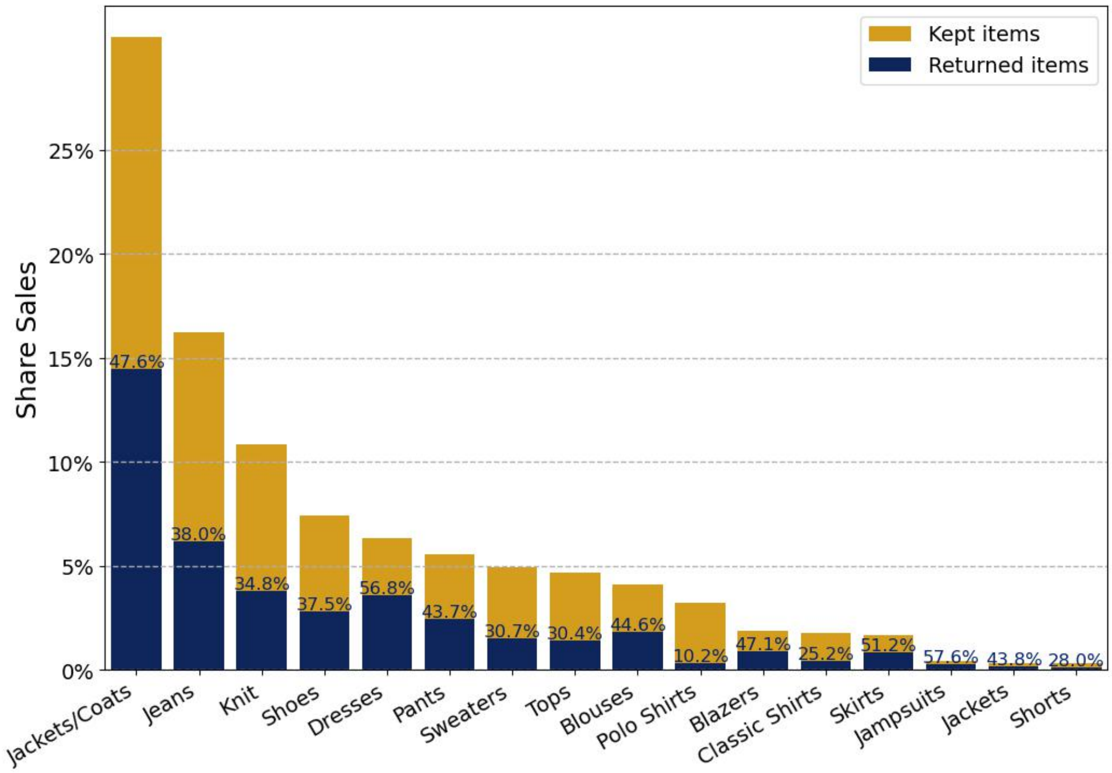


(c) Number of filters used



(d) Number of product-category clicks

Figure 2: Sales Share and Return Rates by Product Category



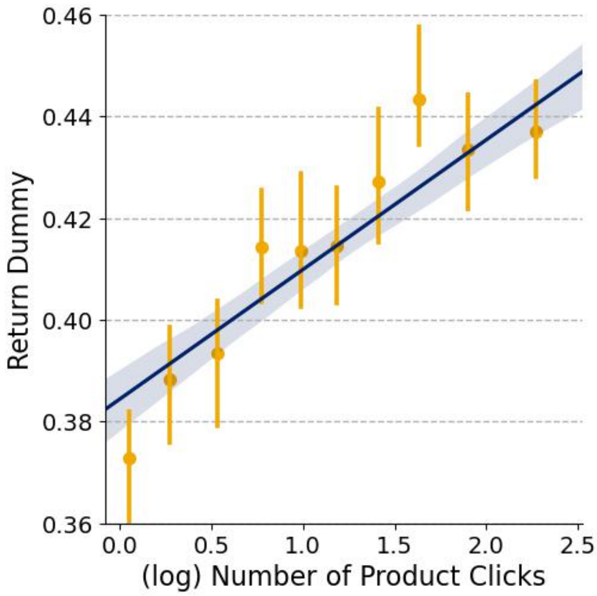
impact of outliers (for example, extremely long sessions).

Holding product characteristics constant, Figure 3a suggests a strong positive correlation between the number of product clicks and the probability of return – customers who click and review more products return, on average, more frequently. Besides the number of clicks, the order matters. Figure 3b suggests that customers who purchased the last clicked product are substantially less likely to return the product.

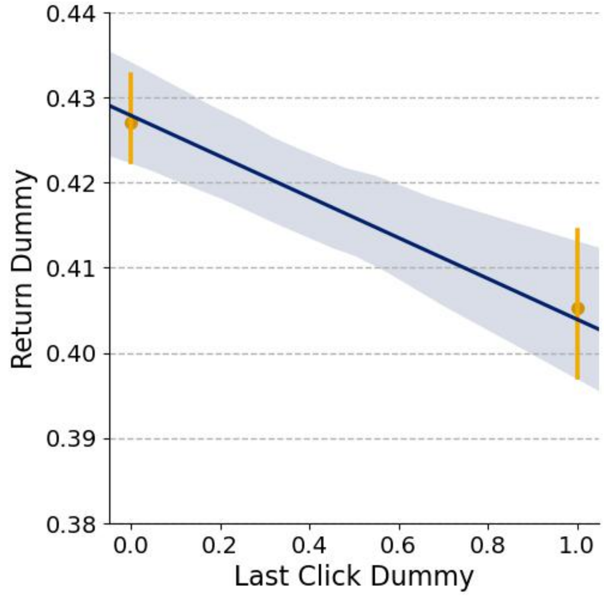
Although we hold product characteristics constant (fixed effects), we can explore relationships among searched products. Figure 4 illustrates two hypothetical browsing sessions. The first customer clicked on six similar long-sleeved T-shirts with solid patterns, while the second customer clicked on two long-sleeved T-shirts with floral patterns, one short-sleeved blouse, a coat, and two dresses. Likely, the first customer was more focused and was explicitly looking for a T-shirt, while the second customer was less focused and was considering various wardrobe choices.

To explore the issue of customer focus, we use deep learning product embeddings. Intuitively, product embeddings summarize information about a product in a K-dimensional vector with the property that similar products have similar product embeddings. For example, the Euclidean distance between two T-shirts of similar green color would be close

Figure 3: Empirically-Based Stylized Facts – Number of Clicks and Whether the Last-Viewed Product Was Purchased are Both Related to Return Probabilities



(a) Number of product clicks



(b) Last click dummy

Figure 4: Two Example Sessions that Vary on Variety

(a) Low variety session

(b) High variety session

to zero, while the distance between a T-shirt and a pair of jeans would be high. Appendix Appendix B provides the details of constructing the embeddings. We use embeddings to evaluate whether customers click on different products (high variety) or similar products (low variety). The embeddings are based on aggregate statistics only. We do not use individual-customer clicks, purchases, or returns and, hence, the embeddings contain no information about relationships among individual-customer clicks, purchases, or returns.

Figure 5 plots the return probability against variety. Customers who click on a larger variety of products are more likely to return the product after the purchase. Figure 5 suggests a potential difference between targeted browsing (looking for a specific product) and casual browsing (browsing for various products). Deep-search customers appear to be less likely to return products, likely because they are either more informed, more focused, or less impulsive.

Embeddings are powerful but are somewhat of a black box. Interpretability is a challenge. Furthermore, although we eliminate within-customer product-purchase correlations by focusing on single-product purchases, account for product-specific fixed effects, and use aggregate data only, we cannot rule out a hypothesis that the product embeddings might contain information on returns. To address the interpretability and potential for the embeddings to contain information on returns or their relationship to clicks, we consider alternative measures of the breadth of the search. These measures count the types of products the customer clicked during the browsing session in Figure 6:

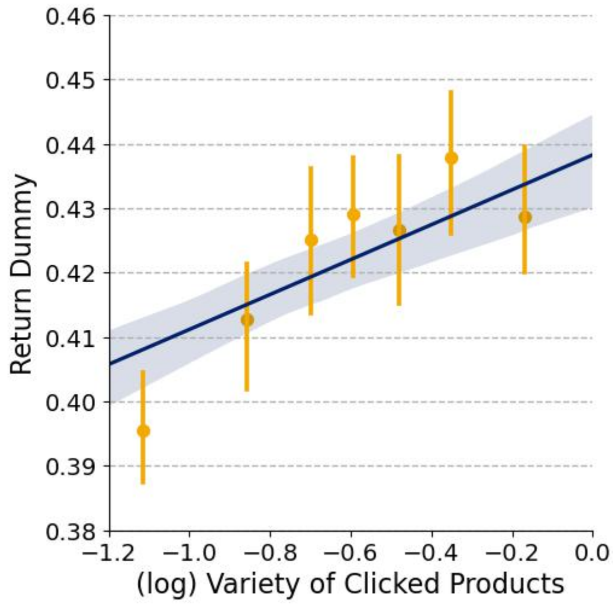
- number of unique categories: T-shirts, jeans, dresses
- number of unique general styles: plus size, regular, etc.
- number of unique materials: cotton, polyester, etc.
- variance in price.

Overall, the interpretable measures are consistent with the more general product embeddings. When a customer searches a higher variety of products, the customer is to more likely to return the purchased product. As an additional robustness check, Appendix Appendix C presents relationships when fixed effects are not removed. The qualitative results are consistent.

4 Model Development

To examine whether a click-to-purchase-to-return model of customer behavior is consistent with the correlative evidence in the stylized facts, we extend models from the developed field of customer search. Specifically, we model customers' click decisions as sequential and rational – customers review products one by one and make a decision to purchase the

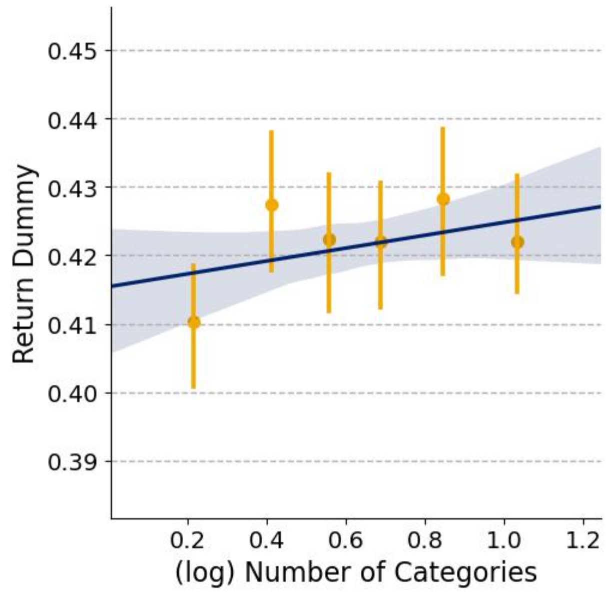
Figure 5: Empirically-Based Stylized Fact – Variety of Clicks Relates to Return Probabilities



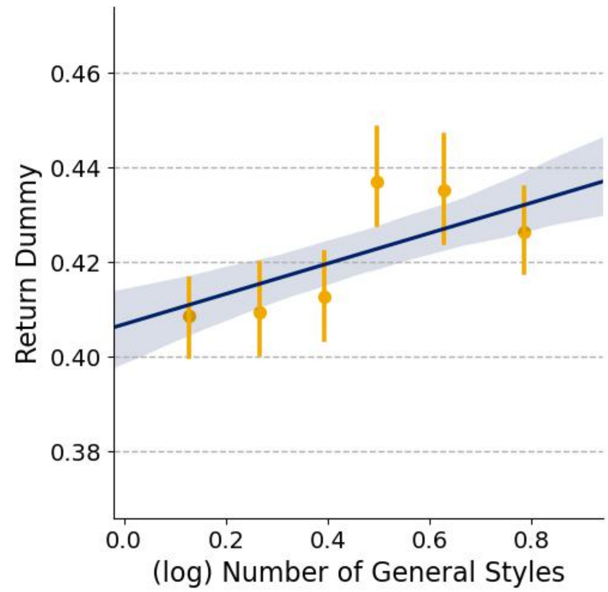
product when the expected value of purchasing exceeds the expected value of reviewing more products. We expand models standard in the literature to allow the customer to anticipate the return decision. (An alternative perspective is that we expand purchase-to-return models to consider search.) This section introduces the formal model and derives the optimal click rules. In subsequent sections, we address parameter estimation, compare the full purchase-journey model to standard models (click-to-purchase, purchase-to-return), and examine whether the stylized facts are consistent with model predictions.

