A Dynamic Theory of Lending Standards[*]

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December 2023

Abstract

We analyze a dynamic credit market where banks choose lending standards, modeled as costly effort to screen out bad borrowers. Tighter standards worsen the borrower pool, increasing banks' incentives to employ tight standards in the future. This dynamic complementarity in lending standards can amplify and prolong downturns, decreasing lending and increasing credit spreads. Because lending standards have negative externalities, the market can converge to a steady state with inefficiently tight lending standards. We discuss the role of optimal policy to avoid this outcome as well as the impact of balance sheet costs on lending standards.

JEL codes: D82, G21, G01, G10

Keywords: Lending standards; Credit cycle; Strategic complementarity; Amplification; Persistence; Policy response

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1 Introduction

Through the allocation of external financing, lending standards play a key role in the economy, determining, for example, which entrepreneurs get initial funding, which firms grow, and which consumers buy houses. Lending standards by many metrics are highly countercyclical, tightening in recessions and loosening in booms. For example, lending standards were loose in the lending boom of the mid-2000's, when credit spreads and default rates were low, and relatively tight during the credit crunch and recession that followed, when spreads and default rates were high. Notably, lending standards were slow to relax following the 2008-09 financial crisis. An important aspect of countercyclical lending standards is that banks use less private payoff-relevant information to condition lending on during good times when lending standards are loose than during market slumps when they are tight (e.g. Howes and Weitzner, 2023). This raises important questions: What determines the dynamics of private information acquisition among lenders? How does it matter for the dynamics of credit markets as a whole? Are market outcomes efficient, and if not, can well-timed government interventions improve matters?

We study the dynamics and efficiency of credit provision in a model of a credit market in which lending standards are endogenous, both influencing and responding to the quality of the borrower pool. We model lending standards as the extent to which banks acquire costly private payoff-relevant information about borrowers and condition their lending on this information. Our model consists of competitive banks and borrowers who are identical conditional on public or readily-available information (e.g. conditional on credit score). Each instant, some borrowers have the opportunity to approach banks in search of a loan to fund an investment project. Projects differ by borrower type: projects of high-quality (low-quality) borrowers have positive (negative) net present value. A bank can simply lend to a borrower who approaches it, or it can condition lending on costly private information about the borrower's type, e.g. an interview with the applicant, verification of reported employment or income, an appraisal of the collateral, or an analysis of the business plan. Critically, borrowers whose loan applications are declined by one bank may apply at another in the future.

These assumptions imply that lending standards are *dynamic strategic complements*: tight lending

standards in the present imply that banks will confront a more adversely-selected pool in the future, which raises their incentive to impose tight lending standards in the future. Our model's dynamics are most interesting when the *ex ante* expected net present value of borrowers' projects is positive, such as in credit markets where borrowers are "prime" or have passed a preliminary evaluation.

Under our assumptions, the lending market exhibits multiple steady states in the single state variable, the *pool quality*, defined as the share of high-quality borrowers in the pool of borrowers. In the *pooling steady state,* the pool quality is high enough that each bank chooses to approve loans without incurring the additional information collection costs, what we call a "normal" lending standard. Low-quality borrowers are funded along with high-quality borrowers, which keeps the pool quality high. In this steady state, the volume of lending is high and loan spreads are low. Conversely, in the *screening steady state,* the pool quality is low enough that banks collect costly private information about borrowers and condition lending on that information, what we call a "tight" lending standard. In this steady state, the tight lending standard keeps the pool quality low, the volume of lending is low, and loan spreads are high.

Our first main result is that lending standards can endogenously lead to credit market hysteresis. While at any point in time the equilibrium of our model is unique, transitory changes in market fundamentals can lead to permanent differences in lending volumes, credit spreads, and default rates. A temporary deterioration in market fundamentals, e.g. a worsening of borrowers' projects, can set in motion a self-reinforcing feedback loop between deteriorating pool quality and tighter lending standards. This feedback loop culminates in a *permanent* shift in the credit market equilibrium, from the pooling steady state to the screening steady state.

Whether these dynamics lead to permanent changes or not, the model's dynamics match the correlation between lending and lending standards observed in credit booms and busts (as discussed subsequently), but the specific mechanism of time-varying, information-sensitive lending standards is supported by direct evidence that banks condition lending on more (less) private assessments during market downturns (booms) (see Lisowsky, Minnis and Sutherland, 2017; Bedayo, Jimenez, Peydro and Vegas, 2020; Howes and Weitzner, 2023). Further, because the information acquisition associated with tighter lending standards raises the cost of loan origination, the model dynamics are also in line with the negative correlation between lending and (labor) productivity of financial

institutions (e.g. Chen, Favilukis, Lin and Zhao, 2023).

Our second main result is that lending standards can be inefficiently tight, providing a rationale for government intervention to relax lending standards. This result follows because tight lending standards have *negative externalities*: a bank that tightens its lending standard today increases the share of low-quality borrowers in the pool in the future which makes the credit market less efficient. Despite the fact that tighter lending standards have the first-order social benefit of leading to less funding of low-quality (negative net present value) projects, the pooling steady state dominates the screening steady state (in a utilitarian sense). Further, in response to a transitory decline in the quality of the pool of borrowers, it can be optimal for the government to intervene temporarily and relax lending standards to avoid getting stuck in the steady state with tight lending standards. An example of such a policy is a temporary loan guarantee program funded by a tax on loan payments. Optimal policy requires collective, i.e. government, action because the pool of borrowers is a common resource and an individual bank cannot recover the short-term losses associated with the policy from the later increased efficiency of the competitive credit market. Interestingly, the dynamics of our model suggest that if a certain "window of opportunity" has passed, it may no longer be optimal for the planner to intervene because the costs incurred during the transition path back to the pooling steady state may be too high. That is, once the economy has neared the steady state with tight lending standards, it may be too costly to move to the steady state with loose lending standards, even though the latter steady state Pareto dominates the former.

While policies that target lending standards improve outcomes, they do not achieve the first best. A first-best policy would eliminate the externality associated with screening by making any private information acquired by banks public. While credit bureaus can potentially address this externality, there are several reasons why they likely fail to do so in many markets (Appendix F).

Our third main result is that there may be an intermediate range of pool quality in which banks restrict lending by rationing credit instead of imposing tight information-based lending standards, a situation we refer to as *slow thawing*. The logic behind this credit rationing is different from the typical credit rationing due to adverse selection (Stiglitz and Weiss 1981, Mankiw 1986). During slow thawing, lending rates fall sufficiently quickly that high-quality borrowers are indifferent between getting funded right away and waiting for their next funding opportunity. This indifference reduces

the surplus from bank lending today, leading to credit rationing as some banks stop lending, which in turn reduces the speed of improvement in lending volumes and credit spreads. The speed of convergence to the pooling steady state is thus non-monotonic, and during the initial slow thawing period, the typical effects of many parameters on lending volumes and interest rates are reversed.

Our fourth main result considers a simple and exogenous version of a balance sheet constraint—an additional costs of lending, such as from regulatory or management concerns about leverage. We find that such costs unambiguously incentivize banks to tighten lending standards. Tighter lending standards can then, as just described, lead to declines in pool quality, hysteresis, and suboptimal market outcomes. When balance sheet costs are large, banks have a greater incentive to screen borrowers in order to lend their limited capital to the most profitable borrowers. This implies that downturns accompanied by financial crises are more likely to lead to the emergence and persistence of tight lending standards, and that there are potential benefits from government policies to relax lending standards. This persistence is a notable feature of financial crises (see Cerra and Saxena, 2008; Reinhart and Rogoff, 2009; Jorda, Schularick and Taylor, 2013; Baron, Verner and Xiong, 2021) and this model prediction contrasts with the predictions of leading macro-finance models (e.g. Gertler and Karadi, 2011) in which periods of high balance sheet costs during financial crises imply high returns on assets and speedy recoveries.

Finally, we extend our model to endogenize the quality of the inflow of borrowers by assuming that expected future looser (tighter) lending standards reduce (raise) the quality of new borrowers entering the pool. In this case, lending standards can be too loose or too tight. Specifically, at high levels of pool quality optimal policy can call for a minimum lending standard which improves the quality of the borrowers approaching all banks.

Related literature. Our four main contributions are about the dynamics of lending. As such, they build on the static models of Fishman and Parker (2015) and Bolton, Santos and Scheinkman (2016) in which a static strategic complementarity leads to multiple equilibria. These papers cannot speak to issues such as hysteresis, the role of dynamically optimal policy, and slow thawing, which we investigate here.

Other static models focus on different aspects of lending standards. In Ruckes (2004), lenders

simultaneously acquire private information about borrowers and then simultaneously quote loan rates. In that setting, lending standards can be strategic substitutes. In Dell'Ariccia and Marquez (2006), there is cream skimming by informed lenders but these lenders are endowed with their information. Hachem (2021) studies lending standards in a static model in which banks can also exert search effort to attract borrowers. When banks are resource constrained, they put too much effort into searching for borrowers and too little effort into checking them upon arrival, so that lending standards are inefficiently loose.

Our model is more closely related to two recent dynamic models which are based on assumptions such that, at times, lending standards are dynamic strategic substitutes. In Hu (2022) and Farboodi and Kondor (2020), this substitutability arises because tight lending standards raise the average quality of newly-entering borrowers.[1] In Hu (2022) economic recoveries can have interesting dynamics such as double-dip recoveries, while Farboodi and Kondor (2020) shows that credit markets can exhibit endogenous cycles in lending standards and borrower quality. In our baseline model, the quality of new borrowers is exogenous. In section 6.2 we endogenize the quality of new entrants and show that intermediate lending standards maximize pool quality and welfare.

A number of related papers study dynamic adverse selection models without information acquisition. Daley and Green (2012, 2016) and Malherbe (2014) analyze models where current markets can break down when high-quality sellers have the incentive to wait for market prices to improve over time as the composition of sellers improves over time. In contrast, during slow thawing in our model, the equilibrium *composition* of borrowers does not change, only the *speed* of lending is reduced.[2] Camargo and Lester (2014) builds on the following strategic complementarity. If buyers offer asset sellers high prices, sellers with high- and low-quality assets sell. Consequently, the average quality of sellers' assets is maintained and there is more incentive for buyers to offer a high price rather than a low price and potentially have to wait. In Asriyan, Fuchs and Green (2017), if future market liquidity and hence prices are expected to be high (low), then prices today are high (low) and the adverse selection problem will be less (more) severe.

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[1]In Farboodi and Kondor (2020), in addition, rejected borrowers go bankrupt and leave the pool of borrowers.

[2]Related, Zryumov (2015) and Caramp (2023) study models where bad sellers strategically enter when market prices are good. This, in and of itself, does not lead to a market shutdown (lower prices positively select entrants), but as Caramp (2023) emphasizes, the bigger presence of bad sellers can raise the likelihood of adverse selection induced market failures in the future.

Finally, our paper is related to information acquisition and adverse selection in secondary markets. Zou (2019) analyzes a dynamic model of trade in which an agent's incentive to collect information is higher if agents in the future are expected to collect information. Gorton and Pennacchi (1990), DeMarzo and Duffie (1999), and Dang, Gorton and Holmstrom (2020), among others, analyze how debt securities minimize adverse selection problems in secondary markets. While the issues of information acquisition are similar, our model is designed to address adverse selection at origination (in primary markets) and abstracts from issues of security design. In Gorton and Ordonez (2014), lenders learn about the value of borrowers' collateral, but the information decays over time, which can trigger information acquisition and a drop in lending. Finally, Lee and Neuhann (2023) present a model with multiple steady states due to a strategic complementarity between borrowers, rather than banks as in our baseline model (but related to our extension in Section 6.2).

2 A Model of Lending Standards

Time is continuous and runs from 0 to infinity, $t \in [0, \infty)$. There are two sets of agents: a unit-mass pool of potential borrowers and a large mass $\mathcal{J}$ of competitive banks; for brevity we will refer to borrowers rather than potential borrowers. All agents are risk neutral with discount rate $\rho > 0$. To simplify the description of our model as much as possible, we build on the search-and-matching literature (Diamond, 1982; Mortensen and Pissarides, 1994) and its application to financial markets (e.g. Duffie, Garleanu and Pedersen, 2005; Golosov, Lorenzoni and Tsyvinski, 2014).

Borrowers. At Poisson rate $\kappa > 0$, each borrower receives a project requiring an up-front investment of 1. Borrowers have no capital and must fund the project externally. If a borrower raises the funds and invests at time t, the project returns a pledgeable cash flow at $t + T$ and a non-pledgeable private benefit $u > 0$ (in present value) to the borrower.

There are two types of borrowers, H and L. Borrowers privately observe their own type and privately observe how long they have been in the borrower pool. Type-H borrowers' projects produce pledgeable cash flow D_H at $t + T$ with positive net present value (NPV): $r_H \equiv e^{-\rho T} D_H - 1 >$

0.[3] Type-L borrowers' projects produce pledgeable cash flow D_L at $t + T$ with negative NPV: $r_L \equiv e^{-\rho T} D_L - 1 < 0$. We assume $r_L + u < 0$; investing in a type-L project is not profitable even including the private benefit. Let $r^\Delta \equiv \frac{r_H - r_L}{-r_L} > 0$ equal the (normalized) *return difference* between the two types' projects.[4]

When a borrower receives a project, the borrower chooses whether to apply to a bank for funding or wait for improved borrowing opportunities. We denote the probability of a type-L borrower applying for funding by φ_t^L, and that of a type-H borrower applying for funding by φ_t^H. A borrower who chooses not to apply for funding loses their project and remains in the borrower pool where, as before, at rate κ a new project is received. Similarly, a borrower who applies for funding but is unsuccessful is also assumed to lose their project and remains in the borrower pool.[5] Finally, a borrower who applies and receives funding exits the pool to run their project.

At Poisson rate $\delta > 0$, a borrower no longer receives projects and exits the pool with a zero payoff. All borrowers who exit the pool are immediately replaced by borrowers who are type-H borrowers with exogenous probability λ and type-L borrowers with probability $1 - \lambda$.[6]

The exit/entry assumption implies that the borrower pool size is constant at 1. This is convenient as it allows the model to only have one state variable: the fraction of type-H borrowers in the pool, or for short *pool quality*, at time t, $x_t \in [0, 1]$. In Section 6.1 we show that our main insights carry over to an environment with a constant inflow rather than a constant pool size.[7] With a pool quality of x_t, the flow of type-H loan applicants is

$$\kappa_{Ht} \equiv \kappa \varphi_t^H x_t, \tag{1}$$

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[3]Note that r_H is the excess return on a type-H project because $\frac{1}{T} \ln(1 + r_H) = \frac{1}{T} \ln(D_H) - \rho$.

[4]While collateral is not explicitly modeled, one can interpret the loan as a collateralized loan and then u as the private benefit net of the loss of collateral (e.g. u could represent the net benefit of purchasing and living in a house until foreclosure; see Appendix E.3) and types as the quality of the collateral rather than the project.

[5]Assuming that previously denied borrowers are able to keep their projects and apply for funding more quickly than at rate κ would only strengthen the results in our paper.

[6]Our analysis is unchanged if we assume that borrowers whose funding applications were previously denied exit at a greater rate than other borrowers. This is because our model is formally equivalent to one in which only denied borrowers exit at rate δ, while others do not exit exogenously.

[7]The only exception are the results in Section 4.2 on slow thawing, which are less tractable in a constant-inflow setting due to there being two state variables, the number of type-H and number of type-L borrowers in the pool.

the flow of type-L loan applicants is

$$\kappa_{Lt} \equiv \kappa \varphi_t^L (1 - x_t), \tag{2}$$

and the total flow of applicants is $\mathcal{K}_t \equiv \kappa_{Ht} + \kappa_{Lt}$.

All agents have common knowledge of the structural parameters of the market and the initial fraction of type-H borrowers in the pool, $x_0 \in [0,1]$ and so can infer past, current, and future x_t.