Figure 7 provides an overview of the full click-to-purchase-to-return model of customer journey. We assume the customer is rational and forward-looking, gains information at a cost while clicking, gains information by purchase and inspection, and incurs a cost if the product is returned. Formally, consider a customer who visits the retailer's website and observes the list of products $\mathcal{V}_i$. By viewing the product list, the customer forms initial impressions: some product-related characteristics x_{ij} (price, category, color, etc.) and an individual pre-click preference shock ξ_{ij}. The customer can click on any of these products to reveal additional post-click information ϵ_{ij}. However, each click requires the customer to incur some costs c_{ij}. (For example, they may need to move the mouse, click, and process the information revealed.) After the click, the customer either continues clicking (if they see other attractive options) or stops to decide whether they like any of the products clicked so far. If the customer decides to terminate the search, they purchase the best product among those clicked or leave the website without a purchase (or with an outside option). If

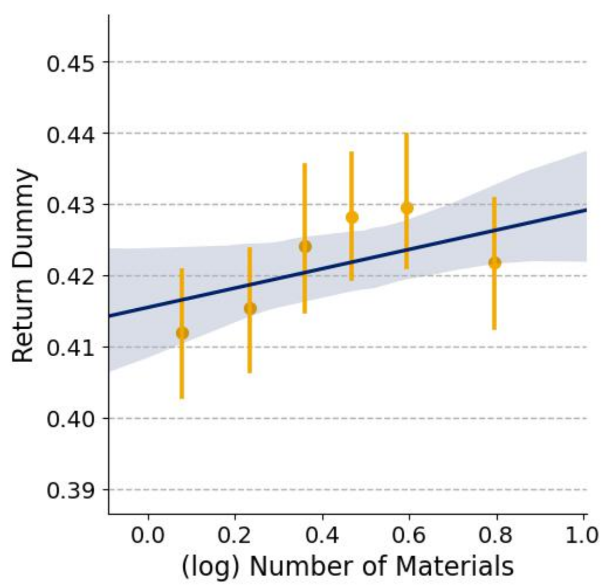
Figure 6: Empirically-Based Stylized Facts Based on Alternative Measures of Variety



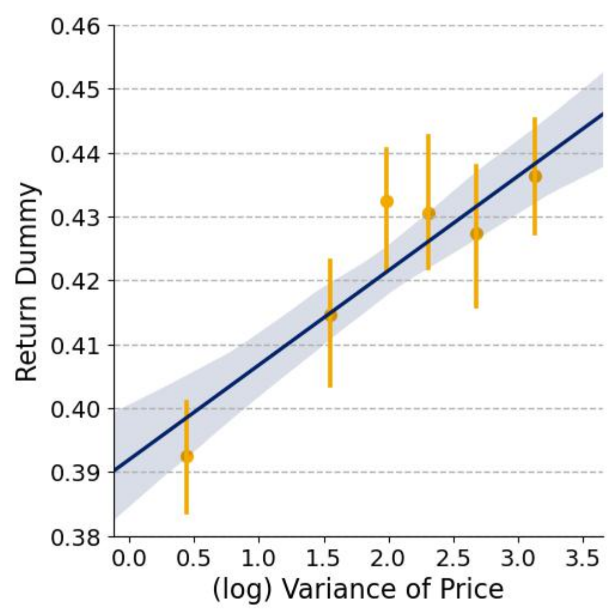
(a) Category



(b) General style



(c) Material



(d) Price

the customer purchases the product, they receive the purchased product and inspect it in more detail at home (e.g., try it on, hold it up, feel the material, and compare its fit to the customer's other fashion products). Inspection reveals additional information (for example, fit with the body type) denoted as the ψ_{ij}. Based on all information accumulated (online clicks and offline inspection), the customer decides whether to keep the product or to incur a return cost R_i (e.g., return label, travel time, etc.) by returning the product to the retailer for a full refund.

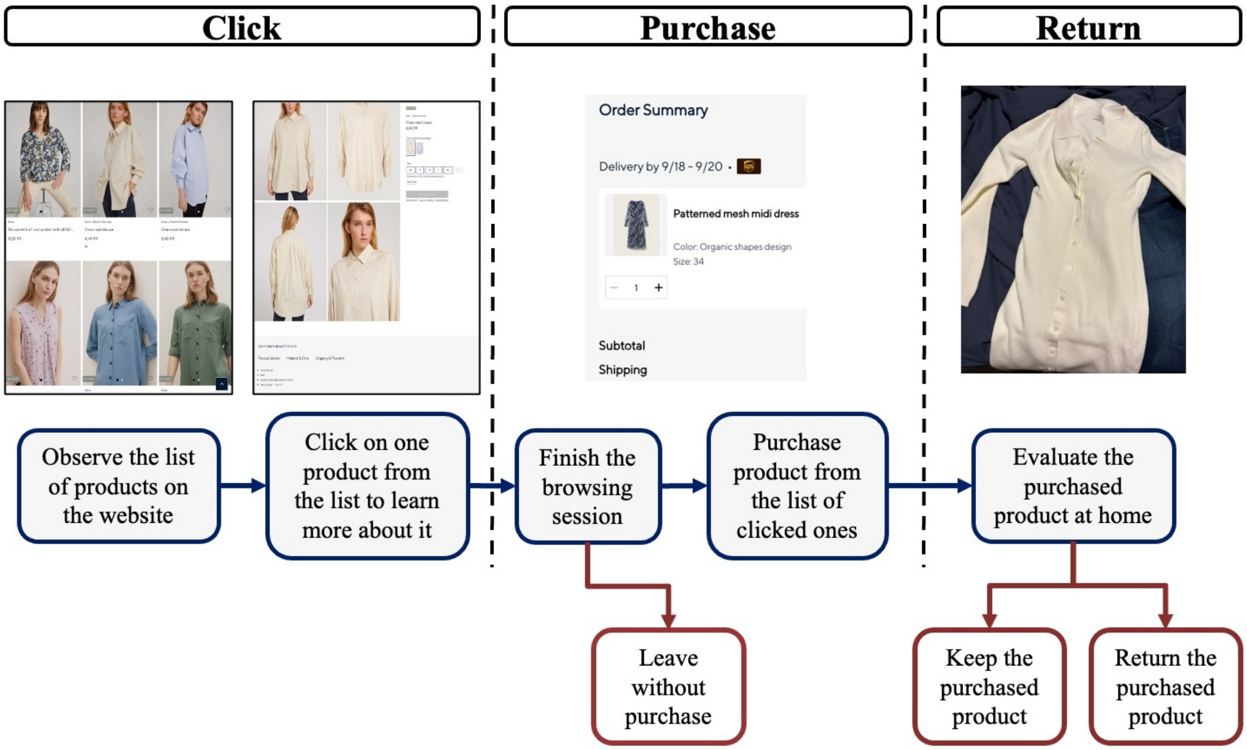
The click-to-purchase-to-return model is based on concepts introduced by Weitzman's model of rational search. In the Weitzman model, the customer sequentially inspects "boxes" to resolve uncertainty. The customer knows the distributions of potential contents of the boxes prior to inspection, not the realized content. In the click-to-purchase-to-return model, the customer resolves some pre-click uncertainty by clicking on a product and resolves further uncertainty by a postpurchase inspection. The customer considers the uncertain outcome of this inspection (return or not) when making a purchase decision and deciding to continue clicking (i.e., browsing the website).

The click-to-purchase-to-return model also serves as an extension to the standard purchase-to-return models. While on the retailer's website, the customer makes the purchase decision under uncertainty about the true fit. Consistent with the standard purchase-to-return models, the click-to-purchase-to-return model predicts that if the variance of ψ_{ij} is close to zero, the customer would make an informed purchase decision and gain little by home inspection, hence the return probability would be low. On the other hand, when most of the customer's learning occurs after the customer receives the product (high variance of ψ_{ij}), the return probability would be higher.

Because the existing click-to-purchase and purchase-to-return models are special cases of the click-to-purchase-to-return model, we can examine whether the restricted models lead to the same or different estimates of the customer behavior parameters. For example, if a click-to-purchase model leads to parameters that differ from a click-to-purchase-to-return model, then in industries where returns (or cancellations) are frequent, managers might make incorrect inferences about which products customers are likely to purchase and return.

The proposed model serves as an extension of the popular Weitzman model of rational search. Specifically, our model assumes that the customer does not infer the true utility upon click: part of the utility ψ_{ij} remains unknown until the customer receives the product at home. While on the retailer's website, the customer makes the purchase decision under uncertainty about the true fit. For example, if the variance of ψ_{ij} is close to zero, there would be almost no discrepancy between how the product looks on the website (online) and at home (offline). Thus, the customer makes a highly informed purchase decision, and the return probability would be low. On the other hand, when most of the customer's learning occurs after they receive the product (high variance of ψ_{ij}), the return probability would be higher as most of the information could not be revealed on the website.

Figure 7: Overview of the Click-to-Purchase-to-Return Model of the Customer Journey



4.1 Click Costs, Utility, and Returns

For ease of notation without loss of generality, we number products such that j indexes the sequence in which customer i clicks on products (for example, $j = 2$ implies the second clicked product, while $j = 0$ implies the outside option, which is always available). The customer's final utility could take one of three possible forms (click costs are paid before the realization of this utility and thus not included in the equation):

$$u_{ij} = \begin{cases} \mu_{ij} + \xi_{ij} + \epsilon_{ij} + \psi_{ij} & \text{purchased and kept a product} j \neq 0 \\ -R_i & \text{purchased and returned a product} j \neq 0 \\ 0 & \text{chose outside option} j = 0 \end{cases} \tag{1}$$

where μ_{ij} is the customer's preference for the attributes x_{ij} of product j. (Recall that j indexes the click order; thus, for every customer, the j^{th} product could be different, requiring a notation that allows the product's attributes to differ by j.) The customer's preference vector varies across customers: $\mu_{ij} = x'_{ij}\beta^u_i$ where $\beta^u_i \sim N(\mu_{\beta^u}, \sigma_{\beta^u})$ is the customer's preference vector.

To make the estimation and identification of the model feasible, we impose additional assumptions on the structure of preference shocks. We follow the standard Weitzman search framework (Weitzman 1979, Anderson et al. 2009), and assume that individual preference shocks $(\xi_{ij}, \epsilon_{ij}, \psi_{ij})$ conditionally on observed product characteristics x_{ij} are independent and normally distributed. Because the utility is invariant to a positive linear transformation, we normalize the variance of after-click and before-click preference shocks to 1. This normalization helps us focus on product returns and on comparisons to standard models.

Expected purchase utility. Because the customer does not observe the "at-home-inspection" shock ψ_{ij} until after the purchase decision, the customer must evaluate the product given the available information by taking an expectation over the unobserved shock: $\omega_{ij} = \mathbb{E}_{\psi_{ij}}[u_{ij}|x_{ij}, \xi_{ij}, \epsilon_{ij}]$. In Appendix Appendix D, we demonstrate that under the assumptions discussed previously, the expected purchase utility takes a simplified form:

$$\omega_{ij} = \sigma_{\psi_{ij}} \mathcal{T}\left(\frac{R_i + \mu_{ij} + \xi_{ij} + \epsilon_{ij}}{\sigma_{\psi_{ij}}}\right) - R_i \tag{2}$$

where $\mathcal{T}(\kappa) = \kappa\Phi(\kappa) + \phi(\kappa)$ and $\Phi(\kappa)$ and $\phi(\kappa)$ are the cumulative distribution and probability density functions of the standard normal distribution, and κ is shorthand for the terms in the parentheses.

Equation (2) demonstrates how the return option indirectly impacts the customers' clicks relative to the standard framework. The distribution of utility depends in part on the distribution of the inspection shock ψ_{ij}, and the distribution of the expected reward is bounded from below by $-R_i$. To examine the face validity of Equation (2), we let $R_i \to \infty$ as would be the case if returns were not allowed. In this case, $\omega_{ij} \to \mu_{ij} + \xi_{ij} + \epsilon_{ij}$, and the model converges to the standard case. It is straightforward to show that $\mathcal{T}(\kappa) \geq \kappa \ \forall \kappa$, which implies that, for any product attribute, the option to return improves the customer's expected click utility. Intuitively, having the option, but not the obligation, to return a product is at least as good as not having the option to return it.

Click and return costs. Let c_{ij} be the click costs incurred by customer i when the customer clicks on product j. Click costs can depend upon the browsing environment; for example, clicking on a product at the top of the website might require less effort. Because j indexes the click order, we write $\log c_{ij} = d_{ij}'\beta^c$ where d_{ij} represents the browsing environment that the customer experiences for the j^{th} product. For clarity, we assume that return costs do not vary by product or customer and write them as $R_i = R$. In a more general case, both click and return costs may vary across customers and products. The main results hold regardless of the parametric form of these costs.

4.2 Optimal Click Strategy when Return is an Option

Because we preserved assumptions of the Weitzman search framework, we can use the standard structure of customer click rules (Ursu 2018). However, the form of the click rules must be updated to account for the changed structure of the reward function. Conceptually, the selection, stopping, and purchase rules retain the property of maximum expected utility, where the return option adds an additional step. We summarize the revised decision rules below and provide more detailed equations in the next section. We provide derivations in Appendix Appendix E.

Selection rule. If the customer is going to click, the customer will choose to click on the product with the highest reservation utility z_{ij} derived from the system in Equation (3):

$$
\begin{aligned}
c_{ij} &= \sigma_{\psi_{ij}} \int_{\theta}^{\infty} \left[\mathcal{T}\left(\frac{R_i + \mu_{ij} + \xi_{ij} + \sigma_{\epsilon_{ij}} t}{\sigma_{\psi_{ij}}} \right) - \mathcal{T}\left(\frac{R_i + \mu_{ij} + \xi_{ij} + \sigma_{\epsilon_{ij}} \theta}{\sigma_{\psi_{ij}}} \right) \right] d\Phi_{\epsilon_{ij}}(t) \\
z_{ij} &= \sigma_{\psi_{ij}} \mathcal{T}\left(\frac{R_i + \mu_{ij} + \xi_{ij} + \sigma_{\epsilon_{ij}} \theta}{\sigma_{\psi_{ij}}} \right) - R_i
\end{aligned}
\tag{3}
$$

The second equation is a 1-to-1 mapping $\theta \to z_{ij}$, but to find z_{ij}, we must first solve the first implicit equation for θ. Intuitively, before the click, customers do not know the value of ϵ_{ij}; thus, their reservation utilities cannot depend on it. By computing the integral in Equation (3), customers estimate the expected difference between expected purchase utilities in Equation (2) with and without a new click.

- Stopping rule. The customer continues to click until their maximal expected utility of clicked options from Equation (2) exceeds the maximal reservation utilities of not-clicked options in Equation (3). This stopping rule is conceptually similar to the standard search framework.
- Purchase rule. When the stopping rule is reached, the customer purchases either the highest-expected utility product from the set of all clicked products or chooses the outside option.
- Return rule. If the customer purchased a product (not the outside option), the customer keeps (does not return) the product if their utility for the chosen-and-inspected product is larger than the negative return costs, $-R_i$.

The return option changes the distribution of rewards and, hence, the reservation utilities for the customer. Adding the return option can also change the order in which the customer clicks on products and the stopping and return rules.

Table 1 summarizes the functional form assumptions discussed in this section and the parameters that need to be estimated. These assumptions make estimation feasible and are

Table 1: Overview of Model Parameters

	Distributional assumptions	Functional forms	Estimated parameters
Customer preference vector	$\beta_i^u \sim \mathcal{N}(\beta^u, \sigma^\beta)$	$\mu_{ij} = x'_{ij}\beta_i^u$	β^u, σ^β
Pre-click preference shock	$\xi_{ij} \sim \mathcal{N}(0, \sigma_{\xi_{ij}})$	$\sigma_{\xi_{ij}} = 1$	
Post-click preference shock	$\epsilon_{ij} \sim \mathcal{N}(0, \sigma_{\epsilon_{ij}})$	$\sigma_{\epsilon_{ij}} = 1$	
Post-purchase preference shock	$\psi_{ij} \sim \mathcal{N}(0, \sigma_{\psi_{ij}})$	$\log \sigma_{\psi_{ij}} = x'_{ij}\beta^\psi$	β^ψ
Click costs		$\log c_{ij} = d'_{ij}\beta^c$	β^c
Return costs		$R_i = R$	R

sufficient to explain the stylized facts. While other distributional and functional form assumptions are certainly possible, we believe these assumptions best enable us to demonstrate the implications of the click-to-purchase-to-return model relative to the standard click-to-purchase and purchase-to-return models.

5 Feasible Estimation of Model Parameters

Because the decision rules in Equation (3) involve integration over the nonlinear function $\mathcal{T}(\kappa)$, we cannot use the estimation strategy discussed in Ursu (2018), where the integral in Equation (3) could be taken. We need to use an alternative estimation strategy. The normalizations in Table 1 assure that the model is identified, but identification does not ensure that a particular estimation strategy can recover the "true" parameters of the model. Furthermore, because the estimation procedure requires the integration of nonlinear functions, there is no guarantee that the procedure will converge in a feasible time. Using synthetic data as a "ground truth" we demonstrate that our proposed estimation strategy recovers the "true" parameters in a feasible time. Synthetic data also enable us to compare the click-to-purchase-to-return model to models that ignore either clicks or returns.

5.1 Likelihood

Let V_i denote the number of products presented to the customer (for example, the number of products the customer sees on the website's main page). The customer clicks on C_i products from this set of products according to the optimal search rules discussed in the previous section. Recall that the index j represents the order in which the customer clicks on the product (e.g., $j = 2$ denotes the second clicked product, and $j = C_i$ denotes the last clicked product). This notation implies that the customer did not click on products with $j > C_i$, enabling us to enumerate the order of non-clicked products randomly.

Consider the customer who was presented with V_i products, clicked on C_i products, purchased a product with index b, and decided to return it. This sequence implies the following constraints where $\mathcal{I}$[constraint] is the indicator function that takes on a value of 1 if the constraint is satisfied:

Click continuation. After clicking on option j, the customer continues clicking on other options if the value of exploring is more than the value of the best option in hand.

$$\forall j < C_i \quad \mathcal{I}\left[z_{i(j+1)} \geq \max_{s=j+2..V_i} z_{is}\right] \mathcal{I}\left[\max_{s=0..j} \omega_{is} < \max_{s=j+1..V_i} z_{is}\right] = 1 \tag{4}$$

Click stopping. The customer stops clicking when the maximal expected utility of clicked options is higher than the value of exploring the remaining options.

$$\mathcal{I}\left[\max_{s=0..C_i} \omega_{is} \geq \max_{s=C_i+1..V_i} z_{is}\right] = 1 \tag{5}$$

Purchase. Given that the customer has clicked C_i products and decides to stop clicking, the customer purchases a product if the expected utility of the purchased product is greater than the expected utility of all other clicked products, including the outside option.

$$\mathcal{I}\left[\omega_{ib} \geq \max_{s=0..C_i} \omega_{is}\right] = 1 \tag{6}$$

Return. Given the customer bought product b, the customer returns the product if the product utility is lower than $-R_i$.

$$\mathcal{I}\left[\mu_{ib} + \xi_{ib} + \epsilon_{ib} + \psi_{ib} \leq -R_i\right] = 1 \tag{7}$$

Equations (4) to (7) define the set of constraints that must be satisfied to observe the given browsing session. Multiplication of the indicator functions for these conditions is the same as requiring all conditions to hold and produces a binary variable W_i that takes 1 if and only if all constraints are satisfied. The case when the customer decides to keep the product or chooses the outside options closely follows the derivations in Equations (4) to (7). In Appendix Appendix G, we demonstrate that the set of Equations (4) to (7) can be rewritten in the more compact form in Equation (8):

$$\begin{aligned} W_i = &\left[\prod_{j=1}^{C_i-1} \mathcal{I}[z_{ij} \geq z_{i(j+1)}]\right] \mathcal{I}\left[z_{iC_i} \geq \max_{s=C_i+1..V_i} z_{is}\right] \left[\prod_{j=0}^{C_i-1} \mathcal{I}[\omega_{ij} \leq \min\{z_{iC_i}, \omega_{ib}\}]\right] \\ &\mathcal{I}[\omega_{iC_i} \leq \omega_{ib}]\mathcal{I}[\omega_{ib} \geq \max_{s=C_i+1..V_i} z_{is}]\mathcal{I}[\mu_{ib} + \xi_{ib} + \epsilon_{ib} + \psi_{ib} \leq -R_i] \end{aligned} \tag{8}$$

Because the researcher does not observe individual shocks $\xi_{ij}, \epsilon_{ij}, \psi_{ij}$, and the individual preference vector β_i^u, we obtain the probability of observing the given click sequence of customer i by integrating out these variables to determine the probability that all constraints are satisfied. This integration produces the log-likelihood function:

$$LL(\beta) = \sum_{i=1}^{N} \log \int W(\xi_i, \epsilon_i, \psi_i, \beta_i^u) \, dF(\xi_i, \epsilon_i, \psi_i) \, dF(\beta_i^u) \tag{9}$$

where $F(\xi_i, \epsilon_i, \psi_i)$ represents the joint distribution of unobservable shocks.

If computations were feasible, we could find the estimates of the parameters by maximizing the log-likelihood function in Equation (9). Unfortunately, Equation (9) highlights two complications prohibiting direct maximization.