Loans. Banks consider lending 1 in exchange for a promised loan payment D_t at time $t + T$. With loan face value D_t, repayment is $\min\{D_t, D\}$, where D is the investment payoff, D_L or D_H, depending on borrower type. Since type-L borrowers have negative-NPV investments, it must be that $D_t > D_L$ for a bank to break even in expectation. Thus, type-L borrowers always default. Nevertheless, given the private benefit u, type-L borrowers have the incentive to finance their project even though they will receive no monetary benefit. The face value D_t is without loss of generality bounded above by D_H as any higher D_t generates no additional repayment. Thus, type-H borrowers never default. We define and from here on work with $r_t \equiv e^{-\rho T} D_t - 1$ which is the *credit spread* charged by banks since $\rho + \frac{1}{T} \ln(1 + r_t)$ is per-period (log) return on a loan that does not default. Note that r_t lies in (r_L, r_H).

Banks. At the beginning of each period t, each bank decides whether to be *active* or *inactive*. We denote by $\mathcal{J}_t$ the mass of active banks. Any active bank then enters the lending market and meets a borrower applying for a loan with probability $\min\{\mathcal{K}_t / \mathcal{J}_t, 1\}$. This expression captures that if $\mathcal{J}_t \geq \mathcal{K}_t$, there are more active banks than borrowers, the probability of meeting a borrower is $\mathcal{K}_t / \mathcal{J}_t$. If instead $\mathcal{J}_t < \mathcal{K}_t$, there are fewer active banks than borrowers, giving banks a probability 1 of meeting a borrower. Vice versa, the probability that a borrower applying for a loan meets a bank is given by

$$\theta_t \equiv \min\{1, \mathcal{J}_t / \mathcal{K}_t\}. \tag{3}$$

At any instant t, any bank can only meet a single borrower, and any borrower can only meet a single bank. Once in such a meeting, bank and borrower bargain over the credit spread r_t. After the

credit spread is set, the bank has the option to impose a (non-contractible) *lending standard* $z_t \in [0, \bar{z}]$, where $\bar{z} \in (0, 1]$ is a parameter. With lending standard z_t, a type-L borrower who applies for a loan is identified as type L with probability z_t, in which case their loan is denied. Otherwise, the borrower's loan is approved. For simplicity a type H borrower is never misidentified as type L (see Appendix E.1 for an extension with both types of identification errors). A bank's cost of using lending standard z_t is $\tilde{c} z_t$, where $c \equiv \frac{\tilde{c}}{-r_L} > 0$ is the (normalized) marginal cost.[8] To ensure that the lending standard has bite, we assume banks cannot observe a borrower's past history of loan applications (see Appendix F for a discussion of credit bureaus).

Optimal lending standard. Conditional on credit spread r_t, the lending standard z_t is chosen so as to maximize bank profits,

$$\Pi_t(r_t) \equiv \max_{z \in [0, \bar{z}]} \frac{\kappa_{Ht}}{\mathcal{K}_t} r_t + \frac{\kappa_{Lt}}{\mathcal{K}_t} (1 - z) r_L - \tilde{c} z. \tag{4}$$

where $\kappa_{Ht}/\mathcal{K}_t$ and $\kappa_{Lt}/\mathcal{K}_t$ are the respective probabilities of facing a type-H or type-L borrower. With a linear screening cost $\tilde{c} z$, the choice of lending standard will be at a corner, $z_t = 0$ or $z_t = \bar{z}$. We discuss convex screening costs in Appendix E.2. For reasons that will become clear, we refer to $z_t = 0$ as a "normal" lending standard and $z_t = \bar{z}$ as a "tight" lending standard.

Borrower's problem. Given the path of credit spreads $\{r_t\}$, borrowers with projects choose the probability with which they will apply for a loan at each time t. Let J_t^H and J_t^L denote the value functions of a type-H and type-L borrower. The optimal strategies for the two satisfy the following Hamilton-Jacobi-Bellman equations:

$$\rho J_t^H = \max_{\varphi_t^H \in [0,1]} \kappa \varphi_t^H \theta_t \left\{ r_H - r_t + u - J_t^H \right\} + \dot{J}_t^H - \delta J_t^H \tag{5a}$$

$$\rho J_t^L = \max_{\varphi_t^L \in [0,1]} \kappa \varphi_t^L \theta_t (1 - z_t) \left\{ (u - J_t^L) \right\} + \dot{J}_t^L - \delta J_t^L, \tag{5b}$$

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[8] We omit bank-specific subscripts from the credit spread r_t and the lending standard z_t since all banks will behave symmetrically in equilibrium.

with transversality conditions $\lim_{t\to\infty} e^{-(\rho+\delta)t} J_t^H = \lim_{t\to\infty} e^{-(\rho+\delta)t} J_t^L = 0$ satisfied. (5a) captures that a type-H borrower receives a project at rate κ, decides to apply with probability φ_t^H and meets a bank with probability θ_t. Conditional on meeting a bank, its payoff is then the project return net of the credit spread plus the private benefit, $r_H - r_t + u$. The HJB for type-L borrowers is similar except that type-L borrowers' projects are only funded with probability $1 - z_t$ conditional on meeting a bank, and their payoff is entirely equal to the private benefit.

Credit spread r_t. Upon meeting a bank, a borrower applying for a loan makes a take-it-or-leave-it offer to the bank, specifying the credit spread r_t. Thus, the credit spread r_t is set so as to impose zero profits for banks,

$$\Pi_t(r_t) = 0. \tag{6}$$

In that sense, the assumption of giving the borrower the power to make a take-it-or-leave-it offer is similar to assuming perfect competition among banks. We denote the surplus between a type-H borrower and the bank by

$$S_t \equiv \frac{\kappa_{Ht}}{\mathcal{K}_t} \left\{ r_H - r_t + u - J_t^H \right\} + \Pi_t(r_t) = \frac{\kappa_{Ht}}{\mathcal{K}_t} \left\{ r_H + u - J_t^H \right\} + \max_{z \in [0,\bar{z}]} \left\{ \frac{\kappa_{Lt}}{\mathcal{K}_t} (1 - z_t) r_L - \tilde{c} z \right\}$$

Given the assumption of a take-it-or-leave-it-offer from borrowers, surplus S_t entirely accrues to the borrower. We assume that banks strictly prefer to be active whenever there is positive surplus in meetings with type-H borrowers, $S_t > 0$; and are indifferent between being active or not if $S_t = 0$. As we show in Appendix E.4, this is a natural assumption that follows directly from bank optimality whenever banks have any small amount of bargaining power.

Evolution of the pool of borrowers. The evolution of the fraction of type-H borrowers in the pool is given by

$$\dot{x}_t = \theta_t \kappa_{Lt} (1 - z_t) \lambda - \theta_t \kappa_{Ht} (1 - \lambda) + \delta(\lambda - x_t). \tag{7}$$

The first term accounts for $\theta_t \kappa_{Lt}(1 - z_t)$ type-L borrowers who are funded and exit the pool (improving pool quality). The second term accounts for $\theta_t \kappa_{Ht}$ type-H borrowers who are funded and exit the pool (reducing pool quality). The third term accounts for exogenous borrower exit and

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replacement by a new borrower with probability λ of being type-H.

Equilibrium. We define an equilibrium as follows:

Definition 1. Given an initial share of type-H borrowers $x_0 \in [0, 1]$ in the pool, an *equilibrium* consists of a path of the fraction of type-H borrowers $\{x_t\}$, credit spreads $\{r_t\}$, the fraction of active banks $\{\theta_t\}$, borrowers' application decisions $\{\varphi_t^H, \varphi_t^L\}$, implied application flows of type-H and type-L borrowers $\{\kappa_{Ht}, \kappa_{Lt}\}$, and screening choices $\{z_t\}$ such that

- $\{\varphi_t^H, \varphi_t^L\}$ solves each borrower type's maximization problem (5) given $\{r_t, z_t, \theta_t\}$,

- $\{\kappa_{Ht}, \kappa_{Lt}\}$ are determined by (1) and (2),

- at each t, z_t solves the bank's maximization problem (4) given $r_t, \kappa_{Ht}, \kappa_{Lt}$,

- all banks are active, $\theta_t = 1$, whenever the surplus is positive, $S_t > 0$, else banks are indifferent, $\theta_t \in [0, 1]$,

- $\{r_t\}$ is determined by the zero profit condition (6),

- $\{x_t\}$ follows the law of motion (7).

A *steady state (equilibrium)* is an equilibrium with $\{x_t, r_t, \theta_t, \varphi_t^H, \varphi_t^L, z_t\}$ constant over time.

To study variation in lending standards, we further make the following parameter assumptions:

Assumption 1. *The cost of bank screening c is not too low or too high:*

$$1 - \lambda < c < 1 - x^s + \bar{z}^{-1} \min \left\{ x^s r^\Delta - 1, 0 \right\},$$

where $x^s = \lambda - \lambda (1 - \lambda) \frac{\bar{z}}{1 - \lambda \bar{z} + \delta \kappa^{-1}}$.

The first inequality in Assumption 1 ensures that the screening cost c is high enough that the lending standard $z = \bar{z}$ does not strictly dominate $z = 0$. The second inequality ensures that c is not so high as to rule out a steady state with $z = \bar{z}$.

3 Steady-state equilibria

We begin by characterizing the steady-state equilibria of our model. Since prices and quantities are constant, we drop the time subscripts for this section. Provided accepting a loan has constant and positive value for a borrower, no borrower has an incentive to wait to borrow in a steady-state equilibrium and therefore $\varphi^H = \varphi^L = 1$. For type-$L$ borrowers, the payoff to borrowing is a constant $u > 0$. For type-H borrowers, (5a), $\dot{J}_t^H = 0$, and the fact that the loan rate is always weakly below the highest pledgeable payoff, $r \leq r_H$, imply that type-H borrowers prefer borrowing over waiting:

$$r_H - r + u - J^H > 0. \tag{8}$$

Under these conditions, all banks are active in a steady state, $\theta = 1$.

The steady-state quality of the pool x and the steady-state lending standard z are jointly determined by the interaction of two forces. On the one hand, the law of motion of x, (7), implies that when $\dot{x} = 0$,

$$x = \lambda - \lambda \frac{(1-\lambda)z}{(1-\lambda z) + \delta \kappa^{-1}}. \tag{9}$$

This equation highlights that tighter lending standards—higher z—are associated with a lower steady-state quality of the pool of borrowers x, as more type-L borrowers are rejected by banks. This effect is greater when the effects of lending standards on the pool are more persistent (low exit rate δ) or when opportunities to invest arise more frequently (high κ) and so borrowers are evaluated more frequently.

On the other hand, banks solve (4) and choose tighter lending standards precisely when the pool is more adversely selected,

$$z = \begin{cases} 0 & \text{if } x > \overline{x} \\ [0, \overline{z}] & \text{if } x = \overline{x}, \quad \text{where } \overline{x} \equiv 1 - c. \\ \overline{z} & \text{if } x < \overline{x} \end{cases} \tag{10}$$

The combination of equations (9) and (10) is illustrated in Figure 1. Both represent downward-

Figure 1: The two forces shaping steady-state equilibria.
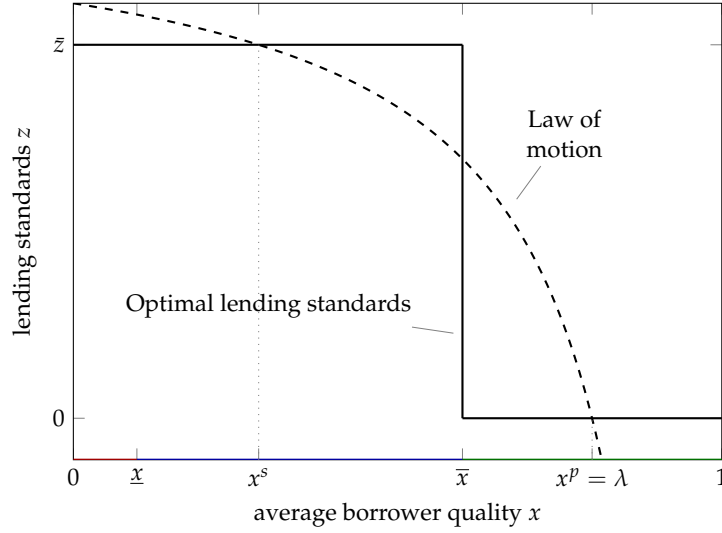
Note: This figure shows two curves whose intersections yield the steady-state pool quality x and the steady-state lending standard z. The solid line represents the optimal choice of the lending standard, (10). The dashed line represents the pool quality x that is caused by any given lending standard z through the law of motion.

sloping relations between x and z, and given Assumption 1 admit three intersections, each of which represents a steady-state equilibrium.[9] This logic is summarized in the following proposition.

Proposition 1 (Steady-state equilibria). *If $\lambda r_H + (1 - \lambda) r_L \geq 0$, then there exist three steady-state equilibria, all with $\theta = 1$:*

(i) *A* pooling *steady state with normal lending standards $z = 0$ and $x = x^p \equiv \lambda$.*

(ii) *A* screening *steady state with tight lending standards $z = \bar{z}$ and $x = x^s \equiv \lambda - \lambda \frac{(1-\lambda)\bar{z}}{(1-\lambda\bar{z})+\delta\kappa^{-1}}$.*

(iii) *A* mixed *steady state with $z = \frac{\lambda - \bar{x}}{\lambda - \lambda\bar{x}}\left(1 + \delta\kappa^{-1}\right) \in (0, \bar{z})$ and $x = \bar{x}$.*

If $\lambda r_H + (1 - \lambda) r_L < 0$, then there exists only one steady-state equilibrium: A screening *steady state with tight lending standards, as in (ii).*

The root of the multiplicity in the first part of the proposition is a *dynamic strategic complementarity* among banks. By (10), banks respond to a lower-quality pool by tightening their lending standards; however, according to (9), tighter lending standards worsen the pool itself, creating an even bigger incentive for banks to tighten their standards in the future. This reasoning rationalizes the existence

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[9]As one might expect, if screening is very expensive or free, only a single steady state will exist. Multiple steady states can also exist for nonlinear screening costs, see Appendix E.2.

of the pooling and screening equilibria in Figure 1. A mixed steady state formally exists but will turn out to be unstable and therefore play no role in the remainder of the analysis.

In the final part of the proposition, the unconditional expected NPV of a project is negative and so no bank will lend without a tight lending standard. The dynamics here are uninteresting: from any initial condition, the credit market converges to the screening steady state. Therefore, going forward, our analysis considers the case of $\lambda r_H + (1 - \lambda)r_L \geq 0$, which is the basis of our statement in the introduction that we focus on prime lending markets.

The pooling and screening steady states have different volumes of lending, credit spreads, and default rates. First, because screening reduces the flow of borrowers that receive funding, fewer projects are funded in the screening steady state—a flow of $\kappa x + \kappa(1 - x)(1 - \bar{z})$—than in the pooling steady state—a flow of κ projects. Second, credit spreads are higher in the screening steady state because the lower pool quality and imposition of lending standards implies that banks' have greater costs of originating loans. Banks charge higher credit spreads to cover these costs. This is true even though banks choose tight lending standards.[10] Finally, in the screening steady state, the quality of funded borrowers is better, leading to a lower default rate, $\frac{(1-x^s)(1-\bar{z})}{x^s+(1-x^s)(1-\bar{z})}$, in the screening steady state than in the pooling steady state, where the default rate is $1 - x^p$. Key to this result is that borrowers exogenously exit at rate $\delta > 0$. Thus, a bank that screens and rejects type-*L* borrowers reduces the probability that they are ever funded, increasing the average quality of funded borrowers. If there was no exogenous exit, $\delta = 0$, the default rate would equal $1 - \lambda$ in any steady state, irrespective of the screening decision, as rejected type-*L* borrowers can remain in the pool until funded.[11]

4   The dynamics of lending standards

The steady state characterization has already touched upon the dynamic strategic interactions that lie at the heart of our paper. We now analyze the full dynamic predictions of our model, outside steady states, beginning with predictions for lending standards z_t and pool quality x_t.

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[10]Proposition 3 has a complete characterization of interest rate spreads.
[11]The results in this paragraph are proved in the Appendix, and are robust to alternative assumptions on the dynamics of the borrower pool, e.g. assuming a constant inflow rate rather than a constant pool size. See Section 6.1.

4.1 Lending standards, pool quality, and credit spreads

Figure 1 shows that at pool qualities greater than $\bar{x}$ (and less than λ), banks choose normal lending standards (solid line) and that tighter lending standards would be required to maintain pool quality constant (dashed line). In this region, bank lending removes a higher share of type-L projects than the share of type-L projects among borrowers entering the pool of borrowers, so pool quality improves over time, $\dot{x} > 0$. A similar argument implies that pool quality is declining at lower pool qualities above x^s and below $\bar{x}$.

For pool qualities below x^s, banks impose tight lending standards. At very low levels of pool quality, even the maximum loan rate does not make profits for a bank. Thus, $\Pi(r_H) = 0$ defines an $\underline{x}$ such that

$$\theta(x) = \begin{cases} 0 & \text{if } x < \underline{x} \\ > 0 & \text{if } x = \underline{x} \end{cases}, \quad \text{where } \underline{x} \equiv \frac{1 - \bar{z} + c\bar{z}}{r^\Delta - \bar{z}}. \tag{11}$$

By Assumption 1, $\underline{x}$ is well-defined and strictly smaller than x^s. Below $\underline{x}$, banks are inactive, $\theta = 0$, and pool quality only improves over time due to the exogenous exit and entry of borrowers. Above $\underline{x}$, but below x^s, pool quality is still sufficiently poor that it continues to improve over time despite banks being active and imposing tight lending standards (see Figure 1).

The following proposition formally states the limiting steady state for each initial pool quality, and along with Propositions 3 and 4 below, completely characterizes equilibrium dynamics.

Proposition 2 (Dynamics of lending standards). *Suppose $x_0 \in [0,1]$ is the initial fraction of type-H borrowers in the pool. There is a unique equilibrium for $\{x_t\}_{t \geq 0}$ in which lending standards are given by (10) and as $t \to \infty$, the credit market converges to*

(i) *the screening steady state, $x_t \to x^s$, if $x_0 < \bar{x}$.*

(ii) *the mixed steady state, $x_t \to \bar{x}$, if $x_0 = \bar{x}$.*

(iii) *the pooling steady state, $x_t \to x^p$, if $x_0 > \bar{x}$.*

Figure 2 illustrates the state space of the credit market and highlights the transitional dynamics (but not the speed of transition) in the three regions of bank behavior: the "no lending" region

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Figure 2: State space and banks' optimal strategies.

No lending   Tight lending standards   Normal lending standards

$0 \quad \underline{x} \qquad x^s \qquad\qquad \overline{x} \qquad x^p = \lambda \qquad 1$

Screening steady state   Pooling steady state

Mixed steady state
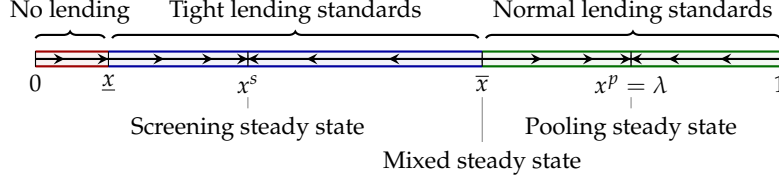
for low pool qualities, where banks are inactive ($\theta_t = 0$) and pool quality improves only due to exogenous exit and replacement; the "tight lending standards" region, where banks screen borrowers $z_t = \overline{z}$ and the market approaches the screening steady state; and the "normal lending standards" region where banks choose $z_t = 0$ and the market approaches the pooling steady state.[12]

A crucial part of the diagram is at $x = \overline{x}$. This point represents a sharp boundary between the regions with tight and normal lending standards, giving rise to an important model prediction—a "bifurcation" property: when x_0 lies above $\overline{x}$, the credit market converges to the pooling steady state with normal lending standards; and when x_0 lies below $\overline{x}$, the self-reinforcing nature of tight lending standards pushes the market to the screening steady state. We further explore the implications of this property in Section 4.3.

How do credit spreads vary over time? Lower x_t implies higher default rates for any given lending standard, suggesting higher credit spreads. But lower x_t can also lead to tight lending standards, which would suggest lower default rates and lower credit spreads, but also raise the cost of lending $\tilde{c}z_t$. As the following proposition shows, the credit spread r_t is in fact uniformly decreasing in pool quality x_t.

Proposition 3 (Equlibrium credit spread)**.** *The equilibrium credit spread $r_t = r(x_t)$ is decreasing in the fraction of type-H borrowers, x, and is given by*

$$
r_t = r(x_t) = \begin{cases} \infty & \text{if } x_t < \underline{x} \\ (-r_L)x_t^{-1}\left\{c\overline{z} + (1-\overline{z})(1-x_t)\right\} & \text{if } \underline{x} \le x_t < \overline{x} \\ (-r_L)x_t^{-1}\left\{1 - x_t\right\} & \text{if } x_t \ge \overline{x} \end{cases} . \tag{12}
$$

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[12]Neither the results up to this point, nor the ones in the next section on efficiency rely on our assumptions on the

Figure 3: Break-even credit spread as function of pool quality x.
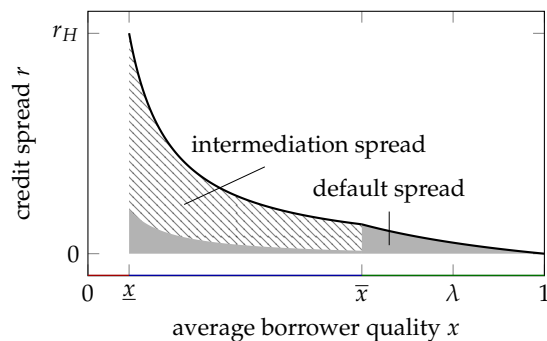
Note: Grey is the component of the credit spread that is due to default risk (the *default spread*). Hatched is the component of the credit spread that is due to intermediation costs (the *intermediation spread*).

We can decompose the expression for $r(x)$ in (12) into a *default spread*, $-r_L x^{-1}(1-z(x))(1-x) > 0$, where $z(x)$ is the optimal screening choice given x, and an *intermediation spread* $-r_L x^{-1} c z(x) \geq 0$. Figure 3 plots the credit spread $r(x)$ and these two components over the state space. The shaded areas in Figure 3 highlight that the default spread changes discretely at $x = \bar{x}$ as banks switch between tight and normal lending standards, but this change is offset by an equally large change in the intermediation spread. The spread rises significantly due to intermediation costs at lower pool qualities x. The decoupling of credit spreads and credit risk in this region of the state space provides a rationale for why, at times, credit spreads may appear to be high given the credit risk.[13]

Together, Propositions 2 and 3 suggest that credit spreads increase over time as an economy converges to the screening steady state from the right, $x \searrow x^s$; and decrease over time as an economy converges to the pooling steady state from the left, $x \nearrow x^p$. This gives rise an interesting possibility: could a type-H borrower have an incentive to wait for lower credit spreads before applying for loans? As we explore next, the answer is yes.

4.2 Slow thawing

A convenient simplification in our steady state analysis was that banks are always active in a steady state, $\theta = 1$. We already saw that this is no longer true in dynamic equilibria, where $x < \underline{x}$ leads to inactive banks, $\theta_t = 0$. We now show that a second region in which some banks remain inactive can

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inflow of new borrowers that keep the pool size constant (see Appendix 6.1).

[13]Factors other than intermediation costs may of course also be at play, such as low liquidity (He and Milbradt, 2014), or high effective risk aversion.

also exist.

To see why, consider a pool quality x just above $\bar{x}$. The pool is set to improve, credit spreads are set to come down over time. If these dynamics are expected to occur sufficiently quickly, a type-H borrower may have an incentive to wait, setting its loan application probability φ_t^H to zero. This would make it unprofitable for banks to be active as they cannot turn a profit by financing the negative NPV projects of type-L borrowers (see Definition 1). Any such reduction in bank activity, pushing θ_t below 1, slows down convergence to the pooling steady state. This is why we refer to these dynamics as *slow thawing*.

Our next proposition shows that in equilibrium, credit markets will thaw at exactly the speed at which type-H borrowers are indifferent between applying and not applying; and at which the marginal bank is indifferent between being active and being inactive.[14]

Proposition 4 (Slow thawing). *Suppose the private benefit is small, $u \to 0$, and $x_0 \in [0,1]$ is the initial fraction of type-H borrowers in the pool. There is a unique equilibrium, in which lending standards are given by Proposition 2, all borrowers apply for loans, $\varphi_t^H = \varphi_t^L = 1$, banks' activity policies satisfy (11), and there exists a threshold $\hat{x} \in (0, x^p)$, such that:*

(i) if $\hat{x} \leq \bar{x}$, then there is no slow thawing region and all banks are active, $\theta(x) = 1$, for $x > \underline{x}$.

(ii) if $\hat{x} > \bar{x}$, then there is a slow thawing region. For $x \in [\bar{x}, \hat{x})$ a positive fraction of banks are inactive

$$\theta(x) = \frac{(\rho + \delta)\,(r_H - r(x))}{-\kappa r'(x)(\lambda - x)} - \delta\kappa^{-1} < 1 \tag{13}$$

where $r(x) = -r_L x^{-1}\,(1 - x) > 0$ and type-H borrowers are indifferent about applying for loans. For $x \geq \hat{x}$, all banks are active, $\theta(x) = 1$, and type-H borrowers strictly prefer to apply for loans.

The cutoff value, $\hat{x}$, is the unique solution to $\theta(\hat{x}) = 1$ in $(0, x^p)$, and the transition speed $\dot{x}$ is given by (7).

The intuition for the expression in (13) comes directly from the indifference condition of type-H borrowers. The HJB of a type-H borrower is given by

$$\rho J_t^H = \max_{\varphi_t^H \in [0,1]} \kappa \theta_t \varphi_t^H \left\{ r_H - r_t - J^H \right\} + j^H - \delta J^H$$

---

[14]To keep the derivations and exposition clear, we focus on the case where the private benefit from running the project, u, is vanishingly small, $u \to 0$, in which case $J_t^L \to 0$.

Figure 4: Slowly thawing credit markets.
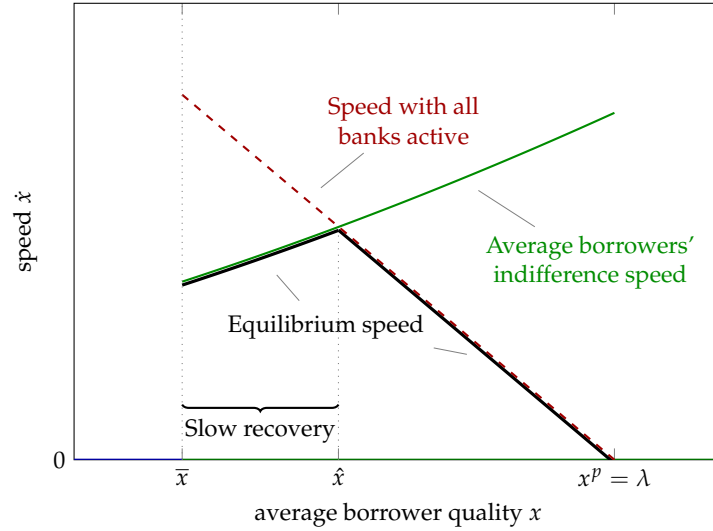
Note. This figure illustrates when there exists a region with "slow thawing" where credit markets recover only very slowly from a crisis. The green solid line represents the speed at which the pool quality needs to improve for type-H borrowers to be exactly indifferent between applying for loans (strictly preferred below the curve) and waiting (strictly preferred above). The red dashed line represents the speed of improvement when all banks are active. The equilibrium speed (black solid line) is the minimum of both curves.

with indifference between applying for a loan or not at pool quality x requiring that $J^H(x) = r_H - r(x)$. Substituting this back into the HJB yields an equation for the speed $\dot{x}$ at which the pool needs to improve for type-H borrowers to remain exactly indifferent,

$$\underbrace{-r'(x)\dot{x}}_{\text{benefit of waiting}} = \underbrace{(\rho + \delta)(r_H - r(x))}_{\text{opportunity cost of waiting}} . \tag{14}$$

When is $\dot{x}$ the equilibrium speed? Precisely when θ_t is such that $\dot{x}$ satisfies the law of motion of x, (7). Together, (14) and (7) give (13).

Figure 4 schematically illustrates the logic of slow thawing. The green solid line represents the speed $\dot{x}$ at which type-H borrowers are indifferent between borrowing now and waiting for the pool to improve. This is an increasing line as the benefit of waiting declines the closer x is to the pooling steady state. The red dashed line represents the speed at which the pool quality improves when all banks choose to be active and all borrowers borrow. Clearly, where this line lies below the green solid line of indifference, it is also equal to the equilibrium speed, shown as the black solid line.

However, for $x < \hat{x}$, the speed $\dot{x}$ with all banks active lies above the solid green indifference curve. In this region, type-H borrowers would prefer to wait if all banks are active, making it unprofitable for banks to be active in the first place. Thus, a fraction $1 - \theta(x)$ of banks choose to be inactive, bringing down the equilibrium speed to match the one along the green solid indifference curve. This leads to a hump-shaped thawing speed: initially little lending due to the threat of type-H borrowers waiting; a period of slow thawing as lending volume and the pool quality accelerate; and finally a period of normal convergence to the steady state.[15]

Slow thawing is thus a mechanism by which lending is slow to recover after crises, based on two intuitive ideas. First, credit spreads rise when the pool of borrowers is temporarily poor, but decline as the pool improves. Second, the more financially sound (type-H) borrowers have an incentive to "wait out" crises until credit spreads come down, while low-quality borrowers choose not to do so. These two ideas amplify each other, as banks increase credit spreads in response to a worse pool of borrowers, further incentivizing type-H borrowers to delay borrowing.[16]

What determines how likely or how strong this period of slow thawing is? How could it be sped up? The following corollary reveals the roles of project payoffs, meeting frequencies, and borrower impatience.