First, to compute the reservation utility z_{ij}, we need to solve Equation (3), which includes integration. Because z_{ij} depends on the model's parameters and needs to be computed for each iteration of the optimization algorithm – it is infeasible to compute the exact value of the integral for each customer-product pair. This is a known issue with Weitzman-like click models; however, in the standard case, the integral could be replaced with a function of the form $c_{ij} = h(z_{ij})$ where h is an invertible function, for example, Ursu (2018). In our case, the argument in the integral is a non-linear function, and there is no known closed form for the integral in Equation (9). To make the estimation feasible and find z_{ij}, we use a trilinear interpolation described in Appendix Appendix F. Intuitively, we compute the exact integral for the predefined grid of points and approximate the values between the grid points by a continuous function. This approach allows us to approximate the integral with any pre-defined accuracy and substantially reduce the computation time.

Second, no known closed-form solution exists to the integral in Equation (9). The inner integral involves integration over $\xi_i, \epsilon_i, \psi_i$ that contains $V_i + C_i + 1$ variables (V_i shocks are realized before clicks, C_i shocks are realized from the set of clicked products, and one shock is realized from the purchased product). Previous research recognizes the problem and relies on two variations of simulation techniques to approximate the integral in Equation (9) and recover the model's parameters. Unfortunately, these methods are not feasible for the click-to-purchase-to-return model.

Accept-reject simulator (Chen and Yao 2017). An accept-reject simulator replaces the true probability P_i with a simulated probability $\hat{P}_i$. For given parameter estimates, the approach simulates B random draws of shocks from corresponding distributions and calculates the share of draws in which $W_i = 1$ (all constraints in Equation (8) are satisfied). The maximum likelihood estimate is the parameter vector corresponding to the largest share of draws. However, for browsing data in fashion retailing P_i is close to zero requiring infeasibly large values of B. Compounding the computational burden, the objective function is not smooth and requires substantially slower non-gradient optimization methods, such as the Nelder-Mead method.

Accept-reject simulator with smoothing (Honka and Chinthalunta 2017; Ursu 2018). Smoothing replaces the sharp constraints in the accept-reject simulation, such as $\mathcal{I}[a < b]$, with a continuous and monotonic function of difference $b - a$. This approach punishes large violations of the constraints but allows small differences. While this approach is often feasible, it is not the case for our data. First, most of the constraints of the form $\mathcal{I}[a < b]$ have arguments a and b bounded from below. For example, ω_{ij} is bounded by $-R_i$ because $\mathcal{T}(\kappa) \to 0$ if $\kappa \to +\infty$. When we attempt to impose these bounds, the difference $b - a$ does not translate well into a probability. Second, returns are represented by a single constraint, and we observe returns only for sessions that end with a purchase. The "return constraint" constitutes a small proportion of all the constraints in the model. With smoothing, violation of the "return constraint" would not have a sufficient impact on the final objective function, effectively reducing the model to a click-to-purchase model rather than the full customer journey.

Partially closed-form integration (this paper). We address the computational burden by formulating the click-to-purchase-to-return model (Table 1) such that some, but not all, variables in the constraints can be integrated with closed-form solutions. For example, from Equation (8), it follows that only the return constraint contains the value of the shock ψ_{ib}. In Appendix Appendix H, we show that sufficiently many constraints can be integrated out for the inner integral in Equation (9). Only one remaining constraint needs to be replaced with a smoothed version. Partially closed-form integration substantially reduces the required number of draws B and allows maximization with a gradient-based algorithm. Appendix Appendix I compares the proposed method with the alternative using the synthetic data.

5.2 Recovering Model Parameters Using Synthetic Data

To examine parameter recovery and the implications of considering the complete customer journey, we use synthetic data (empirically grounded Monte Carlo simulated data). Specifically, we assume that customers behave according to the proposed model and simulate the click, purchase, and return behavior of 5,000 synthetic customers. We consider three different scenarios. The first scenario closely mimics the observed data and allows us to evaluate the ability of partially closed-form integration to recover "true" parameters. The second and third scenarios illustrate situations where submodels that ignore either clicks or returns do not recover "true" parameters and provide misleading interpretations. The models we evaluate are:

- Click-to-purchase-to-return (Scenarios 1, 2, 3). As proposed in Section 4.
- Purchase-to-return (Scenario 2). This model assumes the customer evaluates all products in the available assortment.
- Click-to-purchase (Scenario 3). This model removes the customer's option to return the product.

For all scenarios, we assume that the retailer has an assortment of 100 different products equally split between two categories (we use a dummy coding such that products from the second category have $x_{ij} = 1$). Customers visiting the website see a subsample of these products where different positions on the website have different click costs (products at the top of the list have lower click costs). Each subsample contains 50 randomly selected products where a fraction γ is from the second category and a fraction $1-\gamma$ is from the first category. Table 2 summarizes three scenarios. Parameter values are listed in the upper half of the table, simulated sales based on these parameters are listed in the lower half of the table. We estimate the click-to-purchase-to-return model for all three scenarios to examine whether the proposed estimation strategy can recover "true" parameters. The second and third scenarios illustrate conditions where the reduced models fail to recover known parameters.

Table 2 demonstrates that the parameters of our model can be recovered using the proposed estimation procedure. The proposed model recovers the "true" parameters with reasonable accuracy.

An example when the purchase-to-return only model fails. Scenario 2 illustrates a situation in which the purchase-to-return only model fails to estimate the customer preference β_1^u sign correctly. In Scenario 2, the products in the second category have higher sales even though they are less preferred ($\beta_1^u < 0$), Products in the second category have higher sales because they are featured more often on the website ($\gamma = 0.8 \neq 0.5$). A model that does not account for rational click decisions overestimates the customer preferences for these products. If the retailer was attempting to make assortment decisions based on the purchase-to-return model (and ignoring clicks), the retailer might make incorrect assortment decisions.

An example when the click-to-purchase only model fails. Scenario 3 illustrates a situation in which products from the second category are still a worse alternative but have a higher variance of post-purchase fit ($\beta_1^\psi > 0$). The higher variance of post-purchase fit, combined with a reasonable return cost, makes products in the second category more attractive. With the right choice of parameters, customers benefit more from the products they keep (upper tail of post-purchase shock) than they lose for the products they return. However, when returns are not modeled, the click-to-purchase model attributes this advantage for second-category products to preferences and misestimates the sign of β_1^u.

Table 2: Estimation with Synthetic Data to Test if True Parameters Can be Recovered

		True Values (1)	Click-to-purchase-to-return	True Values (2)	Click-to-purchase-to-return	Purchase-to-return	True Values (3)	Click-to-purchase-to-return	Click-to-purchase
Value of outside option	β_0^u	-4.3	-4.28 (0.04)	-4.3	-4.39 (0.03)	-3.36 (0.03)	-4.3	-4.36 (0.05)	-3.78 (0.02)
Customer preference	β_1^u	0.4	0.45 (0.03)	0.3	0.30 (0.05)	-0.31 (0.20)	-0.3	-0.30 (0.05)	0.19 (0.01)
Preference heterogeneity	σ_1^u	0.3	0.25 (0.03)	0	0.01 (0.01)	0.17 (0.18)	0	0.04 (0.01)	0.02 (0.01)
Click costs (intercept)	β_0^c	-7.0	-6.89 (0.08)	-7.0	-7.12 (0.08)	-	-7.0	-7.08 (0.10)	-6.27 (0.08)
Click costs (slope)	β_1^c	0.7	0.69 (0.01)	0.7	0.71 (0.01)	-	0.7	0.70 (0.01)	0.69 (0.01)
Post-purchase information	β_1^ψ	-0.4	-0.51 (0.04)	0	-0.00 (0.07)	0.07 (0.14)	0.5	0.46 (0.04)	-
Return costs	$\log R$	-1.1	-1.19 (0.10)	-1.1	-1.34 (0.07)	-0.98 (0.09)	-1.1	-1.27 (0.10)	-
Share of Category 2 products γ		0.8	-	0.8	-	-	0.5	-	-
Sales Category 1 (share returned)		382 (0.41)	-	768 (0.39)	-	-	452 (0.37)	-	-
Sales Category 2 (share returned)		196 (0.18)	-	395 (0.38)	-	-	1033 (0.57)	-	-

Note: standard errors in parenthesis using bootstrap with 10 repetitions

The click-to-purchase only model misattributes consumer choice to average pre-inspection preference rather than the variance of post-purchase fit. A retailer might react by making incorrect product-assortment decisions.

Scenarios 2 and 3 illustrate that a model analyzing only part of the full customer journey, by neglecting either search or returns, can lead to incorrect parameter inference and incorrect managerial decisions. We chose only two illustrative scenarios; there are many others, and it is straightforward to develop a scenario when both reduced models fail to accurately recover the values of key parameters that describe the customer's journey. Of course, there are scenarios in which neglecting either clicks or returns has less impact. The important insight is that we cannot know a priori (or even post-estimation with a reduced model) which scenario is empirically relevant.

In the synthetic data, the click-to-purchase-to-return rational model is "ground truth." Furthermore, in the synthetic data (as in our empirical data) the retailer does not use clickstream data to attempt to manage returns. When both of these assumptions hold (ground truth, no endogeneity), then the proposed estimation procedure accurately recovers "true" parameters. We have also shown that modeling the entire customer journey matters. Not modeling the click or return component does more than adding random noise. Not modeling these aspects of customer behavior can, in realistic scenarios, lead to incorrect inferences and potentially incorrect managerial actions.

5.3 Endogeneity and Feasible Modeling

By construction, the synthetic data contain no endogeneity. Products were made available randomly based on the γ parameter independently of customer behavior. In fashion retail, decisions are made for each fashion season. Given EU rules, by the time sufficiently many returns are observed, it is too late to act (Dzyabura et al. 2023). Decisions on which products to feature may be based on observed sales, but returns data are not available in time for these decisions.

Empirically, in our data, the retailer did not select product assortment or return discouragement policies based on individual-customer clickstream data. For estimation based on these data, endogeneity (clickstream-to-policy) is not a major issue. Nonetheless, we gain insight in Section 6 by examining whether a click-to-purchase-to-return model estimated on the data reproduces the stylized facts. If it does not, then there may be unobserved and relevant endogeneity. If it reproduces the stylized facts, then the results are consistent with a hypothesis that endogeneity is of second order – although this does not rule out endogeneity.

Endogeneity would be an issue in estimation if it were feasible for the retailer to select its policies anticipating parameters of the click-to-purchase-to-return model. In this case, the likelihood function would be misspecified. However, if the fashion retailer's policy is based only on observed clicks, purchases, and/or returns (not directly on the model parameters), then all parameter estimates are consistent by the likelihood principle (Bowden and Trippa, 2017; Liu et al., 2007). Parameter estimates might still be biased for small samples (Hadad et al., 2021). Fortunately, the fashion retailer's data provide large samples such that the consistent estimates are extremely close to their asymptotic values.

Endogeneity would also be an issue if the customers behave strategically. For example, the customer might be strategic if, when the customer knows the retailer will make returns more difficult for certain click patterns, then the customer avoids or hides click patterns. If the customer behaves strategically, then such strategic behavior must be modeled. If the customer is not strategic, then the parameter estimates should be sufficient for policy decisions.