Corollary 1 (Determinants of slow thawing). *Fix a quality of the borrower pool $x \in (\overline{x}, x^p)$ and let $\dot{x}$ denote the speed of improvement in the pool's quality. Then in the interior of the slow thawing region:*
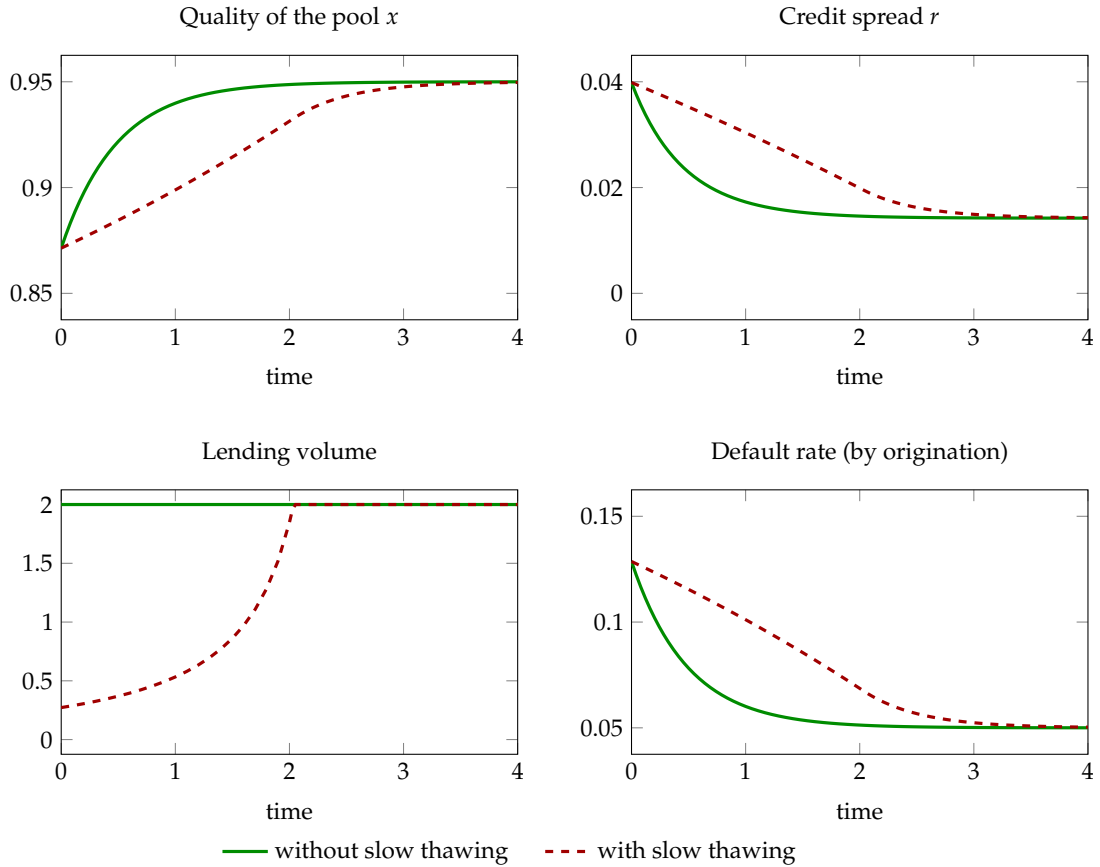
1. *Worse projects always slow down the recovery: $\dot{x}$ falls with lower r_L, r_H.*

2. *Increasing the rate at which borrowers apply for loans does not speed up the recovery: for $x < \hat{x}$, $\dot{x}$ does not rise with κ, while for $x > \hat{x}$, $\dot{x}$ rises with κ.*

3. *More patient borrowers slow down the recovery: $\dot{x}$ falls with lower ρ (holding fixed r_L, r_H).*

When the rate κ at which borrowers receive projects increases (part 2 of Corollary 1), the red line in Figure 4 increases. This naturally increases the speed of the recovery towards the steady state

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[15]Note that Figure 4 does not show $\dot{x}$ just to the left of $\overline{x}$ because it is negative. By Proposition 3, $\dot{x} < 0$ implies $\dot{r} > 0$. With spreads rising over time, there is no incentive to delay and so no region of slow thawing.

[16]It is inessential for slow thawing that type-H borrowers and lenders are all indifferent. In a model in which banks need to pay heterogeneous costs to be active, and in which borrowers have heterogeneous patience, there will, in general, be both a positive mass of borrowers that strictly prefer to wait during slow thawing, as well as a positive mass of lenders that strictly prefer to be inactive.

Quality of the pool x · Credit spread r · Lending volume · Default rate (by origination)

—— without slow thawing   - - - with slow thawing

Note. The plots compare two transitions back to the pooling steady state. Green solid is a transition without "slow thawing", where type-H borrowers always accept current loan offers and banks do not ration credit; red dashed is a transition with slow thawing, where banks ration credit in equilibrium. The parameters used for this simulation are as follows: $\rho = \delta = 0.05$, $\lambda = 0.95$, $r_L = -0.27$, $r_H = 0.13$, $\tilde{c} = 0.035$, $\kappa = 2$, $\bar{z} = 0.8$.

outside the slow-thawing region. Inside that region, however, it has no effect. In fact, even when the recovery is immediate outside the slow-thawing region, $\kappa \to \infty$, the transition inside the region is slow and entirely determined by the indifference condition (14). The reason for this is that banks are not finding it profitable to lend more, as the pool is adversely selected, and are thus holding back lending.

Greater patience, lower ρ, makes type-H borrowers more willing to wait, shifting down the indifference curve in Figure 4 and slowing down the recovery. In practice, the patience parameter reflects how timely the borrower's need for funding is. Thus, paradoxically, when good borrowers are less desperate for funding, the recovery takes longer.

Figure 5 juxtaposes the transitional dynamics with slow thawing (dashed red line) and the transitional dynamics without slow thawing (solid green line). The latter was computed by ruling out slow thawing by assumption, imposing $\varphi_t^H = \theta_t = 1$ and dropping equilibrium equation (8). Instead we assume that borrowers are myopic in the sense that they mechanically always approach the competitive banking sector and accept the loan rather than wait for their next project. As is visible in the figure, slow thawing can greatly slow the transition back to the pooling steady state, and lead to a relatively low lending volume as well as elevated credit spreads and default rates.[17]

4.3 Hysteresis

Propositions 1 and 2 imply that a tightening of lending standards can turn a temporary worsening of credit market fundamentals into permanently lower lending volume, higher intermediation costs, and higher interest rate spreads. To illustrate these dynamics, consider a temporary decline in the size and quality of the pool of borrowers. From time 0 to time T', we reduce the inflow of new type-H borrowers into the pool by a fraction μ. In Appendix C, where we derive the equations for this section, we show that the average quality of new borrowers entering the pool is now

$$
\lambda_t \equiv \begin{cases} (1 - \mu(1 - x_t))\lambda & t \leq T' \\ \lambda & t > T' \end{cases}
\tag{15}
$$

which is notably lower than λ before time T'. As a result, the fraction of type-H borrowers, x_t, evolves according to

$$
\dot{x}_t = \theta_t \kappa (1 - x_t)(1 - z_t)\lambda_t - \theta_t \kappa x_t (1 - \lambda_t) + (\lambda_t - x_t)\frac{\delta}{N_t}
\tag{16}
$$

The last term in (16) is adjusted for by the total pool size N_t, which falls temporarily during this episode, in line with the law of motion

$$
\dot{N}_t = \delta(1 - N_t) - 1_{\{t \leq T'\}}\mu \lambda N_t \left(\frac{\delta}{N_t} + \theta_t \kappa x_t + \theta_t \kappa (1 - x_t)(1 - z_t) \right)
\tag{17}
$$

---

[17]Note that a similar region with slow thawing can also appear in the region between $\underline{x}$ and x^s and slow down the convergence to the screening steady state from the left.

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Figure 6: Hysteresis.

Quality of the pool x · Credit spread r · Lending volume · Default rate (by origination)

— short downturn - - - long downturn
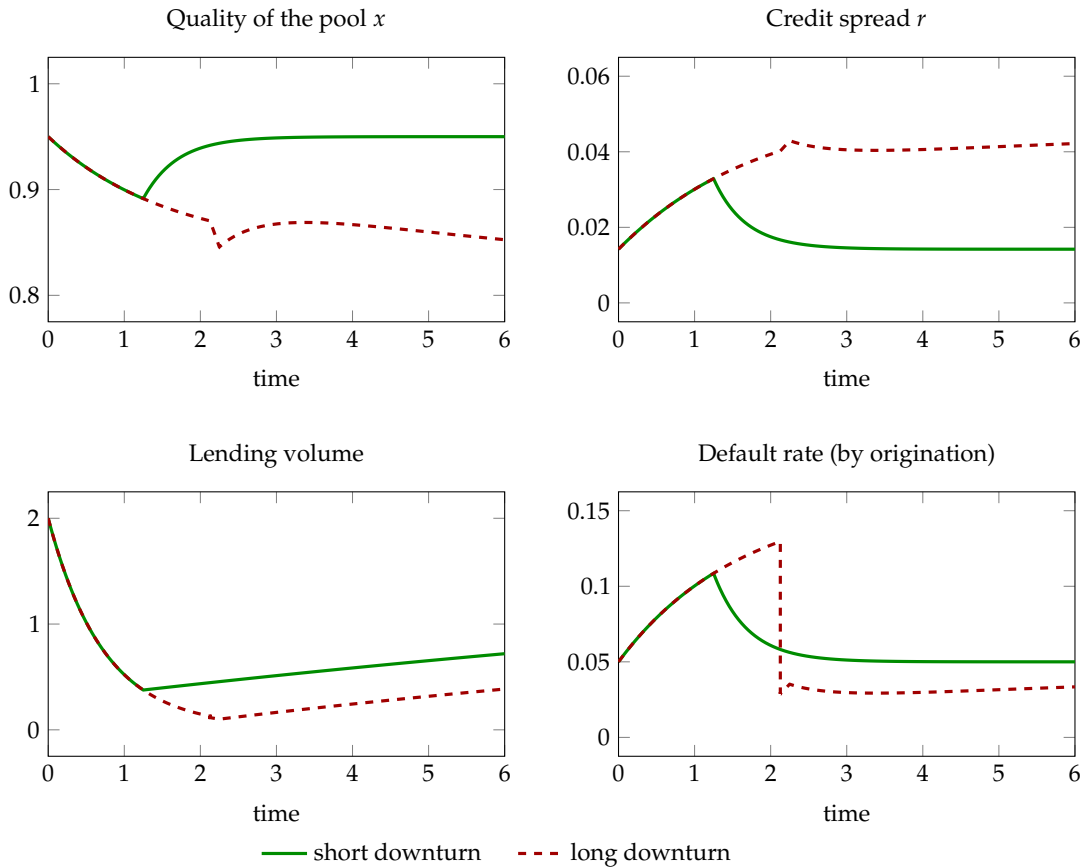
Note. This figure shows a credit market in response to a temporary reduction in the inflow of type-H borrowers: green solid (1.25 periods long reduction), red dashed (2.25 periods long reduction). The parameters used for this simulation are as follows: $\rho = \delta = 0.05$, $\lambda = 0.95$, $r_L = -0.27$, $r_H = 0.13$, $\tilde{c} = 0.035$, $\kappa = 2$, $\bar{z} = 0.8$, $\mu = 0.70$.

Figure 6 shows the credit market response when the decline in the quality of new pool entrants is short-lived and when it is longer, for $T' = 1.25$ and $T' = 2.25$. As the solid green line shows, the short-lived decline in pool quality is associated both with a slight decrease in lending volume, as fewer borrowers enter the pool, and with an increase in interest rate spreads, as pool quality declines and the ex-post default rate increases. When the downturn in fundamentals ends, the increase in the demand for loans and the increase in the quality of borrowers both lead to increased lending and decreased interest rate spreads. Ultimately, the credit market returns to its previous equilibrium with normal lending standards after the short downturn.

By contrast, in the longer downturn (dashed red line) lending standards remain tight even after the inflow of new projects rises back to its initial value. As a consequence, the credit market continues to deteriorate even after the fundamental recovers. The longer period of decline in the

average quality of borrowers leads to an abrupt decline in lending when banks tighten lending standards roughly at $t = 2$. Following this tightening, pool quality deteriorates more rapidly, credit spreads increase, and lending volumes continue to contract. Because lending standards are tight, when the slowdown ends at $T' = 2.25$ and more type-H borrowers start to enter the pool of borrowers again, lending volumes increase, but interest rate spreads remain high and lending never fully recovers. Such hysteresis does not follow from shorter or smaller downturns, consistent with lending standards normalizing following the milder 2001 U.S. recession but not the 2008-2009 Great Recession.

More generally, these dynamics match stylized facts about the pro-cyclicality of credit conditions (e.g those discussed in Greenwood and Hanson, 2013) and evidence on the counter-cyclicality of the use of private information in lending decisions (Lisowsky, Minnis and Sutherland, 2017; Bedayo, Jimenez, Peydro and Vegas, 2020; Howes and Weitzner, 2023): credit contractions feature tighter information-based lending standards, lower lending volumes, higher credit spreads conditional on default probability, and lower default than implied by public information and stable lending standards. Further, a salso noted in the Introduction, the model dynamics also match the negative correlation between lending and (labor) productivity of financial institutions (Chen, Favilukis, Lin and Zhao, 2023) because the collection of information associated with tighter lending standards involves more cost per loan originated (which also then raises interest rate spreads).

Our finding of hysteresis is not limited to declines in borrower quality λ. Indeed, similar dynamics would follow from a downturn in which recovery rates after loans default, r_L, worsen temporarily. In this case, banks would maintain normal lending standards for small declines in r_L, leading to no lasting damage to credit markets. In contrast, deep downturns, in which r_L falls enough to trigger tight lending standards, can cause permanent declines in lending volume and increases in interest rate spreads as banks incur intermediation costs.

Is this permanent shift in lending standards efficient? In the next section, we show that while it is individually optimal for banks to tighten lending standards during a long downturn, it can be socially sub-optimal.[18]

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[18]It turns out that in the long-downturn in Figure 6, optimal policy maintains normal lending standards (Appendix B).

5 Socially optimal lending standards

Because one bank's lending standard affects the future pool of borrowers for all banks, equilibrium lending standards will, in general, not be efficient. The first-best allocation would allow the planner to fund only type-H borrowers. However, such a policy requires the planner to observe individual borrowers' types. This section instead characterizes the constrained efficient allocation.

Our concept of constrained efficiency only allows the planner to control banks' activity and screening decisions, subject to borrowers' application decisions, without having access to superior information relative to banks. The planner's objective is to maximize the sum of all agents' utilities.[19] Throughout this section, we continue to focus on the algebraically simpler case where $u \to 0$. Further, we assume that the planner can set the path of market interest rates $\{r_t\}$, and therefore prevent type-H borrowers from waiting, avoiding slow thawing. We discuss relaxing this assumption below.

The constrained efficient planning problem is then given by

$$\max_{z_t \in [0,\bar{z}], \theta_t \in [0,1]} \int_0^\infty e^{-\rho t} \kappa \theta_t \left\{ x_t r_H + (1 - z_t)(1 - x_t) r_L - \tilde{c} z_t \right\} dt \tag{18}$$

subject to the law of motion of x_t, (7). The solution to this problem can be characterized as follows.

Proposition 5 (Second-best policy)**.** *There exists a threshold $\bar{x}^* \in [0, \bar{x})$ such that the planner sets:*

$$z_t = \begin{cases} \bar{z} & \text{if } x_t < \bar{x}^* \\ 0 & \text{if } x_t > \bar{x}^* \end{cases} \tag{19}$$

For any $x_t \in (\bar{x}^*, \bar{x})$, equilibrium lending standards are (second-best) inefficiently tight.
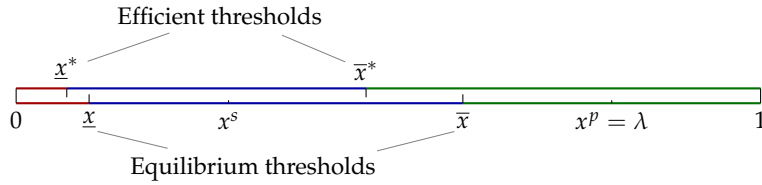
For any $\bar{x}^* > x^s$, the optimal policy for bank activity is given by

$$\theta_t = \begin{cases} 0 & \text{if } x_t < \underline{x}^* \\ 1 & \text{if } x_t > \underline{x}^* \end{cases}$$

for some $\underline{x}^* \in [0, \bar{x}^*)$.

---

[19]Since borrowers and banks are risk-neutral, this is without loss when transfers between agents are feasible.

Figure 7: Constrained efficient vs. equilibrium lending standards.

Proposition 5 shows that the constrained efficient lending standard takes a similar form to the privately-optimal policy: when the pool quality is relatively high, $x > \overline{x}^*$, normal lending standards, $z = 0$, are optimal; and when $x < \overline{x}^*$, tight lending standards, $z = \overline{z}$, are optimal. But the cutoffs for the optimal policy and for the market equilibrium differ: there exists a region in the state space, $(\overline{x}^*, \overline{x})$, where equilibrium lending standards are too tight relative to the constrained-efficient outcome.[20] The private and social thresholds are shown in Figure 7.

We discuss policies that implement the constrained optimum in Appendix B. For example, the constrained optimum can be implemented in practice using a government-funded loan insurance program.