Our current data and the current retailer situation do not enable us to test whether the customers behave strategically or whether the retailer sets policy based on the parameters of the model. However, all hope is not lost. An analysis strategy from the product-development literature might be useful in a multiyear study. Specifically, product-development managers use metrics such as speed-to-market, customer satisfaction, and component reuse to manage product-development teams. They set weights for the metrics based on the current "operating point" recognizing that doing so is a linear approximation to a non-linear surface. The product-development team reacts to the metrics, adjusts its behavior and the system moves to a new "operating point." Over many years, the linear approximations act like a gradient search to find the global set of optimal weights for the metrics. Given the highly non-linear relationships in the click-to-purchase-to-return model, such a solution strategy might help the retailer find the best policy even if either the customers are strategic or the retailer's policies are based on model parameters.

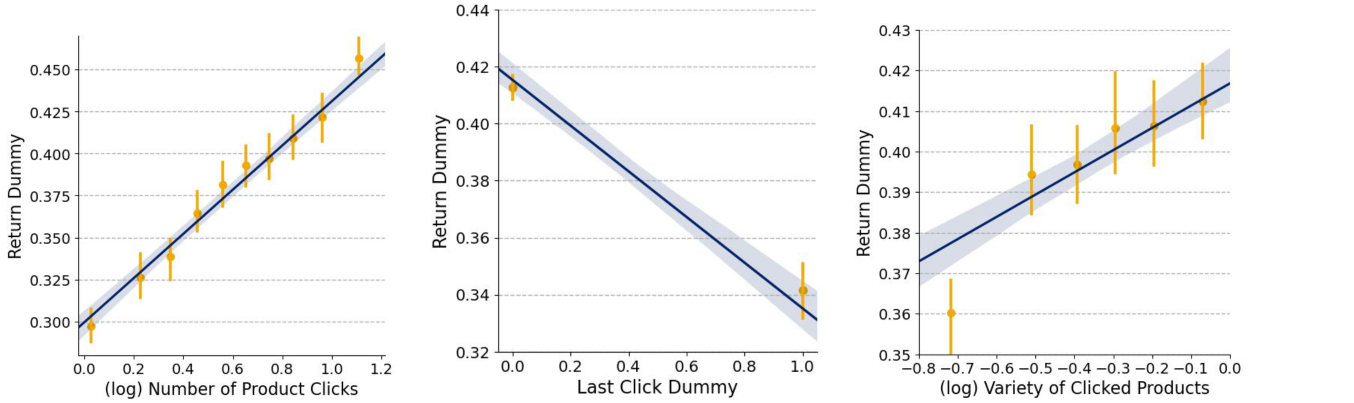
Finally, to the best of our knowledge, a forward-looking rational click-to-purchase-to-return model of customer behavior is just barely feasible to estimate. Any feasible model with endogeneity modeled explicitly would need to make assumptions to simplify the click-to-purchase-to-return model. Tradeoffs between a model that does not explicitly address endogeneity and a model that addresses endogeneity but simplifies the purchase journey is an open research question best addressed empirically. This is an important area to explore. We hope that our synthetic-data analyses illustrate the value of modeling the full purchase journey and, in doing so, encourage research to explore modeling tradeoffs.

6 Click-to-Purchase-to-Return Model is Consistent with Stylized Facts

6.1 Replication of Empirically-Based Stylized Facts with the Click-to-Purchase-to-Return Model

Using the observed fashion retailer's data, we estimate the parameters of the click-to-purchase-to-return model. For each customer in the retailer's data, we create a digital twin that faces exactly the same website environment but makes click, purchase, and return decisions according to the estimated model. We compare the behavior of the observed customers from our retailer's data (Figures 3 and 5) with those customers who act according to the estimated click-to-purchase-to-return model (Figure 8). As with observed customers, the plots for the digital twins have slopes in the same direction, and the slopes have roughly the same magnitude. As we would expect with any model, there is less noise in the relationships and some of the relationships are attenuated (lower slope).

Figure 8: Replication of Stylized Facts



(a) Number of product clicks **(b)** Last clicked product **(c)** Click variety

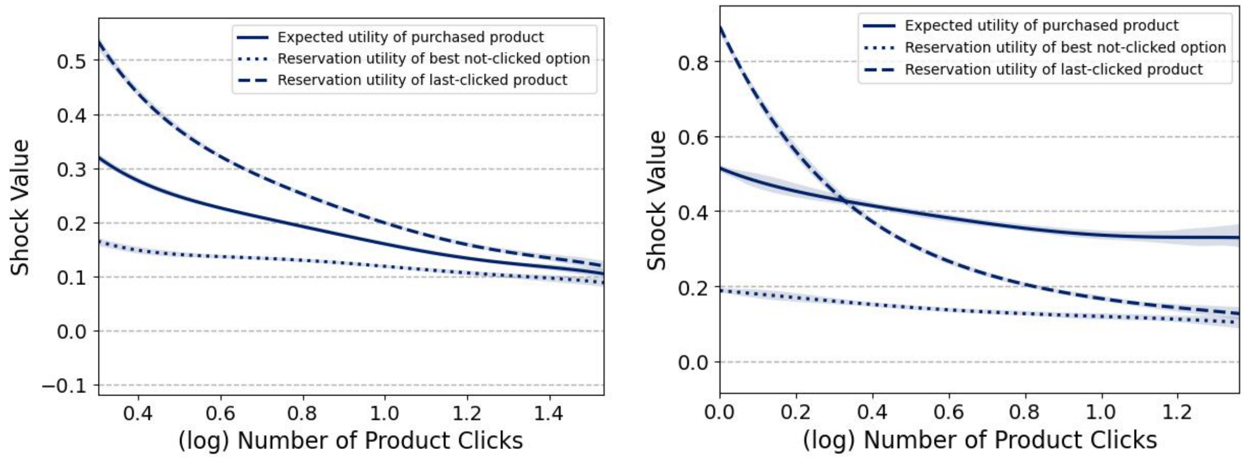
6.2 Why the Click-to-Purchase-to-Return Model Replicates the Empirically-Based Stylized Facts

Although the click-to-purchase-to-return model is complex, non-linear, and dynamically forward-looking, estimation with synthetic data uncovers "true" parameters well, and the analysis of model-based digital twins reproduces the stylized facts. It is possible (although complicated) to argue why each model parameter is identified. Rather than detail every argument, we sample those arguments to illustrate why the model successfully reproduces the stylized facts.

Number of clicked products. To visualize why the model has these properties (and to provide insight as to why we can identify parameters), consider why the number of clicks informs the researcher about the hidden variables z_{ij} and ω_{ij}. According to the click-to-purchase-to-return model, the customer's click sequence is fully determined by the values of reservation utility z_{ij} and expected purchase utility ω_{ij}. The return decision is implicitly related through the constraints imposed by optimal decision rules. The number of clicks the customer made during the browsing session informs the researcher about the values of hidden variables z_{ij} and ω_{ij}.

To illustrate the mechanism, we consider two cases depending on whether or not the customer purchased the last clicked product. At the moment of purchase, the customer has just clicked on the last product C_i with the reservation utility z_{iC} and knows the expected utility of the chosen product ω_{ib}. Both these variables are higher than the reservation utility of the best not-clicked option. If the customer did not purchase the last-clicked product (Case 1), the expected utility of purchasing product b would always be bounded from above by the value of reservation utility of the last-clicked product z_{iC}. Intuitively, the customer still found it beneficial to continue the search. Because they click in the decreasing order

Figure 9: Replication Stylized Facts: Number of Clicked Products



(a) Case 1: purchased not last-clicked product **(b)** Case 2: purchased last-clicked product

of reservation utilities, customers who made more clicks would, on average, have a lower value of the reservation utility of the last clicked product C_i, or, similarly, a smaller lower bound on ω_{ib}. Figure 9a illustrates this dependence. If the customer purchased the last-clicked product (Case 2), then the upper bound on the expected utility of the purchased product does not exist. Similar to the previous case, customers with longer sessions would end the click sequence with lower reservation utility, z_{iC}. Because the reservation utility is negatively correlated with the number of clicks, the final utility would also be lower and the return probability higher. Figure 9b illustrates this dependence. Intuitively, customers who click on many products struggle to find a product they like. Customers who purchased a product after many clicks would, on average, choose a worse option than the customer who discovered a great product after a smaller number of clicks. Thus, on average, the length of the search session informs retailers about customers who are struggling with the decision and are more likely to return.

Last clicked product. We can make related arguments based on the click-to-purchase-to-return model to explain why customers who purchased the last clicked product are less likely to return the product. Simplifying, the last-clicked product's expected utility is not bounded from above and could potentially reveal a higher utility value. For example, if the customer found a "dream T-shirt" with high expected utility, the customer would be less likely to return it.

Variety. At the beginning of the customer's search, customers vary based on their individual preference vectors β^i and the realizations of pre-click preference shocks ξ_{ij}. Customers who click on similar products tend to have higher $x_{ij}\beta^i$ components of utility relative to the ξ_{ij} components. On the other hand, customers who click on a high variety of products have higher ξ_{ij} relative to $x_{ij}\beta^i$. (Unless, by luck, multiple products happen to vary on x_{ij} in a way that the $x_{ij}\beta^i$ happens to be high – a less common case.) With a high $x_{ij}\beta^i$ component, the post-inspection shock matters less and the customer is less likely to return the product.

7 Summary and Future Directions

7.1 Summary and Potential Managerial Actions

Product returns pose a significant challenge for online retailers, resulting in substantial financial losses due to refunds, reverse logistics, and product processing costs. Standard research on managing returns focuses primarily on purchase and return behaviors, often overlooking valuable information from the customer's prepurchase journey. Because customers spend considerable time clicking on the website before making a purchase decision, these actions reveal information that carries predictive signals that could help retailers proactively manage and mitigate returns.

Understanding the full customer journey, from initial website clicks to the decision to keep or return a product improves standard models and could improve managerial decision-making. In this paper, we explore a click-to-purchase-to-return framework that integrates customer behavior from the initial browsing stage to the post-purchase decision of whether to return or keep a product. By analyzing data from a major European apparel retailer, we demonstrate how specific prepurchase customer click patterns provide valuable insights into the likelihood of product returns, a perspective that enhances insights from standard purchase-to-return and click-to-purchase models.

Our rational model aligns with empirically observed stylized facts and enables a deeper exploration of the mechanisms that connect customer click behavior to product returns. Specifically, we find that purchasing the last-clicked product, browsing fewer products, and browsing a more focused variety of products are associated with a lower probability of returns. Standard frameworks based on purchase-to-return or click-to-purchase do not completely capture these relationships. We illustrate how standard models may result in inaccurate estimation of parameters that characterize both customer preferences and the products they select. We are cautious not to claim causality and have avoided explicit evaluation of managerial strategies. However, the rational click-to-purchase-to-return model provides insight that has the potential to be relevant to retailers.

Retailers could use the click-to-purchase-to-return model to suggest policies aimed at reducing returns. By observing customer clicks, the retailer may actively classify the customer-session into a high-risk or low-risk of return and treat the customer-sessions differently. For example, if the customer purchased a product after making ten product clicks and reviewing a wide variety of different products (these patterns are associated with a high return likelihood), the retailer could offer the customer a gift card or a coupon for the next purchase, contingent on not returning the product. This offer would increase utility to the customer so that the utility would be higher than the return cost, thus decreasing the return likelihood. The click-to-purchase-to-return model identifies a range of reasonable gift-card and coupon values and their expected impact. The firm could test different triggers of customer behavior using randomized control experiments. Because small changes in the return probability lead to a considerable increase in profitability (Thomasson, Emma 2013), there is substantial potential for improved retailer policies.

After the click-to-purchase-to-return model is tested further and causality assured (say based on randomizing the sequence of displayed products), retailers might seek to enhance customer experience by optimizing their websites. Retailers might adjust the display order of products so that high-utility items appear closer to the top of the page (Ursu, 2018). Section 5 suggests that randomized controlled experiments are best analyzed with a model that reflects the full customer journey from click-to-purchase-to-return. The full-journey model is more accurate relative to standard models and provides both more "levers" to pull and a greater combination of output variables to optimize.

Fortunately, the click-to-purchase-to-return model is based on data to which online retailers routinely have access – browsing, purchase, and returns data from their own websites. The model does not require third-party sources which may or may not be available and may or may not violate privacy restrictions. Given the practical feasibility of estimating the click-to-purchare-to-return model in real-world settings, returns-conscious retailers can benefit from the proposed approach with minimal additional costs.

7.2 Future Research

By far the most critical next steps are to establish causality beyond a reasonable doubt. Causal experiments can test the rational click-to-purchase-to-return model and, if necessary, elaborate the model. By construction, there is no endogeneity in the synthetic data and, by practice, there is no retailer endogeneity in the retailer's data. But once new policies are implemented, customers may behave strategically, closing the loop. This is an important area of research.

We demonstrated a practical means to estimate the click-to-purchase-to-return model, but estimation required various assumptions on the functional form of various probability distributions. Although we believe these assumptions are benign, future research could explore the sensitivity of the model to these assumptions. Research might also extend the model to other aspects of the customer journey such as the impact of pre-website-visit marketing and/or the generation of post-purchase word-of-mouth. There are many exciting areas of research to explore based on the customer's journey.

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Appendix A Data Preprocessing and Additional Information

Overall data cleaning. Search data is typically noisy; therefore, we preprocessed the data to obtain better estimates of the model parameters. We took the following steps in the preprocessing:

1. Removed non-fashion products (e.g., linen, towels) and kids' apparel. These products constitute a small proportion of the data and are not the retailer's focus (95% of purchases are adult fashion products).
2. Removed browsing sessions without product listing views (each product listing contains up to 96 products presented to the customer; see Figure 7 for the example). This could happen if the customer comes to the website from a third-party website and lands directly on the product page. These sessions do not represent the true customer search process at the retailer's website, and we are not able to recover the set of products from which the customer was choosing.
3. Removed browsing sessions that have more than 50 pages viewed, products clicked, or products purchased.
4. Removed browsing sessions where customers were viewing product listings of size greater than 48. Our retailer allows the customer to view 48 or 96 products in one listing, most customers view 48 products (default option).
5. Removed browsing sessions that have not clicked products after a page view and sessions that have clicked products before a page view. This implies we kept only sessions with the clean search process: the customer views the product page and selects a product to click on it. The alternative could happen if the customer found a product through an alternative means (from a third-party website) and in this case, it is impossible to infer the set of products from which he or she was choosing.

Selecting single-item orders. In the paper, we consider orders where the customer purchased at most one product. However, there are two additional steps used to obtain the representative data sample:

1. Sessions without a purchase. After we selected transactions with only one item purchased, we randomly subsampled sessions without a purchase to preserve the relative purchase rate.
2. Orders with one product but multiple sizes or several identical units. In this case, we split the order into several orders with the same search session, and only one unit was purchased. However, these orders could have different return outcomes. This approach allows us to keep more data and thus improve the estimation quality. Using alternative approaches does not lead to substantial changes.

The preprocessing procedures do not change the main message of the paper and are aimed at obtaining a representative data sample that balances the quality and quantity of data. In practice, the retailer may implement different preprocessing procedures, which could change the parameter values, but qualitative findings would remain similar.

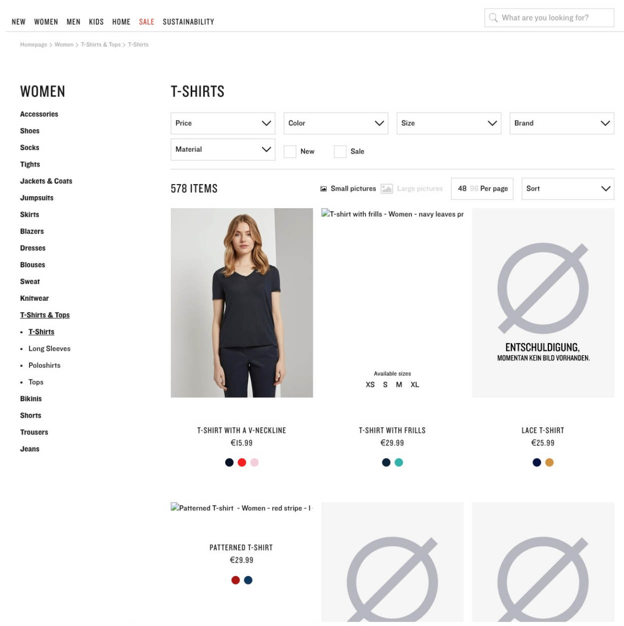
Screenshot of the retailer's website. The customer browsing the retailer's website during our observational period would observe the information in Figure A.1. By 2024, our retailer had updated the design of their websites. To provide the most accurate information, we use Wayback Machine (https://web.archive.org/) to capture the version of the website as of 2020. Unfortunately, some product pictures were not stored by the platform.

Appendix B Deep Learning Embeddings

In the paper, we mentioned that during the estimation, we used deep learning product embeddings to address the issue of data's high dimensionality. Our procedure for extracting the product embeddings could be summarized in the following steps:

1. Creating product base features:
   (a) Combine product quantitative characteristics (category dummy, price, brand).
   (b) Use ResNet model to generate product image embeddings (2048-dimensional vectors) and PCA transformation to extract 64 components.
   (c) Concatenate (a) and (b).
2. Computing aggregate product-level outcomes:
   (a) Click rate cr_j – the ratio of clicks to views.
   (b) Purchase rate pr_j – the ratio of sales to clicks.
   (c) Return rate rr_j – the ratio of returns to sales.

Figure A.1: Screenshot of Retailer's Website



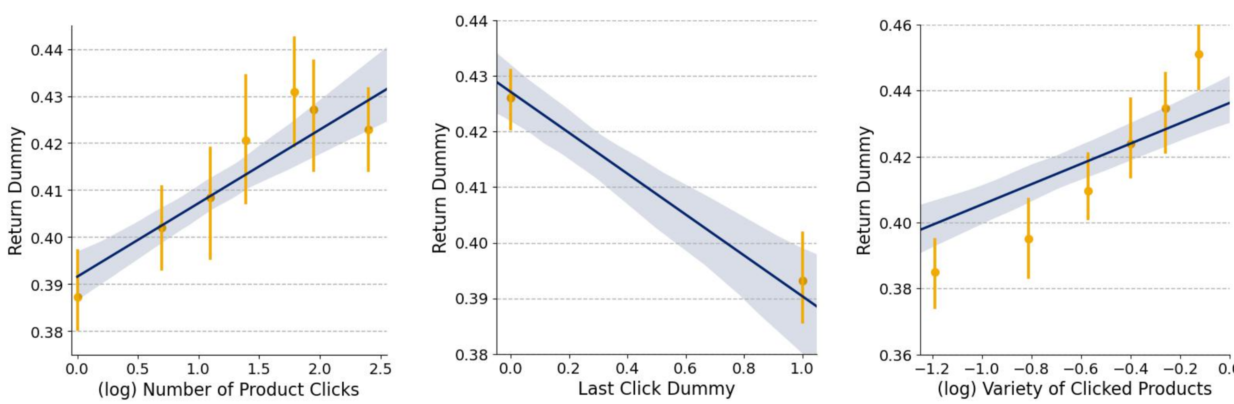
3. Training the neural network to produce 8-dimensional product embeddings e_j such that $e_j^{cr}, e_j^{pr}, e_j^{rr}$ minimize the prediction error of log-ratio of cr_j, pr_j, rr_j respectively.

These embeddings represent the product in terms of its three key characteristics: click rate, purchase rate, and return rate. Other ways exist to construct product embeddings; however, exploring these options goes beyond the scope of the paper. We leave this exercise to the retailer, who may have different available data. We only note that this approach is sufficient to support the paper's main message.

Appendix C Empirically-Based Stylized Facts without Fixed Effects

We replicate the main results from Section 3 in Figure C.1 without using the fixed effects. However, to simplify the comparison, we subtracted the mean values from the variables of interest. Intuitively, it is equivalent to using uniform fixed effects (only one product type). The results are qualitatively the same.

Figure C.1: Replication of Empirically-based Stylized Facts without Fixed Effects



(a) Number of product clicks **(b)** Last clicked product **(c)** Click variety

Appendix D Derivation of Expected Purchase Utility

Without loss of generality, we drop all indices and subscripts in this section to preserve readability. In the section discussing the model, we wanted to find the expected utility of purchasing a product from the website. In this case, the customer knows the variables μ, ϵ, and ξ and computes the expected utility in Equation 1 over ψ:

$$\begin{aligned}\omega &= \mathbb{E}_\psi\left[(\mu+\xi+\epsilon+\psi+R)\cdot\mathcal{I}(\mu+\xi+\epsilon+\psi+R\geq 0) - R \mid \mu,\xi,\epsilon\right] \\ &= \mathbb{E}_\psi\left[\zeta\cdot\mathcal{I}(\zeta\geq 0)\mid \mu,\xi,\epsilon\right] - R = \sigma_\psi\cdot\mathcal{T}\left(\frac{\mu+\xi+\epsilon+R}{\sigma_\psi}\right) - R\end{aligned} \tag{D.1}$$

where $\zeta \mid \mu,\xi,\epsilon \sim \mathcal{N}(\mu+\xi+\epsilon+R;\sigma_\psi)$ and formula for the expectation of truncated normal distribution was used; $\mathcal{T}(\kappa) = \kappa\Phi(\kappa)+\varphi(\kappa)$ with $\Phi(\kappa)$ and $\varphi(\kappa)$ being CDF and PDF of standard normal distribution respectively.

Appendix E Derivation of Reservation Utilities for Model with Product Returns

In the original paper, Weitzman (1979) demonstrated that the reservation utility z for a product could be found from Equation (E.1) where we drop the individual i and product j indices for compactness:

$$c = \int_z^\infty (u-z)dF(u) \tag{E.1}$$

In the section discussing the model, we demonstrated that the return option changes the reward distribution; thus, in this case, we need to find the distribution of the expected purchase utility from Equation (D.1). Notice that the customer observes only μ and ξ before clicking. Therefore the randomness in Equation (D.1) comes from the post-click preferences shock ϵ:

$$\begin{aligned}F(u) &= \mathbb{P}[\omega(\epsilon)\leq u\mid\mu,\xi] = \mathbb{P}\left[\sigma_\psi\cdot\mathcal{T}\left(\frac{\mu+\xi+\epsilon+R}{\sigma_\psi}\right) - R\leq u\mid\mu,\xi\right] \\ &= \mathbb{P}\left[\tilde{\mu}+\epsilon+R\leq\sigma_\psi\mathcal{T}^{-1}\left(\frac{R+u}{\sigma_\psi}\right)\mid\mu,\xi\right] = \Phi\left[\sigma_\psi\mathcal{T}^{-1}\left(\frac{R+u}{\sigma_\psi}\right)-\tilde{\mu}-R\right]\end{aligned} \tag{E.2}$$

where $\tilde{\mu} = \mu+\xi$, and the assumption that $\epsilon\sim\mathcal{N}(0;\sigma_\epsilon)$ was used.