The result in Proposition 5 does not imply that tight lending standards should always be prevented. In fact, tight lending standards are constrained optimal whenever the pool quality is sufficiently poor, as normal standards would involve lending to all type-*L* borrowers that have accumulated in the pool, and thus be very costly. This result holds despite the fact that all projects are ex ante positive expected NPV, and is an important implication of our dynamic perspective that would be lost in a static context. Despite this, the welfare in the screening steady state is always lower than welfare in the pooling steady state.

Corollary 2. *When both steady states exist (a result of Assumption 1), the screening steady state has strictly lower welfare than the pooling steady state.*

If $\delta = 0$, this result would be a simple consequence of the fact that screening borrowers is costly and the quality of funded borrowers is independent of the steady state (see the discussion following Proposition 1). But with $\delta > 0$, screening borrowers has a social benefit because a share of them are

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[20]The logic that lending standards are only inefficiently tight in the region $(\overline{x}^*, \overline{x})$ follows from the linearity of the cost function. If banks' screening costs were nonlinear such that the optimal $z^*(x)$ were continuous and strictly decreasing, then lending standards could be generically too tight.

Figure 8: Early interventions dominate late ones.

Quality of the pool x at intervention

Welfare after intervention
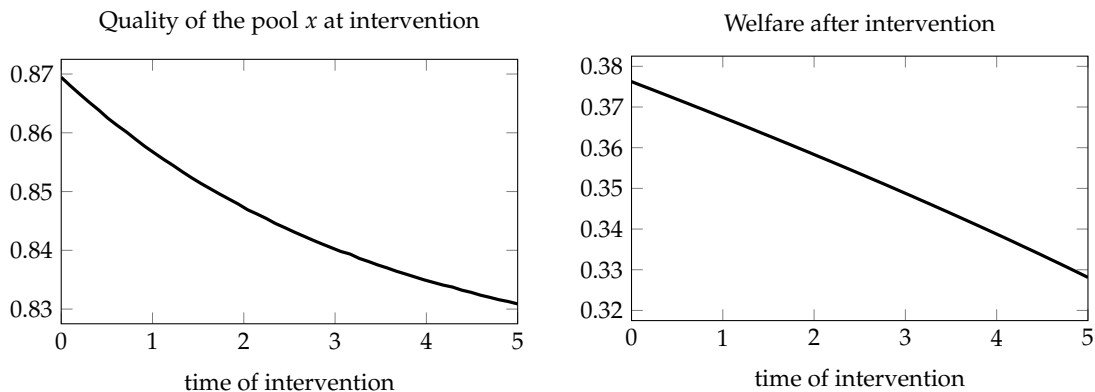
Note. Figure shows how an intervention implementing normal lending standards affects a credit market that is transitioning towards the screening steady state. Horizontal axis: time at which the intervention starts (0 corresponds to the immediate, constrained efficient intervention). The parameters used for this simulation are as follows: $\rho = \delta = 0.05$, $\lambda = 0.95$, $r_L = -0.27$, $r_H = 0.30$, $\tilde{c} = 0.035$, $\kappa = 1$, $\bar{z} = 0.8$.

never funded. Still, the corollary shows that welfare in the pooling steady state is higher.

There are two important dynamic implications that follow from the existence of a non-empty interval $(\bar{x}^*, \bar{x})$ where the market equilibrium diverges from the constrained optimum.

1. Intervention timing matters. Figure 8 illustrates the welfare consequences of intervening in a credit market that starts at a given $x_0 \in (\bar{x}^*, \bar{x})$ for various intervention start times (on the horizontal axis). An intervention is assumed to require banks to set normal, rather than tight, lending standards. As Figure 8 shows, the later the time of intervention is, the lower is the quality of the pool of borrowers when the policy switches from screening to pooling (left panel). Later intervention times thus increase the short-run losses incurred at the start of the intervention and are therefore welfare-inferior to early interventions. In fact, after a sufficiently long time, if x_t has fallen below $\bar{x}^*$, intervening may even be welfare-dominated by not intervening at all. In this situation, it would be socially optimal to allow the credit market to converge to the screening steady state, despite its having lower steady-state welfare than the pooling steady state (Corollary 2).

2. Better screening technology may be detrimental to welfare. Consider a decline in the cost $\tilde{c}$ of operating tight lending standards. Such a cost reduction necessarily raises efficiency in any steady-state equilibrium. In a dynamic equilibrium, however, it can decrease welfare as it raises both the privately optimal ($\bar{x}$) and socially optimal ($\bar{x}^*$) thresholds between tight and normal lending standards. If, for example, a market is just recovering from a crisis, with x_0 just above $\bar{x}$, such a

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technological improvement may cause $\bar{x}$ to rise above x_0, thereby preventing a recovery and leading to a reduction in welfare. If $\bar{x}^*$ also rises above x_0 then it is too costly for policy to mitigate this decline in welfare.

A decrease in cost $\tilde{c}$ represents an improvement in private information technology. What happens if instead public information technology (e.g. credit reporting) improves? A crude way to capture such a change is as an increase in δ, the probability that borrowers exit. While the exit of borrowers who have never been rejected has no effect on equilibrium as they are replaced in the pool by an equal measure of new identical borrowers, a greater exit rate of rejected borrowers does matter for equilibrium. A larger exit rate δ unambiguously improves pool quality at every point in time and is therefore beneficial for welfare. Thus, the welfare effects of improving public information are positive (see Appendix F for a discussion of credit bureaus).

6   Extensions

We now show how our core model can serve as the basis for richer analyses of lending market dynamics by extending our analysis in three ways. First, we relax our assumption that the pool size is constant, and instead assume a constant inflow rate. This allows for a richer description of booms and busts in lending volumes. Aside from this added feature, we show that all our main results are robust to this modification. The second extension builds on the first by introducing an additional channel through which borrower pool quality is endogenous: the quality of new borrowers who enter the pool is lower when future lending standards are expected to be less tight. In this case, optimal policy can involve setting a minimum lending standard. The third extension introduces a positive cost to a bank of making a loan, motivated by the idea that banks' balance sheets may be constrained, especially during periods of heightened financial distress. The extension shows that such a cost of lending can give rise to high and persistent credit spreads, as in the dynamics in Section 4.3.

6.1 Non-constant pool size

In our first extension, we illustrate that our baseline model does not require a constant pool size and, in fact, gives rise to the same steady states and dynamics when instead inflow rates are assumed to be constant at δ. We endogenize inflow rates in the subsequent extension. For simplicity, we focus on the case of active banks, $\theta_t = 1$, throughout this section.

Without a constant pool size, there are two state variables: m_{Ht}, the number of type-H borrowers in the pool and m_{Lt}, the number of type-L borrowers in the pool. The laws of motion of the state variables are given by

$$\dot{m}_{Ht} = \delta\lambda - \delta m_{Ht} - \kappa m_{Ht} \tag{20}$$

$$\dot{m}_{Lt} = \delta(1 - \lambda) - \delta m_{Lt} - \kappa(1 - z_t)m_{Lt}. \tag{21}$$

The first term in both laws of motion reflects the constant inflow of $\delta\lambda$ type-H borrowers and $\delta(1 - \lambda)$ type-L borrowers. The second term captures the constant exit probability of borrowers in the pool. The final term is the flow rate of borrowers who receive a loan.

Since the first equation is independent of z_t, m_{Ht} converges to a steady state that is independent of lending standards,

$$m_H^* = \frac{\delta\lambda}{\delta + \kappa}. \tag{22}$$

We make the natural assumption that m_H starts out at this steady state initially, and thus remains there forever, $m_H = m_H^*$. We continue to denote the share of type-H borrowers in the pool by $x_t \equiv m_H/(m_H + m_{Lt})$. Given (22), the law of motion of x_t can easily be shown to be given by

$$\frac{\dot{x}_t}{x_t/\lambda} = \kappa(1 - x_t)(1 - z_t)\lambda - \kappa x_t(1 - \lambda) + \delta(\lambda - x_t). \tag{23}$$

Observe that the right hand side of (23) is identical to the one of (7) after substituting out κ_{Ht} and κ_{Lt} and setting $\theta_t = 1$. Thus, the only difference between our baseline constant-pool-size model and the constant-inflow one in this section is that the speed here is altered by a factor x_t/λ on the left hand side of (23). In particular, our results on steady states and transitions between them carry

over one-for-one to the model in this section.[21] In Appendix D, we show that steady state welfare considerations are also similar in the two models.

6.2 Endogenous borrower quality and lending standards that are too loose

So far in the model, looser lending standards lead to an unambiguously better pool quality over time. This is the case because, with looser lending standards, low types are more likely to obtain funding and leave the pool, improving its quality. One could imagine, however, that the ease of finding financing might attract more low-quality borrowers into the pool. We introduce this idea via an "entry margin" in our model and show that the relationship between efficiency and lending standards becomes non-monotonic. In particular, it can be constrained optimal for a planner to set a minimum lending standard $\underline{z}$, in which case the pooling steady state no longer has $z = 0$.

Except for one addition, we work with the model introduced in the previous subsection. While we leave the inflow rate $\delta\lambda$ of high type borrowers unchanged, we assume that there is a fringe of low type borrowers with positive and heterogeneous entry costs, measured in units of the utility benefit u.[22] We denote by $\psi_i \cdot u > 0$ the entry cost of low type borrower i, and assume that ψ_i has a cdf $\Psi(x)$. An individual borrower i then enters at date t if

$$J_t^L > \psi_i \cdot u$$

and the overall mass of low-type entrants is $\Psi(J_t^L/u)$. The term $\Psi(J_t^L/u)$ replaces the term $\delta(1-\lambda)$ in (21), changing the law of motion for x_t is from (23) to

$$\frac{\dot{x}_t}{x_t/\lambda} = -\Psi(J_t^L/u)\frac{\delta+\kappa}{\delta}x_t + \lambda(1-x_t)(\delta+\kappa(1-z_t)). \tag{24}$$

The law of motion (24) captures that when the value of being a low type, J_t^L, is higher, there is more entry of low type borrowers, consequently reducing pool quality. We next analyze how this changes the steady states of the model, relative to the ones shown in Figure 1, as well as the transitional

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[21]What becomes harder to analyze with a non-constant pool size is slow thawing, since $\theta_t < 1$ affects both type-H and type-L borrowers, that is, m_H is no longer constant at m_H^*.

[22]Allowing for endogenous entry of high type borrowers would only amplify the mechanisms in our paper, as high type borrowers would then be less likely to enter when pool quality is low, which further reduces pool quality. In that sense, our assumption of only allowing low type borrowers to enter endogenously can be viewed as conservative.
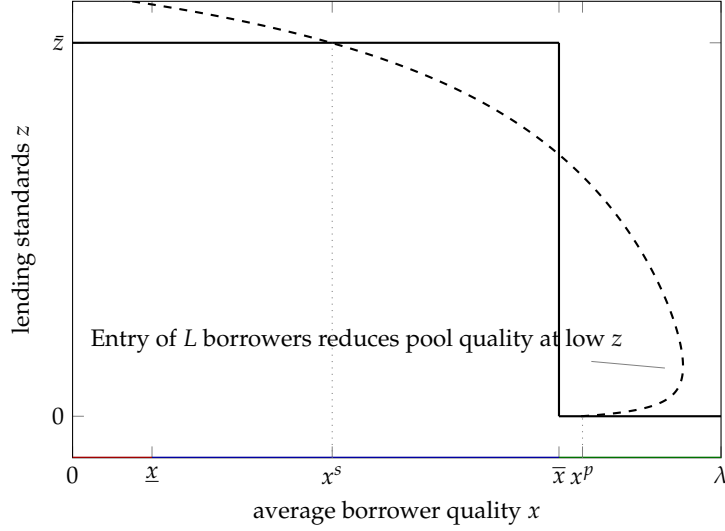
dynamics and welfare considerations.

Steady states. In steady state, the value of being a low type borrower is

$$J^L = \frac{(1-z)\,\kappa}{\rho + \delta + (1-z)\,\kappa} u,$$

which is strictly decreasing in z. Thus, looser lending standards raise the value of J^L and draw in more low type borrowers. Solving for the law of motion locus, for which $\dot{x}_t = 0$, we now find

$$x(z) = \frac{\lambda\delta(1 - \frac{\kappa z}{\delta+\kappa})}{\lambda\delta(1 - \frac{\kappa z}{\delta+\kappa}) + \Psi(J^L/u)} \tag{25}$$

rather than (9). In Figure 9 we illustrate how this affects the steady states in our model. Now the law of motion locus (25) can be backward bending, with the exact shape depending on the shape of $\Psi(x)$.

While the screening steady state is unchanged, a positive lending standard z can be optimal in the pooling steady state. In this case, when lending standards are loose, that is, z is low, rapid entry of low types pollutes the pool and reduces steady state pool quality x. This shifts the pooling steady state to the left. Starting at the pooling steady state, a backward bending locus implies that a policy that requires banks to modestly *tighten* lending standards can *raise* pool quality. Requiring banks to

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tighten lending standards by too much, however, reduces pool quality again. As we show below, this is crucial when thinking about welfare.

Transitions. We demonstrate in Appendix D that the model with entry can be analyzed using a traditional phase diagram in (x, J^L) space. We obtain that the pooling and screening steady states, whenever they exist, are still stable, while the middle steady state is unstable.

Efficiency. Assuming all banks set lending standards to z, steady state welfare $\mathcal{W}(z)$ is given by[23]

$$\kappa^{-1}\mathcal{W}(z) = \frac{m_H^*}{x(z)}\left(x(z)r_H + (1-z)(1-x(z))r_L - \tilde{c}z\right)$$

where, compared to expression (18) in the constant-pool-size model, an extra factor $\frac{m_H^*}{x(z)}$ appears. Where is $\mathcal{W}(z)$ is maximized? The first order condition $\mathcal{W}'(z) = 0$ is equivalent to

$$1 - x(z) + [(1-z) + cz]\frac{x'(z)}{x(z)} = c$$

When the law of motion locus is downward sloping, just like before, $x'(z) < 0$. Then, the pooling steady state, which exists if $c > 1 - x(0)$, must be the steady state with the highest welfare, as the left hand side is strictly below the right hand side, or, $\mathcal{W}'(0) < 0$. When the law of motion locus is backward bending, however, $x'(z)$ can be positive. In that case, it is possible that $\mathcal{W}'(0) > 0$ and that there is an interior maximum of $\mathcal{W}(z)$.

To illustrate this case, we numerically plot $\mathcal{W}(z)$ for a model without endogenous entry (dashed line) and one with endogenous entry (solid line) in Figure 10. In the baseline model without endogenous entry, it is most efficient to have $z = 0$. In the model with endogenous entry, by contrast, the socially optimal lending standard can be positive.

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[23]For simplicity, we again consider the limit $u \to 0$ here.

Figure 10: Steady state welfare $\mathcal{W}(z)$ with and without entry.
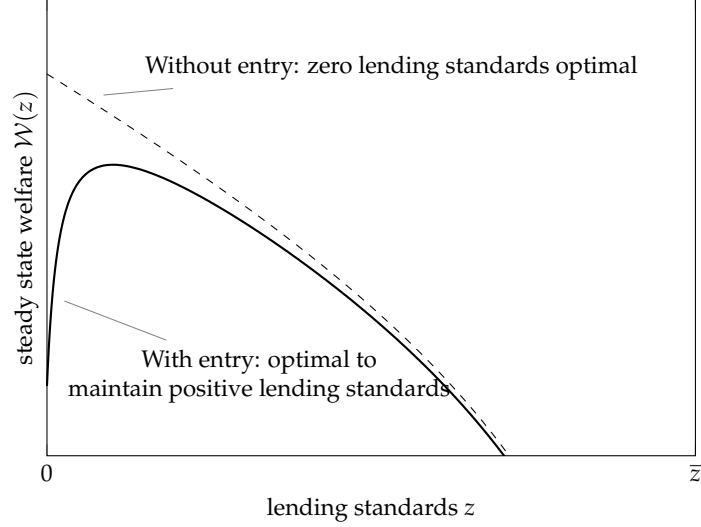
6.3 Balance sheet costs and lending standards

In this section, we explore how regulatory or market-based imposed costs that constrain lending, henceforth "balance sheet costs," interact with equilibrium lending standards.[24] To simplify the math, we work in the limit $u \to 0$ and assume no slow thawing, $\varphi_t^L = \varphi_t^H = \theta_t = 1$ and $\mathcal{K}_t = \kappa$. Without loss, we assume for this section that the total mass of banks $\mathcal{J}$ is equal to κ. Thus, each (infinitesimal) bank meets on average one (infinitesimal) borrower per instant.