Next, we plug in the distribution from Equation (E.2) in Equation (E.1) and obtain:

$$\begin{aligned}c &= \int_{\sigma_\psi\mathcal{T}^{-1}\left(\frac{R+z}{\sigma_\psi}\right)-\tilde{\mu}-R}^{\infty}\left(\sigma_\psi\cdot\mathcal{T}\left(\frac{\tilde{\mu}+t+R}{\sigma_\psi}\right)-R-z\right)d\Phi(t) \\ &= \sigma_\psi\int_\theta^\infty \mathcal{T}\left(\frac{\tilde{\mu}+R+t}{\sigma_\psi}\right)-\mathcal{T}\left(\frac{\tilde{\mu}+R+\theta}{\sigma_\psi}\right)d\Phi(t)\end{aligned} \tag{E.3}$$

where we used the substitution $z = \mathcal{T}\left(\frac{\tilde{\mu}+R+\theta}{\sigma_\psi}\right) - R$ for compactness.

Appendix F Approximating the Solution to the Equation

In the paper, we made an assumption that $\sigma_\epsilon = 1$. Thus, from Equation (E.3) it could be seen that the reservation utility is a function of three parameters: $z^* = f(\tilde{\mu}+R,\sigma_\psi,c) = f(x_1,x_2,x_3)$. Finding this function for each customer-product combination during the optimization algorithm is not feasible as it involves many integration steps.

To circumvent the computational burden, we used the trilinear interpolation technique. Specifically, for three-dimensional variables (x_1,x_2,x_3), we constructed a grid of values and computed the exact reservation utilities for each element of the grid. Notice that in this case, the space of possible values of (x_1,x_2,x_3) is divided into 3-dimensional cubes. For each of these cubes, we know the exact values of reservation utilities in eight vertices. For any vector within the cube, we approximate the reservation utility function $f(x_1,x_2,x_3)$ as:

$$
\begin{aligned}
f_{\text{true}}(x_1, x_2, x_3) &\simeq f_{\text{approx}}(x_1, x_2, x_3) \\
&\simeq \alpha_0 + \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \alpha_4 x_1 x_2 + \alpha_5 x_2 x_3 + \alpha_6 x_1 x_3 + \alpha_7 x_1 x_2 x_3
\end{aligned} \tag{F.1}
$$

where we require $f_{\text{true}}(x_1, x_2, x_3) = f_{\text{approx}}(x_1, x_2, x_3)$ at the grid (or cube vertices) points. Because $f_{\text{approx}}(x_1, x_2, x_3)$ has eight parameters and eight constraints, the linear system has a unique solution for each cell.

Appendix G Derivation of Equivalent Set of Constraints on Model Parameters

After combining Equations (4) to (7), we can compute the variable W_i. For compactness and without loss of generality, we drop the customer index i:

$$
\begin{aligned}
W = &\prod_{j=0}^{C-1} \left[\mathcal{I}\left[\max_{s=0..j} \omega_s < \max_{s=j+1..V} z_s \right] \mathcal{I}\left[z_{j+1} \geq \max_{s=j+2..V} z_s \right] \right] \\
&\times \mathcal{I}\left[\max_{s=0..C} \omega_s \geq \max_{s=C+1..V} z_s \right] \\
&\times \mathcal{I}\left[\omega_b \geq \max_{s=0..C} \omega_s \right] \mathcal{I}\left[\mu_b + \xi_b + \epsilon_b + \psi_b \leq -R \right]
\end{aligned} \tag{G.1}
$$

Consider the part of the equation related to click continuation:

$$
\begin{aligned}
P_1 &= \prod_{j=0}^{C-1} \mathcal{I}\left[z_{j+1} \geq \max_{s=j+2..V} z_s \right] \\
&= \prod_{j=0}^{C-1} \prod_{s=j+2..V} \mathcal{I}\left[z_{j+1} \geq z_s \right] \\
&= \mathcal{I}\left[z_C \geq \max_{s=C+1..V} z_s \right] \prod_{j=1}^{C-1} \mathcal{I}\left[z_j \geq z_{j+1} \right]
\end{aligned} \tag{G.2}
$$

Notice that Equation (G.2) is a necessary condition for $W = 1$. Thus, we can assume that these inequalities hold in further derivations. Specifically, it follows that

$$
\forall j \leq C: \max_{s=j+1..V} z_s = z_{j+1} \tag{G.3}
$$

and we can rewrite another click continuation constraint as:

$$
\begin{aligned}
P_2 &= \prod_{j=0}^{C-1} \mathcal{I}\left[\max_{s=0..j} \omega_s < \max_{s=j+1..V} z_s \right] \\
&= \prod_{j=0}^{C-1} \left[\prod_{s=0..j} \mathcal{I}\left[\max_{s=0..j} \omega_s < z_{j+1} \right] \right] \\
&= \prod_{j=0}^{C-1} \mathcal{I}\left[\omega_j < z_C \right]
\end{aligned} \tag{G.4}
$$

Similarly, we simplify the click-stopping and purchasing decision constraints:

$$
\begin{aligned}
P_3 &= \mathcal{I}\left[\max_{s=0..C} \omega_s \geq \max_{s=C+1..V} z_s\right] \prod_{j=0}^{C} \mathcal{I}\left[\omega_b \geq \omega_j\right] \\
&= \mathcal{I}\left[\omega_b \geq \max_{s=C+1..V} z_s\right] \prod_{j=0}^{C} \mathcal{I}\left[\omega_j < \omega_b\right]
\end{aligned} \tag{G.5}
$$

Finally, after combining all equations for P_1, P_2, and P_3, we obtain the simplified version of variable W from Equation (8):

$$
\begin{aligned}
W = &\left[\prod_{j=1}^{C-1} \mathcal{I}\left[z_j \geq z_{j+1}\right]\right] \mathcal{I}\left[z_C \geq \max_{s=C+1..V} z_s\right] \prod_{j=0}^{C-1} \mathcal{I}\left[\omega_j \leq \min\{z_C, \omega_b\}\right] \\
&\times \mathcal{I}\left[\omega_C \leq \omega_b\right] \mathcal{I}\left[\omega_b \geq \max_{s=C+1..V} z_s\right] \mathcal{I}\left[\mu_b + \xi_b + \epsilon_b + \psi_b \leq -R\right]
\end{aligned} \tag{G.6}
$$

Appendix H Derivation of Semi-Closed Form Likelihood

As in the previous sections, we drop the customer-related index i for compactness. Recall the set of constraints that must be satisfied to observe a given customer sequence from Equation (G.6):

$$
\begin{aligned}
W(\xi, \epsilon, \psi) = &\left[\prod_{j=1}^{C-1} \mathcal{I}\left[z_j \geq z_{j+1}\right]\right] \mathcal{I}\left[z_C \geq \max_{s=C+1..V} z_s\right] \prod_{j=0}^{C-1} \mathcal{I}\left[\omega_j \leq \min\{z_C, \omega_b\}\right] \\
&\times \mathcal{I}\left[\omega_C \leq \omega_b\right] \mathcal{I}\left[\omega_b \geq \max_{s=C+1..V} z_s\right] \mathcal{I}\left[\mu_b + \xi_b + \epsilon_b + \psi_b \leq -R\right]
\end{aligned} \tag{H.1}
$$

where ω_j is a function of unobserved to researcher shocks ξ_j and ϵ_j; z_j is a function of unobserved to researcher shock ξ_j.

To obtain the likelihood function, one would integrate out all unobserved shocks from the variable $W(\xi, \epsilon, \psi)$

$$
\begin{aligned}
\int \cdots \int W(\xi, \epsilon, \psi) dF(\xi, \epsilon, \psi) &= \int \cdots \int W(\xi, \epsilon, \psi) \left[\prod_{j=1}^{V} dF_{\xi_j}(\xi_j)\right] \left[\prod_{j=1}^{C} dF_{\epsilon_j}(\epsilon_j)\right] dF_{\psi_b}(\psi_b) \\
&= \int \cdots \int W(\xi, \epsilon, \psi) \left[\prod_{j=1}^{V} d\Phi(\xi_j)\right] \left[\prod_{j=1}^{C} d\Phi(\epsilon_j)\right] d\Phi\left(\frac{\psi_b}{\sigma_{\psi_b}}\right)
\end{aligned} \tag{H.2}
$$

where we used the assumption that all shocks are independent and normally distributed

Notice that only the last inequality in Equation (H.1) depends on ψ_b and we can replace the integral with the expression:

$$
\mathcal{R}_b(\xi_b, \epsilon_b) = \int \mathcal{I}\left[\mu_b + \xi_b + \epsilon_b + \psi_b \leq -R\right] d\Phi\left(\frac{\psi_b}{\sigma_{\psi_b}}\right) = 1 - \Phi\left(\frac{R + \mu_b + \xi_b + \epsilon_b}{\sigma_{\psi_b}}\right) \tag{H.3}
$$

where $\mathcal{R}_b(\xi_b, \epsilon_b)$ explicitly reflects that it depends on two unobserved shocks ξ_b and ϵ_b.

Next, the set of shocks $\{\xi_j\}_{j=C+1}^{V}$ appear only in two constraints and could be simplified to:

$$\begin{aligned}\int\cdots\int\mathcal{I}\left[z_C\geq\max_{s=C+1..V}z_s\right]\mathcal{I}\left[\omega_b\geq\max_{s=C+1..V}z_s\right]\prod_{j=C+1}^{V}d\Phi(\xi_j)\\=\int\cdots\int\mathcal{I}\left[\min\{z_C,\omega_b\}\geq\max_{s=C+1..V}z_s\right]\prod_{j=C+1}^{V}d\Phi(\xi_j)\\=\prod_{j=C+1}^{V}\int\mathcal{I}\left[\min\{z_C,\omega_b\}\geq z_j\right]d\Phi(\xi_j)\\=\prod_{j=C+1}^{V}\int\mathcal{I}\left[z_j^{-1}\left(\min\{z_C,\omega_b\}\right)\geq\xi_j\right]d\Phi(\xi_j)\\=\prod_{j=C+1}^{V}\left[1-\Phi\left(z_j^{-1}\left(\min\{z_C,\omega_b\}\right)\right)\right]\end{aligned}\tag{H.4}$$

where we used the fact that $z_j(\xi_j)$ is an invertible function for each j, and index j reflects the fact that the function would depend on estimable parameters of the model. Notice that the simplified form depends only on three unobserved shocks: ξ_C through z_C, ξ_b and ϵ_b through ω_b (if $b=C$ or $b=0$ it is only two shocks).

The constraints related to the purchase decision: $\prod_{j=0}^{C-1}\mathcal{I}\left[\omega_j\leq\min\{z_C,\omega_b\}\right]\times\mathcal{I}\left[\omega_C\leq\omega_b\right]$. To simplify them, we need to consider three separate cases: choosing an outside option, the last searched option, and all else.