We do not derive balance sheet costs from first principles. Rather, for simplicity, we model balance sheet costs as an additional cost $\omega > 0$ imposed per loan a bank makes. Given lending standard z_t and pool quality x_t, each bank makes on average $x_t + (1 - x_t)(1 - z_t)$ loans per instant. If a bank pools, $z_t = 0$, the bank makes exactly one loan. Imposing tighter lending standards, $z_t > 0$, reduces the average number of loans made.

The presence of balance sheet costs alters the optimal choice of lending standards for banks. Indeed, banks now solve the modified problem

$$\Pi_t(r_t) \equiv \max_{z \in [0, \bar{z}]} x_t r_t + (1 - x_t)(1 - z) r_L - \tilde{c}z - \omega\left(x_t + (1 - x_t)(1 - z)\right) \tag{26}$$

where the last term highlights captures the balance sheet cost. As the next proposition shows, this

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[24]The 2020 working-paper version of this paper contains an analysis of the dynamics of lending and credit spreads when banks face a binding constraint on market-wide lending that relaxes exogenously over time.

modifies the optimal screening threshold.

Proposition 6. *Balance sheet costs lead to tighter lending standards. For $\omega > 0$, banks impose tight lending standards for all $x_t < \overline{x}^c$ and set normal lending standards, $z = 0$, for all $x_t > \overline{x}^c$ where the cutoff pool quality $\overline{x}^c$ is strictly greater than the cutoff value with $\omega = 0$:*

$$\overline{x}^c = 1 - \frac{c}{1 + \omega/|r_L|} > \overline{x}$$

For $x_t = \overline{x}^c$, banks are indifferent over lending standards.

Proposition 6 highlights a fundamental effect of balance sheet costs, whether they are direct monetary costs or indirect costs of financial regulation: they incentivize banks to impose tighter lending standards in order to avoid costs and preserve balance sheet capacity. This result does not require assumptions that discourage loans to type-L borrowers or penalize future losses on bad loans; in fact, our cost is simply on the loan volume at origination. Proposition 6 shows that the result still follows as with $\omega > 0$, greater scrutiny when lending reduces balance sheet costs as a side benefit.

The fact that $\overline{x}^c > 1 - c = \overline{x}$ whenever $\omega > 0$ means that there is now a greater range of pool qualities over which dynamics lead towards the screening steady state. Thus, even temporarily increased balance sheet costs can have long-lasting effects on the economy.

6.4   Discussion of modeling assumptions

Appendix E discusses several modeling assumptions. Appendix E.1 considers the possibility that in addition to mistakenly lending to type-L borrowers—the assumption in our baseline model,—banks can also mistakenly reject type-H borrowers. Appendix E.2 considers the case with a strictly convex cost of screening, rather than a linear cost. Appendix E.3 discusses the introduction of collateral. Appendix E.5 discusses miscellaneous other assumptions.

7 Concluding remarks

We develop a dynamic theory of lending standards, based on two intuitive features of credit markets. First, tighter lending standards lead to the rejection of unprofitable loan applications. Second, it is not costless for banks to identify unprofitable applicants, even those previously rejected by other banks. These two features give rise both to a dynamic strategic complementarity in lending standards, which leads to more persistence in lending standards than in fundamentals; and to negative externalities from tight lending standards, implying that lending standards can be too tight for too long after negative shocks.

These two central features provide guidance regarding which markets our theory applies to. The first feature is likely true for *any* lending standard. The second, however, focuses our model on markets in which borrowers likely shop for loans from multiple lenders and in which lenders can, at some cost, gather soft or private information about the default probability or the loss in the event of default. By contrast, markets in which borrowers have limited ability or need to approach multiple banks, in which the outcome of lending depends purely on public information, or in which the borrower can be forced to bear significant losses in the event of default are unlikely to be subject to the conclusions from our theory.

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A Dynamic Theory of Lending Standards — Appendix

A   Proofs and derivations

A.1   Steady state equilibria: Proof of Proposition 1

The three pairs (x, z) mentioned in Proposition 1 are solutions to (9) and (10) if $\lambda > \bar{x}$, $x^s < \bar{x}$, and $\frac{\lambda - \bar{x}}{\lambda - \lambda \bar{x}}\left(1 + \delta \kappa^{-1}\right) < \bar{z}$. The first two of these hold by Assumption 1 and the third is a straight consequence of the second.

   We claim that the three pairs indeed constitute equilibria, with $\theta = 1$, $\varphi^a = \varphi^r = 1$ and with r pinned down by Proposition 3. To prove this, first note that the law of motion (7) as well as the bank's maximization problem (4) are satisfied due to (9) and (10). The zero profit condition (6) pins down the interest rate (see our proof to Proposition 3). Finally, in any steady state a type-H borrower strictly prefers a loan today, that is,

$$r_H - r + u - J^H > 0,$$

and since $r \le r_H$ (which holds since $x^s \ge \underline{x}$ with $\underline{x}$ as in (11) due to Assumption 1) we have that $\theta = 1$ and $\varphi^a = \varphi^r = 1$.

A.2   Proof of claims in text following Proposition 1

The flow of projects being funded in the pooling steady state is κ, compared to $\kappa x^s + \kappa(1 - x^s)(1 - \bar{z})$ in the screening steady state. The credit spread result follows directly from Proposition 3 and the fact that $r(x)$ is strictly decreasing in x. The equilibrium default rate is given by

$$\frac{\kappa(1 - x)(1 - z)}{\kappa(1 - x)(1 - z) + \kappa x} = \left(1 + \frac{x}{(1 - x)(1 - z)}\right)^{-1}$$

which can further be simplified to

$$(1 - \lambda)\left(1 + \frac{\lambda z \delta \kappa^{-1}}{(1 + \delta \kappa^{-1})(1 - z)}\right)^{-1}.$$

Thus, when $\delta = 0$, the equilibrium default rate is always equal to $1 - \lambda$, irrespective of the steady state.

A.3   Proof of Proposition 2

Begin with $x_0 \in (\bar{x}, \lambda]$. In that case, $z = 0$ is is the optimal bank strategy (see (4)). Conjecture that $\theta = 1$ and all borrowers apply for loans. Therefore the law of motion of x, (7), reads

$$\dot{x}_t = \kappa(1 - x_t)\lambda - \kappa x_t(1 - \lambda) + \delta(\lambda - x_t) = (\kappa + \delta)(\lambda - x_t) > 0$$

which is positive for any $x_t < \lambda$, implying convergence to the pooling steady state. This is the speed outside of any slow thawing region. However, if at the conjectured $\theta = 1$, $\dot{x}$ is large enough, then

type$-H$ borrowers would prefer to apply for loans rather than waiting. If this is the case, then θ is lower so that $\dot{x}$ is such that type$-H$ borrowers are indifferent between applying for loans and waiting. Importantly, because discounting is positive, $\dot{x}$ is still positive at this indifference condition, as can be verified using the indifference condition as stated in the proof of the Proposition 4

Next turn to $x_0 \in [\underline{x}, \overline{x})$. In that case, $z = \overline{z}$ is the optimal bank strategy (see (4)), and therefore the law of motion of x, (7), reads

$$\dot{x}_t = \kappa(1 - x_t)(1 - \overline{z})\lambda - \kappa x_t(1 - \lambda) + \delta(\lambda - x_t) = (\delta - \kappa \overline{z}\lambda + \kappa)\left(x^s - x_t\right)$$

implying convergence to the screening steady state.

For $x_0 < \underline{x}$, note that $\theta_t = 0$ and so

$$\dot{x}_t = \delta(\lambda - x_t) > 0$$

implying that the pool quality improves until it crosses $\underline{x}$ and thereafter converges to the screening steady state.

The case of $x_0 = \overline{x}$ is straightforward as $\overline{x}$ is already a steady state.

A.4 Proof of Proposition 3

The zero profit condition (6) implies that

$$\Pi(R) = \kappa_H r + \kappa_L (1 - z) r_L - (\kappa_H + \kappa_L)\tilde{c}z = 0.$$

Reformulating this we obtain

$$\kappa x r / r_L + \kappa(1 - x)(1 - z) + \kappa c z = 0$$

$$r = -r_L \frac{cz + (1 - x)(1 - z)}{x}$$

which proves Proposition 3.

A.5 Proof of Proposition 4

Define $\theta(x)$ as in (13) and define $\hat{x}$ implicitly as the unique value of $x < \lambda$ with $\theta(x) = 1$. Such a value exists since $\theta(x)$ is strictly increasing and continuous in x with $\theta(0) = -\delta \kappa^{-1} < 0$ and $\lim_{x \to \lambda} \theta(x) = \infty$.

Assume $\hat{x} > \overline{x}$. Conjecture for any $x_0 \in [\overline{x}, \hat{x})$ that the equilibrium is one with $\theta_t = \theta(x_t)$. To verify the conjecture, we need to show that type-H borrowers are indifferent between taking a loan and waiting. Assuming $u \to 0$ in (5a), this is equivalent to

$$J_t^H = r_H - r(x_t)$$

with

$$\rho J_t^H = \dot{J}_t^H - \delta J_t^H.$$

Putting the two together, we obtain (14),

$$-r'(x)\dot{x} = (\rho + \delta)(r_H - r(x)).$$

The law of motion for x with $\theta < 1$ is $\dot{x}_t = (\kappa\theta + \delta)(\lambda - x)$, which, together with (14) yields (13) and therefore confirms that type-H borrowers are, by construction, precisely indifferent.

A.6 Proof of Proposition 5

We prove Proposition 5 in two steps. First, we determine the efficient screening policy $z^*(x)$ conditional on banks operating. Then we determine the optimal behavior for banks to operate $\theta^*(x)$.

A.6.1 Optimal screening policy $z^*(x)$

To do so, let $V(x,z)$ denote the present value of welfare if the current state of the market is x and the screening policy is z from hereafter, that is,

$$V(x,z) \equiv \frac{\rho x + \alpha^z x^z}{\rho + \alpha^z} r_H + (1-z)\left(1 - \frac{\rho x + \alpha^z x^z}{\rho + \alpha^z}\right) r_L - \tilde{c}z. \tag{A.1}$$

where $\alpha^z \equiv \kappa + \delta - \lambda\kappa z$ and $x^z \equiv \lambda - \lambda\frac{(1-\lambda)z}{(1-\lambda z)+\delta\kappa^{-1}}$. Also, denote by

$$v(x,z) \equiv \rho\left\{xr_H + (1-z)(1-x)r_L - \tilde{c}z\right\} \tag{A.2}$$

the flow value of policy z at state x. Finally, we call

$$d(x,z) \equiv \kappa(1-x)(1-z)\lambda - \kappa x(1-\lambda) + \delta(\lambda - x) \tag{A.3}$$

the derivative of x at state x under policy z (see the law of motion in (7)). Observe that

$$\rho V(x,z) = v(x,z) + V_x(x,z)d(x,z) \tag{A.4}$$

as well as

$$d(x^s, \bar{z}) = 0 \quad d(x^p, 0) = 0. \tag{A.5}$$

We first prove the following helpful lemma.

Lemma 1. *We have:*

1. *If $\lambda\kappa r^\Delta \geq \rho + \kappa + \delta$, pooling is strictly optimal for any state x, i.e. $z^*(x) = 0$.*

2. *If $\lambda\kappa r^\Delta < \rho + \kappa + \delta$, $V(x,z)$ has negative cross-partials, $V_{xz} < 0$.*

3. *If $\lambda\kappa r^\Delta < \rho + \kappa + \delta$ and $V(x,0) > V(x,z_1)$ for some $z_1 > 0$, then also $V(x,0) > V(x,z_2)$ for any $z_2 \in (0,z_1)$.*

Proof. Assume $\lambda\kappa r^\Delta \geq \rho + \kappa + \delta$. Suppose pooling were not strictly optimal for every state x. First, if $d(x,z^*(x))$ is ever negative for some $x < \lambda$, there must be a steady state at some $x^0 \in [0,\lambda)$ with some $z^0 = z^*(x^0) > 0$. This cannot be optimal since

$$V(x^0, z^0) < V(x^0, 0)$$

is equivalent to (after a few lines of algebra)

$$-(\rho + (1-\lambda)\alpha^p - \rho x^0)\left(r^\Delta\kappa\lambda - (\rho + \kappa + \delta)\right) < (\rho + \alpha^s)(\rho + \alpha^p)c$$

A-3

which is true since the left hand side is negative. Second, assume $d(x, z^*(x))$ is positive everywhere. Then, $x^p = \lambda$ is still the unique steady state. Let $\mathbf{V}(x)$ be the optimal value function. It has to hold that

$$r\mathbf{V}(x) = v(x, z^*(x)) + \mathbf{V}'(x)d(x, z^*(x)). \tag{A.6}$$

Rearranging,

$$\mathbf{V}'(x) = \frac{r\mathbf{V}(x) - v(x, z^*(x))}{d(x, z^*(x))} \equiv \mathbf{F}(\mathbf{V}(x), x).$$

Compare this to the ODE describing the value of pooling,

$$V_x(x, 0) = \frac{rV(x, 0) - v(x, 0)}{d(x, 0)} = \mathbf{F}^0(V(x, 0), x)$$

Observe that $\mathbf{F}(V, x) > \mathbf{F}^0(V, x)$ for any x for which $z^*(x) > 0$.[A-1] Since $\mathbf{V}(x^p) = V(x^p, 0)$, it must be that $\mathbf{V}(x) < V(x, 0)$ for some x if there is a positive measure where $z^* > 0$. This contradicts our assumption that $\mathbf{V}(x)$ is the optimal value function. Thus, pooling is optimal for every state.

Assume $\lambda \kappa r^\Delta < \rho + \kappa + \delta$. Simple algebra based on (A.1) implies that

$$V_x = \frac{\rho}{\rho + \alpha^z} r_H + (1 - z)\frac{r}{\rho + \alpha^z} r_L > 0$$

and

$$V_{xz} = \rho \frac{\lambda \kappa r^\Delta - (\rho + \kappa + \delta)}{(\rho + \alpha^z)^2 r_L} < 0.$$

For point 3 in the lemma, fix $x \in [0, \lambda]$. Define the positive constants

$$c_0 \equiv \frac{\rho x + \lambda \kappa + \lambda \delta}{\lambda \kappa}, \quad c_1 \equiv \frac{\rho + \kappa + \delta}{\lambda \kappa}, \quad c_2 \equiv r^\Delta, \quad c_3 = -r_L$$

Then, after some algebra, we can write

$$V(x, z) = (1 - z)r_L - \tilde{c}z + c_3 \frac{c_0 - z}{c_1 - z}(c_2 - z)$$

$$= r_L - \tilde{c}z + c_3(c_0 + c_2 - c_1) + c_3 \frac{(c_1 - c_2)(c_1 - c_0)}{c_1 - z}$$

c_1 is always greater than c_0. Also, given $\lambda \kappa r^\Delta < \rho + \kappa + \delta$, c_1 is greater than c_2. Therefore, $V(x, z)$ is a convex function in z, and thus in particularly quasi-convex, from which the stated property follows. $\square$

In the next lemma, we narrow down the set of optimal policies using the necessary (but not sufficient) first order conditions of (18).

Lemma 2. *Describe an efficient screening policy $z^*(x)$ by the following general form: Let $I_1, I_2, I_3 \subset [0, \lambda]$ be (possibly empty) connected intervals such that $I_1 \leq I_2 \leq I_3$, $I_1 \cup I_2 \cup I_3 = [0, \lambda]$, and*

- *$z^*(x) = \bar{z}$ for $x \in I_1$*

- *$z^*(x) \in [0, \bar{z}]$ for $x \in I_2$*

[A-1] Note that $\mathbf{V}'(x) > 0$ by a simple envelope argument.

Figure A.1: Phase diagram for constrained efficient problem
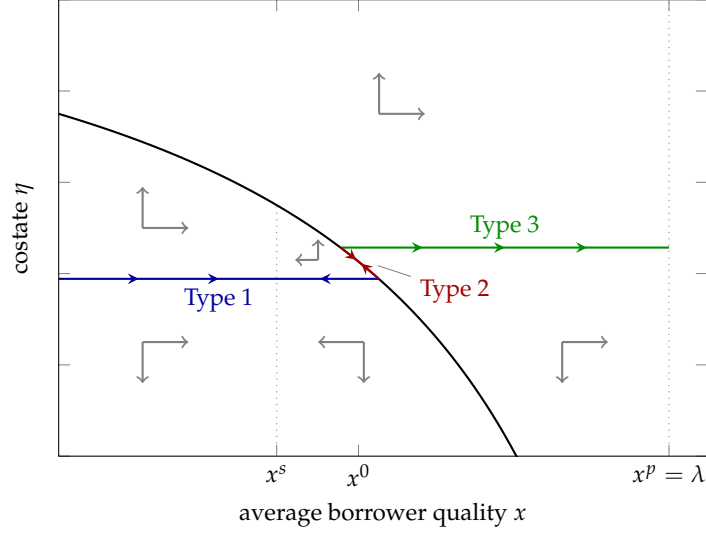
- $z^*(x) = 0$ for $x \in I_3$

where we construct I_1 to be the largest connected interval where $z^*(x) = \bar{z}$, and similarly I_3 for $z^*(x) = 0$.
Then: If I_2 is non-empty (and thus $z^*(x)$ not bang-bang), there exists a $x^0 \in I_2$ with $d(x^0, z^*(x^0)) = 0$.

Proof. We begin by writing down the necessary first order conditions of (18). Denoting by η the costate of x, we have the law of motion

$$\dot{\eta} = \rho\eta - \kappa \left\{ r_H - (1 - z_t)r_L \right\}$$

as well as the first order condition for z, showing that $z_t = \bar{z}$ if

$$\kappa \left\{ -(1 - x_t)r_L - \tilde{c} \right\} - \eta \left(\kappa(1 - x_t)\lambda \right) > 0 \tag{A.7}$$

and $z_t = 0$ if (A.7) hold with "$<$" inequality; with equality, z_t can be anywhere in $[0, \bar{z}]$.

Together with the law of motion of x in (7), this gives a system of two ODEs. We first note that there are three possible steady states. The two steady states for $z = 0$ and $z = \bar{z}$, as well as a third one, $z = z^0$ pinned down by $\dot{\eta} = \dot{x} = 0$ and (A.7) holding with equality,

$$\frac{-r_L}{\lambda} - \frac{\tilde{c}/\lambda}{1 - x^0} = \frac{\kappa}{\rho} \left\{ r_H - (1 - z^0)r_L \right\} \tag{A.8}$$

where

$$x^0 = \lambda \frac{1 - z^0 + \delta\kappa^{-1}}{(1 - \lambda z^0) + \delta\kappa^{-1}}. \tag{A.9}$$

Observe that, after substituting (A.9) into (A.8), the left hand side of (A.8) is increasing in z^0, while the right hand side is decreasing, so there is at most a single solution to (A.8).

Now consider the phase diagram in Figure A.1. As can be seen, there are 3 types of candidate optimal paths. Type 1 converges to x^s, with $z = \bar{z}$ along the path and constant η; Type 3 converges to

A-5

x^p, with $z = 0$ along the path and constant η; and finally type 2 converges to x^0 as in (A.9). Observe that the second type of paths only works if z^0, x^0 exist, solving (A.8) and (A.9).

This implies that, unless the optimal policy $z^*(x)$ is bang-bang, there has to exist a x^0 with $d(x^0, z^*(x^0)) = 0$. □

With this result in mind, we assume in the following that $\lambda \kappa r^\Delta < \rho + \kappa + \delta$ and characterize $z^*(x)$.

Lemma 3. *Assume $\lambda \kappa r^\Delta < \rho + \kappa + \delta$. The efficient screening policy $z^*(x)$ is to screen if $x < \overline{x}^*$ and to pool if $x > \overline{x}^*$, where*

$$V(\overline{x}^*, 0) = V(\overline{x}^*, \overline{z}) \tag{A.10}$$

as long as the solution to that equation is greater or equal to x^s. Otherwise, $\overline{x}^*$ is determined by

$$v_z(\overline{x}^*, 0) + V_x(\overline{x}^*, 0)d_z(\overline{x}^*, 0) = 0. \tag{A.11}$$

Proof. First, notice that $\overline{x}^*$ is indeed well-defined, in that if the solution to (A.10) is x^s, then (A.11) is also solved by x^s. Assume

$$V(x^s, 0) = V(x^s, \overline{z}).$$

Combining (A.4) and (A.5), we can rewrite $V(x^s, 0)$ and $V(x^s, \overline{z})$ and obtain

$$v(x^s, 0) + V_x(x^s, 0)d(x^s, 0) = v(x^s, \overline{z}) + V_x(x^s, \overline{z})d(x^s, \overline{z}).$$

Since $d(x^s, \overline{z}) = 0$, this can be combined into

$$v(x^s, \overline{z}) - v(x^s, 0) + V_x(x^s, 0)\left(d(x^s, \overline{z}) - d(x^s, 0)\right) = 0 \tag{A.12}$$

which is equivalent to (A.11) as v and d are linear in z. Moreover, going these steps backwards, if $\overline{x}^* < x^s$, then (A.12) holds with inequality and therefore

$$V(x^s, 0) > V(x^s, \overline{z}). \tag{A.13}$$

Now we proceed to our main argument, a proof by contradiction. We distinguish four possible cases.

Case 1: There exists $x > \overline{x}^*$ with $x \geq x^s$ where pooling is not optimal. If true, by Lemma 2, this would require there to be at least one point $x^0 \in [\overline{x}^*, \lambda)$ where the planner strictly prefers to remain at x^0 forever (by choosing strategy $z^0 \in (0, \overline{z}]$ such that $d(x^0, z^0) = 0$) over pooling. In math,

$$V(x^0, z^0) > V(x^0, 0).$$

Since V has a negative cross-partial $V_{xz} < 0$ (Lemma 1), this implies that $V(\overline{x}^*, z^0) > V(\overline{x}^*, 0)$ and $V(x^s, z^0) > V(x^s, 0)$, which, by point 3 in Lemma 1, is contradicting either (A.10) or (A.13).

Case 2: There exists $x < \overline{x}^*$ with $x \geq x^s$ where screening is not optimal. If true, by Lemma 2, this would require there to be at least one point $x^0 \in (x^s, \overline{x}^*]$ where the planner strictly prefers to remain at x^0 forever (by choosing strategy $z^0 \in [0, \overline{z})$ such that $d(x^0, z^0) = 0$) over screening. In math,

$$V(x^0, z^0) > V(x^0, \overline{z}).$$

Since V has a negative cross-partial $V_{xz} < 0$ (Lemma 1), this implies that $V(\overline{x}^*, z^0) > V(\overline{x}^*, \overline{z})$, which by point 3 in Lemma 1, contradicts (A.10).

Case 3: There exists $x > \overline{x}^*$ with $x \leq x^s$ where screening is optimal. If true, this would require there to be at least one point $x^0 \in [\overline{x}^*, x^s]$ where the planner strictly prefers to screen with some intensity $z^0 > 0$ in the current instant while pooling is chosen thereafter. That is,

$$v(x^0, z^0) + V_x(x^0, 0)d(x^0, z^0) > v(x^0, 0) + V_x(x^0, 0)d(x^0, 0).$$

Due to linearity of this equation, it also has to hold with $z^0 = \overline{z}$, and therefore also expressed as derivative,

$$v_z(x^0, 0) + V_x(x^0, 0)d_z(x^0, 0) > 0. \tag{A.14}$$

Since this is a linear equation in x^0, to be consistent with (A.11), it must be that (A.14) in fact holds for any $x^0 > \overline{x}^*$, including $x^0 = x^p = \lambda$. In that case, however, (A.14) simplifies to $v_z(x^p, 0) + V_x(x^p, 0)d_z(x^p, 0) > 0$, which is false, since $V_x(x, 0) > 0$, $d_z(x, 0) < 0$ and $v_z(x^p, 0) = -\kappa r_L (c - (1 - \lambda)) < 0$ by Assumption 1.

Case 4: There exists $x < \overline{x}^* \leq x^s$ where pooling is optimal. Let $\mathbf{V}(x)$ be our conjectured value function left of $\overline{x}^*$. By design, $\mathbf{V}(x)$ solves

$$\rho \mathbf{V}(x) = v(x, \overline{z}) + \mathbf{V}'(x)d(x, \overline{z})$$

where $d(x, \overline{z}) = \alpha^{\overline{z}}(x^s - x)$ and $\mathbf{V}'(x)$ solves

$$(r + \alpha^{\overline{z}})\mathbf{V}'(x) = v_x(x, \overline{z}) + \mathbf{V}''(x)d(x, \overline{z}).$$

This ODE can be solved explicitly, giving[A-2]

$$\mathbf{V}'(x) = \rho r_L \left(\frac{r^\Delta}{\rho + \alpha^p} - \frac{r^\Delta - \overline{z}}{\rho + \alpha^{\overline{z}}} \right) \left(\frac{x^s - x}{x^s - \overline{x}^*} \right)^{-\beta} + \rho r_L \frac{r^\Delta - \overline{z}}{\rho + \alpha^{\overline{z}}}$$

where $\beta = 1 + \frac{\rho}{\alpha^{\overline{z}}}$. The coefficient on the first term is positive, since we assumed $r^\Delta \lambda \kappa < \rho + \kappa + \delta$. Thus, $\mathbf{V}'(x)$ is bounded above by

$$\mathbf{V}'(x) \leq \mathbf{V}'(\overline{x}^*) = r(1 - R_L)\frac{r^\Delta}{\rho + \alpha^p}. \tag{A.15}$$

Could it ever be that the planner prefers pooling in this region? If so, we would have an $x < \overline{x}^*$ with

$$v_z(x, 0) + \mathbf{V}'(x)d_z(x, 0) < 0$$

which due to (A.15) and the fact that $d_z(x, 0) = -\kappa\lambda(1 - x) < 0$ implies that

$$v_z(x, 0) + \mathbf{V}'(\overline{x}^*)d_z(x, 0) < 0.$$

[A-2]Note that $v_x(x, \overline{z})$ is a constant in x.

Using the expressions in (A.2) and (A.3) we then see that this cannot hold as the left hand side is zero at $\overline{x}^*$ (by definition), and has a negative slope throughout,

$$v_{xz} + \mathbf{V}'(\overline{x}^*)d_{xz} = \rho r_L \left[-1 + \frac{r^\Delta \lambda \kappa}{\rho + \alpha^p} \right] < 0$$

where again we used $r^\Delta \lambda \kappa < \rho + \kappa + \delta$. This is a contradiction: there cannot be an $x < \overline{x}^*$ where pooling is optimal. $\qquad \square$

A.6.2 Optimal bank operation policy $\theta^*(x)$

Next we focus on the optimal policy $\theta^*(x)$ for banks to operate. We prove the following result.

Lemma 4. *If it is strictly optimal to have banks operate at $\overline{x}^*$, the optimal policy describing when banks operate is bang-bang, that is,*

$$\theta^*(x) = \begin{cases} 0 & x < \underline{x}^* \\ 1 & x > \underline{x}^* \end{cases} \tag{A.16}$$

The threshold $\underline{x}^*$ is the supremum of all $x \in [0, \lambda]$ that solve

$$v(x, z^*(x)) + \mathbf{V}'(x) \left(\kappa (\lambda - x) - \kappa (1 - x) z^*(x) \lambda \right) < 0 \tag{A.17}$$

where $\mathbf{V}(x)$ is the value function associated with the optimal screening policy $z^*(x)$.

Proof. Let $\underline{x}^*$ be defined as in (A.17) and let $\mathbf{V}(x)$ be the value function conditional on banks operating with screening policy $z^*(x)$. If it is optimal for the planner to follow the bang-bang policy (A.16), then its value function for $x \geq \underline{x}^*$ is given by $\mathbf{V}(x)$, whereas for $x < \underline{x}^*$ the value function solves

$$\rho \mathbf{V}(x) = \mathbf{V}'(x) \delta(\lambda - x)$$

which can be solved to express the marginal value in state x as

$$\mathbf{V}'(x) = \mathbf{V}'(\underline{x}^*) \left(\frac{\lambda - x}{\lambda - \underline{x}^*} \right)^{-1 - \rho/\delta}.$$

Observe that this is increasing in x. To prove that the bang-bang policy (A.16) is indeed optimal, we need to prove that

$$\max_{z \in [0, \overline{z}]} v(x, z) + \mathbf{V}'(x) d(x, z, 1) \leq \max_{z \in [0, \overline{z}]} \mathbf{V}'(x) d(x, z, 0) \tag{A.18}$$

for $x < \underline{x}^*$, where

$$d(x, z, \theta) \equiv \theta \kappa (1 - x)(1 - z) \lambda - \theta \kappa x (1 - \lambda) + \delta(\lambda - x)$$

is the speed at which the pool improves given (x, z, θ). Simplifying (A.18), we obtain

$$\max_{z \in [0, \overline{z}]} v(x, z) + \mathbf{V}'(x) \left[\kappa \lambda (1 - z) - \kappa x (1 - z \lambda) \right] \leq 0.$$

The left hand side of this inequality has a negative cross-partial in (x, z), since $v_{xz} < 0$ and $\mathbf{V}'(x)(1 - x) \propto (\lambda - x)^{-\rho/\delta} \frac{1-x}{\lambda - x}$ increases in x. Thus, given that $z = \overline{z}$ is optimal for $x = \underline{x}^*$, it is also optimal for any $x < \underline{x}^*$.

The problem then reduces from (A.18) to showing that for $x < \underline{x}^*$

$$F(\lambda - x) \equiv v(x, \bar{z}) + \mathbf{V}'(x) \left[\kappa \lambda (1 - \bar{z}) - \kappa x (1 - \bar{z}\lambda) \right] < 0. \tag{A.19}$$

To see this, we first show that $F(y)$ is quasi-concave (only has a single local maximum) and therefore can at most have two roots. $F(y)$ is of the form

$$F(y) = -F_0 y + F_1 y^{-\alpha-1}(y - y_0) + const$$

where $\alpha = \rho/\delta > 0$, $F_0 = \rho r_H + \rho(1 - \bar{z})r_L > 0$, $F_1 = \kappa(1 - \lambda\bar{z})\mathbf{V}'(\underline{x}^*)(\lambda - \underline{x}^*)^{1+\alpha} > 0$, $y_0 = \lambda - x^s > 0$. F can only ever have a single local maximum as long as these parameters are positive:

$$F'(y) = 0 \quad \Leftrightarrow \quad y^{-\alpha-2}\left[(1 + \alpha)y_0 - \alpha y\right] = F_1/F_0$$

The left hand side of this equation is strictly decreasing for $y \in (0, (1 + \alpha)y_0/\alpha)$ with range $(0, \infty)$ and thus admits a unique solution for any $F_1/F_0 > 0$. This establishes that $F(y)$ is quasi-concave.