- *Choose an outside option (or $b=0$).* All shocks $\{\epsilon_j\}_{j=1}^{C}$ could be integrated out because only ω_j depends on these shocks, and there is no return constraint, hence Equation (H.3) could be ignored

$$\begin{aligned}\int\cdots\int\mathcal{I}[\omega_C\leq\omega_b]\prod_{j=0}^{C-1}\mathcal{I}[\omega_j\leq\min\{z_C,\omega_b\}]\prod_{j=1}^{C}d\Phi(\epsilon_j)\\=\int\cdots\int\mathcal{I}[\omega_C\leq\omega_0]\prod_{j=0}^{C-1}\mathcal{I}[\omega_j\leq\min\{z_C,\omega_0\}]\prod_{j=1}^{C}d\Phi(\epsilon_j)\\=\int\cdots\int\mathcal{I}[\omega_C\leq\omega_0]\mathcal{I}[\omega_0\leq z_C]\prod_{j=1}^{C-1}\mathcal{I}[\omega_j\leq\min\{z_C,\omega_0\}]\prod_{j=1}^{C}d\Phi(\epsilon_j)\\=\mathcal{I}[\omega_0\leq z_C]\left[\int\mathcal{I}[\omega_C\leq\omega_0]d\Phi(\epsilon_C)\right]\left[\prod_{j=1}^{C-1}\int\mathcal{I}[\omega_j\leq\min\{z_C,\omega_0\}]d\Phi(\epsilon_j)\right]\\=\mathcal{I}[\omega_0\leq z_C]\left[\int\mathcal{I}[\epsilon_C\leq\omega_C^{-1}(\omega_0)]d\Phi(\epsilon_C)\right]\left[\prod_{j=1}^{C-1}\int\mathcal{I}[\epsilon_j\leq\omega_j^{-1}(\min\{z_C,\omega_0\})]d\Phi(\epsilon_j)\right]\\=\mathcal{I}[\omega_0\leq z_C]\Phi\left(\omega_C^{-1}(\omega_0)\right)\prod_{j=1}^{C-1}\Phi\left(\omega_j^{-1}\left(\min\{z_C,\omega_0\}\right)\right)\\=\mathcal{I}[\omega_0\leq z_C]\mathcal{H}^0(\xi_C)\end{aligned}\tag{H.5}$$

  where the last row highlights that the integral depends only on one unobserved shock ξ_C.
- *Choose the last clicked option (or $b=C$).* All shocks $\{\epsilon_j\}_{j=1,j\neq b}^{C}$ could be integrated out because only ω_j depends on these shocks. Notice that $\epsilon_b=\epsilon_C$ also enters the Equation (H.3) and thus could not

be directly integrated out:

$$
\begin{aligned}
&\int \cdots \int \mathcal{R}_b(\xi_b, \epsilon_b)\mathcal{I}[\omega_C \leq \omega_b] \left[\prod_{j=0}^{C-1} \mathcal{I}[\omega_j \leq \min\{z_C, \omega_b\}]\right] \prod_{j=1}^{C} d\Phi(\epsilon_j) \\
&= \int \cdots \int \mathcal{R}_C(\xi_C, \epsilon_C)\mathcal{I}[\omega_C \leq \omega_C] \left[\prod_{j=0}^{C-1} \mathcal{I}[\omega_j \leq \min\{z_C, \omega_C\}]\right] \prod_{j=1}^{C} d\Phi(\epsilon_j) \\
&= \int \mathcal{R}_C(\xi_C, \epsilon_C)\mathcal{I}[\omega_0 \leq \min\{z_C, \omega_C\}] \left[\prod_{j=1}^{C-1} \int \mathcal{I}[\omega_j \leq \min\{z_C, \omega_C\}] d\Phi(\epsilon_j)\right] d\Phi(\epsilon_C) \\
&= \int \mathcal{R}_C(\xi_C, \epsilon_C)\mathcal{I}[\omega_0 \leq \min\{z_C, \omega_C\}] \left[\prod_{j=1}^{C} \Phi\left(\omega_j^{-1}\left(\min\{z_C, \omega_C\}\right)\right)\right] d\Phi(\epsilon_C) \\
&= \mathcal{I}[\omega_0 \leq z_C] \int \mathcal{R}_C(\xi_C, \epsilon_C)\mathcal{I}[\omega_0 \leq \omega_C] \left[\prod_{j=1}^{C} \Phi\left(\omega_j^{-1}\left(\min\{z_C, \omega_C\}\right)\right)\right] d\Phi(\epsilon_C) \\
&= \mathcal{I}[\omega_0 \leq z_C] \int_{\omega_C^{-1}(\omega_0)}^{+\infty} \mathcal{R}_C(\xi_C, \epsilon_C) \left[\prod_{j=1}^{C} \Phi\left(\omega_j^{-1}\left(\min\{z_C, \omega_C\}\right)\right)\right] d\Phi(\epsilon_C) \\
&= \mathcal{I}[\omega_0 \leq z_C]\mathcal{H}^C(\xi_C)
\end{aligned} \tag{H.6}
$$

- *Choose other option (or $0 < b < C$).* All shocks $\{\epsilon_j\}_{j=1, j \neq b}^{C}$ could be integrated out because only ω_j depends on these shocks. Notice that ϵ_b also enters the Equation (H.3) and thus could not be directly integrated out:

$$
\begin{aligned}
&\int\cdots\int \mathcal{R}_b(\xi_b,\epsilon_b)\mathcal{I}[\omega_C\le\omega_b]\left[\prod_{j=0}^{C-1}\mathcal{I}[\omega_j\le\min\{z_C,\omega_b\}]\right]\prod_{j=1}^{C}d\Phi(\epsilon_j)\\
=&\int\cdots\int \mathcal{R}_b(\xi_b,\epsilon_b)\mathcal{I}[\omega_C\le\omega_b]\mathcal{I}[\omega_b\le z_C]\left[\prod_{j=0,j\neq b}^{C-1}\mathcal{I}[\omega_j\le\min\{z_C,\omega_b\}]\right]\prod_{j=1}^{C}d\Phi(\epsilon_j)\\
=&\int\cdots\int \mathcal{R}_b(\xi_b,\epsilon_b)\mathcal{I}[\omega_C\le\omega_b]\mathcal{I}[\omega_b\le z_C]\mathcal{I}[\omega_0\le\min\{z_C,\omega_b\}]\\
&\times\left[\prod_{j=1,j\neq b}^{C-1}\mathcal{I}[\omega_j\le\min\{z_C,\omega_b\}]\right]\prod_{j=1}^{C}d\Phi(\epsilon_j)\\
=&\int \mathcal{R}_b(\xi_b,\epsilon_b)\mathcal{I}[\omega_b\le z_C]\mathcal{I}[\omega_0\le\min\{z_C,\omega_b\}]\\
&\times\left[\int\cdots\int \mathcal{I}[\omega_C\le\omega_b]\left[\prod_{j=1,j\neq b}^{C-1}\mathcal{I}[\omega_j\le\min\{z_C,\omega_b\}]\right]\prod_{j=1,j\neq b}^{C}d\Phi(\epsilon_j)\right]d\Phi(\epsilon_b)\\
=&\int \mathcal{R}_b(\xi_b,\epsilon_b)\mathcal{I}[\omega_b\le z_C]\mathcal{I}[\omega_0\le\min\{z_C,\omega_b\}]\\
&\times\left[\Phi\left(\omega_C^{-1}\left(\omega_b\right)\right)\prod_{j=1,j\neq b}^{C}\Phi\left(\omega_j^{-1}\left(\min\{z_C,\omega_b\}\right)\right)\right]d\Phi(\epsilon_b)\\
=&\mathcal{I}[\omega_0\le z_C]\int_{\omega_b^{-1}(\omega_0)}^{\omega_b^{-1}(z_C)}\mathcal{R}_b(\xi_b,\epsilon_b)\left[\Phi\left(\omega_C^{-1}\left(\omega_b\right)\right)\prod_{j=1,j\neq b}^{C}\Phi\left(\omega_j^{-1}\left(\min\{z_C,\omega_b\}\right)\right)\right]d\Phi(\epsilon_b)\\
=&\mathcal{I}[\omega_0\le z_C]\mathcal{H}^b(\xi_b,\xi_C)\text{ if }0<b<C
\end{aligned}
\tag{H.7}
$$

After combining the equation, we can rewrite the original Equation (H.2) as:

$$
\int\cdots\int W(\xi,\epsilon,\psi)dF(\xi,\epsilon,\psi)=\int\cdots\int\left[\prod_{j=1}^{C-1}\mathcal{I}\left[z_j\ge z_{j+1}\right]\right]\mathcal{I}[\omega_0\le z_C]\mathcal{H}^b(\xi_b,\xi_C)\left[\prod_{j=1}^{C}d\Phi(\xi_j)\right]\tag{H.8}
$$

Equation (H.8) has C binary indicators. This still could result in inefficient estimation, as for the list of generated random variables $\{\xi_j\}_{j=1}^{C}$ substantially proportion realizations of $W(\xi,\epsilon,\psi)$ would still be equal to 0. To circumvent this problem, we notice that $\left[\prod_{j=1}^{C-1}\mathcal{I}\left[z_j\ge z_{j+1}\right]\right]\mathcal{I}[\omega_0\le z_C]$ has a chain like structure, thus, we can sample random variables in a more efficient way:

1. Sample random shock ξ_C such that $\omega_0\le z_C(\xi_C)$ and denote it as ξ_C^g. Store the probability of the event $\mathbb{P}\left(\omega_0\le z_C(\xi_C)\right)=\mathbb{P}\left(\xi_C\le z_C^{-1}(\omega_0)\right)=\Phi\left(z_C^{-1}(\omega_0)\right)$
2. Sample random shock ξ_{C-1} such that $z_{C-1}(\xi_{C-1})\ge z_C(\xi_C^g)$ and denote it as ξ_{C-1}^g. Store the probability of the event $\Phi\left(z_{C-1}^{-1}\left(z_C(\xi_C^g)\right)\right)$
3. Repeat Step 2 until random shock ξ_1^g is generated

This procedure allows sampling random shocks more efficiently from the corresponding truncated distributions. This ensures that $\{\xi_j^g\}_{j=1}^{C}$ are such that $\left[\prod_{j=1}^{C-1}\mathcal{I}\left[z_j\ge z_{j+1}\right]\right]\mathcal{I}[\omega_0\le z_C]=1$. Therefore, Equation (H.8) could be approximated by:

$$
\int\cdots\int W(\xi,\epsilon,\psi)dF(\xi,\epsilon,\psi)=\frac{1}{G}\sum_{g}\mathcal{H}^b(\xi_b^g,\xi_C^g)\Phi\left(z_C^{-1}(\omega_0)\right)\prod_{j=1}^{C-1}\Phi\left(z_j^{-1}\left(z_{j+1}(\xi_{j+1}^g)\right)\right)\tag{H.9}
$$

Appendix I Comparison of Alternative Estimation Methods Using Synthetic Data

We discussed various estimation approaches used in previous research in Section 5. Table I.1 reports the estimation results using Scenario (1) parameters. It demonstrates that previous research methods do not recover the true parameter values. Our purpose is to demonstrate that at least one method is feasible. The detailed investigation of the best suitable method goes beyond the scope of the paper.

Table I.1: Alternative Methods to Estimate the Model Parameters Using Synthetic Data

		True Values (1)	Proposed Approach	AR Simulator	AR Simulator (smoothed)
Value of outside option	β_0^u	-4.3	-4.28	-4.38	-4.73
Customer preference	β_1^u	0.4	0.45	0.20	0.00
Preference heterogeneity	σ_1^u	0.3	0.25	0.00	0.00
Click costs (intercept)	β_0^c	-7.0	-6.89	-7.87	-6.98
Click costs (slope)	β_1^c	0.7	0.69	0.58	0.00
Post-purchase information	β_1^ψ	-0.4	-0.51	0.17	0.00
Return costs	$\log R$	-1.1	-1.19	1.99	0.00