Since $F(y)$ is quasi-concave, it admits at most two roots, $y_1 < y_2$, in between which $F(y)$ is positive, and negative outside of $[y_1, y_2]$. Root y_2 must correspond to $\lambda - \underline{x}^*$: if y_1 were to correspond to $\lambda - \underline{x}^*$, $\underline{x}^*$ would not be the supremum of x with $F(\lambda - x) < 0$ since for any $\epsilon > 0$ small enough, $F(\lambda - (\underline{x}^* - \epsilon)) > 0$. But if $y_2 = \lambda - \underline{x}^*$, then $F(\lambda - x) < 0$ for any $x < \underline{x}^*$, which proves (A.19). $\quad\square$

A.7 Proof of Corollary 2

By Assumption 1, $c \geq 1 - \lambda$. Therefore, welfare in the screening steady state is bounded above,[A-3]

$$W^s = x^s r_H - (1 - \bar{z})(1 - x^s) r_L - \tilde{c}\bar{z} = x^s \frac{r_H}{-r_L} - (1 - \bar{z})(1 - x^s) r_L - c\bar{z}$$

$$\leq x^s \frac{r_H}{-r_L} - (1 - x^s)(1 - \bar{z}) - (1 - \lambda)\bar{z} = x^s \left(\frac{r_H}{-r_L} + 1 - \bar{z} \right) - (1 - \lambda\bar{z})$$

Welfare in the pooling steady state is $W^p = \lambda(\frac{r_H}{-r_L} + 1) - 1$. Observe that W^s increases in x^s, so W^s can only ever be above W^p if x^s is as large as possible. Clearly, given the formula for x^s, x^s is largest as a function of δ if $\delta = \infty$ where $x^s = \lambda$. In that case, we find

$$W^s \leq x^s \left(\frac{r_H}{-r_L} + 1 - \bar{z} \right) - (1 - \lambda\bar{z}) < \lambda \left(\frac{r_H}{-r_L} + 1 - \bar{z} \right) - (1 - \lambda\bar{z}) = \lambda \left(\frac{r_H}{-r_L} + 1 \right) - 1 = W^p$$

Therefore, welfare of the pooling steady state always dominates that of the screening steady state.

A.8 Proof of Proposition 6

This proposition follows directly from noticing that the problem (26) is still linear in z, so that $z = \bar{z}$ is strictly preferred if and only if

$$(1 - x_t)(-r_L) + \omega(1 - x_t) - \tilde{c} > 0$$

Rearranging yields

$$x_t < 1 - \frac{c}{1 + \omega/(-r_L)} \equiv \bar{x}^c$$

[A-3]We define all welfare expressions here as multiples of κ, for expositional clarity. κ multiplies both W^s and W^p equally.

where the threshold $\bar{x}^c$ always strictly exceeds $\bar{x} = 1 - c$. This proves Proposition 6.

B   Implementing the constrained optimum

Optimal policy implements normal lending standards only when $x \in (\bar{x}^*, \bar{x})$. While, there are several ways a government or a regulator could implement the constrained efficient outcome, we focus on a temporary government-funded loan insurance program. Since such an intervention entails short-run losses and the model's banking sector is competitive, either the government or type-H borrowers have to bear these losses. An example of such a policy is a government-funded loan insurance program in which the government provides an insurance benefit $b > 0$ (in present value) to be paid to a bank when a borrower defaults. This policy incentivizes banks to use normal lending standards as long as

$$\frac{b}{-r_L} > 1 - \frac{c}{1-x}.$$

This condition is satisfied for $b = 0$ in the region $x > \bar{x}$ where pooling is privately optimal. It requires nonzero insurance benefits $b = b(x) > 0$ when $x < \bar{x}$. As a function of the pool quality, $b(x)$ is decreasing in x. This means a typical intervention starting from some $x_0 < \bar{x}$ requires large insurance benefits early on, which are then phased out over time.[A-4]

Shadow Banks and limits to implementation.   In practice, policies like government-funded loan subsidies or insurance programs are rarely undertaken for the entire financial sector, but instead usually apply only to certain types of institutions, such as traditional banks but not shadow banks for example. So consider a setting where the government can affect the lending decisions of only a fraction $\eta \in [0, 1)$ of financial institutions. What is the optimal policy under such circumstances? We focus on the case without bank inactivity, $\theta = 1$. We further assume that the share η of traditional banks always charge the same interest rates as their shadow bank competitors. This setting implies that the planner can no longer set the lending standard z_t to any number in $[0, \bar{z}]$. Instead, z_t needs to lie in $[(1-\eta)z(x_t), (1-\eta)z(x_t) + \eta\bar{z}]$, where $z(x_t)$ is the privately optimal lending standard (10).

The new constraint makes optimal policy. In particular, there is now a threshold $\bar{x}^*(\eta)$ that depends on η above which the planner desires to set normal lending standards. Crucially, for low levels η, $\bar{x}^*(\eta)$ will be equal to $\bar{x}$, implying the planner prefers not to intervene at all. To see this, observe that for any $x < \bar{x}$, shadow banks apply tight lending standards, pushing the quality x down, towards the screening steady state. Even if the traditional banks are made to apply normal lending standards, it may not be possible to achieve $\dot{x} > 0$. Even if $\dot{x} > 0$ is possible, with $z = 0$ in the traditional banking sector, it make take so long to reach $\bar{x}$ that the policy is too costly to be worth implementing.. Thus, the government can lack the "firepower" to get to pooling and optimally choose not to intervene at all. In this way, a large shadow banking sector can constrain optimal government policy.[A-5]

[A-4]Another way to implement the constrained optimum that does not have a dynamic aspect to policy would be to *require* that, whenever $x \in (\bar{x}^*, \bar{x})$, all loans made at each point in time are placed into a common pool from which each bank receives a proportionate payout as the loans mature. Such mandated securitization requires only that a loan origination is observable and contractible, not that a rejection is observable. Under this policy, no individual bank has the incentive to tighten lending standards when $x \in (\bar{x}^*, \bar{x})$ since they receive no benefits from placing a higher-quality loan into the securitized pool.

[A-5]If the government intervenes for low η, it is forced to cover increasingly large losses as shadow banks impose tight lending standards, reducing the pool quality. Traditional banks then face a more adversely selected pool. This point may have been relevant for the failure of the government sponsored mortgage agencies in the U.S. in 2008.

Returning to the example of the credit market downturn in Section 4, the parameters in Figure 6 are such that optimal policy in the long downturn is to maintain normal lending standards. Such a policy can be implemented, for example, with a loan guarantee program, as just described. Under optimal policy, the long downturn in Figure 6 would instead look like an extended version of the short one. With no tightening of lending standards at $t = 3$, the lending volume would continue its smooth decline during the fourth year and then slowly rise back to one following the end of the recession, like a longer version of the two-year downturn. Similarly, the rise in credit spreads and (ex post) defaults continues to slow, and starts to recover rapidly at the end of year 4. In short, the credit market deteriorates by less and recovers more quickly.

C   Appendix to Section 4.3

In this section, we derive equations (15), (16) and (17), used in Section 4.3. We use the notation m_{Ht} for the mass of type-H borrowers in the pool at time t; m_{Lt} for the mass of type-L borrowers; and $k_t \equiv \theta_t \kappa m_{Ht} + \theta_t \kappa (1 - z_t) m_{Lt}$ for the mass of borrowers that receive a loan at date t. Moreover, we let μ_t be the time-varying reduction in the inflow of type-H borrowers,

$$\mu_t = \begin{cases} \mu & t \leq T' \\ 0 & t > T' \end{cases}.$$

The law of motion for m_{Ht} is then given by

$$\dot{m}_{Ht} = (1 - \mu_t)(\delta\lambda + k_t\lambda) - \delta m_{Ht} - \theta_t \kappa m_{Ht}. \tag{A.20}$$

The first two terms capture exogenous entry as well as endogenous entry to replenish funded borrowers. The terms are reduced by a factor $1 - \mu_t$ due to the shock. The last two terms are exogenous exit and endogenous exit due to being funded. The law of motion for m_{Lt} is similar,

$$\dot{m}_{Lt} = \delta(1 - \lambda) + k_t(1 - \lambda) - \delta m_{Lt} - \theta_t \kappa(1 - z_t)m_{Lt}. \tag{A.21}$$

With (A.20), (A.21), the total size of the pool $N_t \equiv m_{Ht} + m_{Lt}$ evolves according to

$$\dot{N}_t = (1 - \mu_t)(\delta\lambda + k\lambda) - \delta m_{Ht} - \kappa m_{Ht} + \delta(1 - \lambda) + k(1 - \lambda) - \delta m_{Lt} - \kappa(1 - z_t)m_{Lt}$$

which we can simplify to

$$\dot{N}_t = \delta(1 - N_t) - \mu_t \lambda N_t \left(\frac{\delta}{N_t} + \theta_t \kappa x_t + \theta_t \kappa (1 - x_t)(1 - z_t) \right)$$
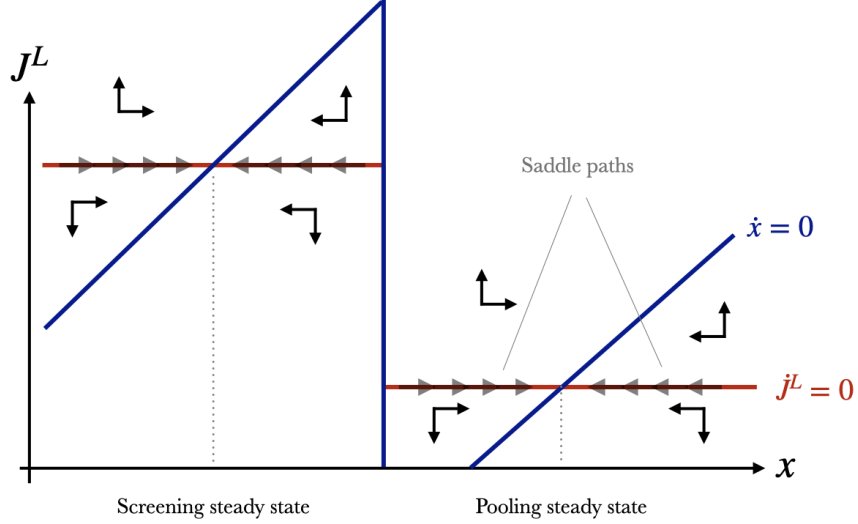
The law of motion for pool quality, $x_t \equiv \frac{m_{Ht}}{m_{Ht} + m_{Lt}}$, can be computed as

$$\dot{x}_t = \frac{\dot{m}_{Ht}}{m_{Ht}} x_t - \frac{\dot{N}_t}{N_t} x_t$$

which, after some algebra, ends up simplifying to

$$\dot{x}_t = \kappa \theta_t (1 - z_t)(1 - x_t)\lambda_t - \kappa \theta_t x_t (1 - \lambda_t) + (\lambda_t - x_t)\frac{\delta}{N_t}.$$

Figure D.1: Phase diagram for the transitional dynamics in the model with entry.

The entry quality parameter λ_t is given by

$$\lambda_t = (1 - \mu_t (1 - x)) \lambda.$$

D   Appendix to Section 6

D.1   Transitions in the model with entry

The dynamic equilibrium of the model with entry is characterized by the following three equations: (i) the HJB (5b) for J_t^L

$$(\rho + \delta + \kappa (1 - z_t)) J_t^L = \kappa(1 - z_t)u + \dot{J}_t^{L}, \tag{A.22}$$

(ii) the evolution of pool quality (24),

$$\frac{\dot{x}_t}{x_t/\lambda} = \kappa(1 - x_t)(1 - z_t)\lambda - \kappa x_t(1 - \lambda) + \delta(1 - x_t) - \Psi(J_t^L/u), \tag{A.23}$$

(iii) and bank optimality (10)

$$z_t = \begin{cases} 0 & \text{if } x_t > \overline{x} \\ \overline{z} & \text{if } x_t < \overline{x} \end{cases}.$$

We can write bank optimality as $z_t = \overline{z}1_{\{x_t>\overline{x}\}}$ and substitute it into the HJB (A.22) and the law of motion (A.23) to arrive at a system of two ODEs. We illustrate the dynamics in a standard phase diagram in Figure D.1.

We see that there is a unique saddle path to one of the two stable steady state equilibria (screening and pooling) at every point in the state space. The middle steady state equilibrium is unstable, as expected.

D.2 Welfare with non-constant pool size

The social planning problem in the constant-inflow-rate model is

$$\max_{z_t \in [0,\bar{z}]} \int_0^\infty e^{-\rho t} \kappa \left\{ m_H^* r_H + (1 - z_t) m_L r_L - \tilde{c} z_t (m_L + m_H) \right\} dt$$

subject to the law of motion for m_L, (21). One can show that it has the exact same properties as the planning problem in Section 5. Relative to the privately optimal threshold $\bar{x} = 1 - c$, which corresponds to

$$\overline{m}_L = \frac{\lambda}{1 + \kappa \delta^{-1}} \frac{c}{1 - c}$$

there exists a socially optimal threshold $\bar{x}^* \equiv \frac{m_H^*}{m_H^* + \overline{m}_L^*}$ where $\overline{m}_L^*$ is determined by

$$\underbrace{(1 - \bar{z} + c\bar{z}) \frac{\rho \overline{m}_L^* + \alpha^s m_L^s}{\rho + \alpha^s} + c\bar{z} m_H^*}_{\text{Average social cost from lending to type-}L\text{ when screening}} = \underbrace{\frac{\rho \overline{m}_L^* + \alpha^p m_L^p}{\rho + \alpha^p}}_{\text{Average social cost from lending to type-}L\text{ when pooling}} \tag{A.24}$$

Here, we define the transition speeds for m_L under pooling and screening by $\alpha^p \equiv \kappa + \delta$ and $\alpha^s \equiv \kappa(1 - \bar{z}) + \delta$. The associated steady state values for m_L are given by $m_L^p = \frac{\delta(1-\lambda)}{\delta + \kappa}$ and $m_L^s = \frac{\delta(1-\lambda)}{\delta + \kappa(1-\bar{z})}$. Similar to Section 5, one can show here, too, that the social planner marginally prefers more pooling, that is,

$$\overline{m}_L^* > \overline{m}_L$$

The reason is identical to that in Section 5.

An especially simple welfare result is the comparison of steady state welfares across steady states. Here, the question is whether it is the case that welfare in the pooling steady state exceeds that in the screening steady state. In the context of this model, this is satisfied if

$$(1 - \bar{z} + c\bar{z}) m_L^s + c\bar{z} m_H^* > m_L^p$$

After some algebra, this simplifies to

$$1 + \delta^{-1}\kappa > (1 - \lambda)c^{-1} + \lambda \delta^{-1} \kappa \bar{z}$$

which is necessarily the case given our Assumption 1: $c > 1 - \lambda$. Thus, Corollary 2 also carries over to this model.

E Discussion of modeling assumptions

E.1 Misidentifying type-H borrowers as type-L borrowers

In our baseline model, the screening technology is asymmetric: a type-L borrower may be incorrectly identified as a type-H but a type-H is always correctly identified. Suppose, instead, that both types of errors are possible. In particular, assume that a bank setting a lending standard $z \in [0, \bar{z}]$, at cost

$\tilde{c}z$ as before, allows it to obtain a binary signal with values "h" and "l", distributed as follows:

$$Prob(\text{signal} = l|\text{type-}L) = p + z(1 - p) \qquad (A.25)$$
$$Prob(\text{signal} = l|\text{type-}H) = p - zp \qquad (A.26)$$

This signal structure is chosen such that: (a) with $z = 0$, no information is revealed by the signal, as its distribution is independent of the borrower type; (b) our baseline model is nested with $p = 0$, in which case type-H borrowers are never misidentified; and (c) any $p > 0$ allows an arbitrary degree of misidentification of type-H borrowers—in fact, $p = 0.5$ treats both types of borrowers entirely symmetrically.[A-6]

Given this signal structure, the probability of observing type-H and type-L with signal realization "h" given some pool quality x is[A-7]

$$Prob(\text{type-}H \text{ and signal} = h) = (1 - p(1 - z))x$$
$$Prob(\text{type-}L \text{ and signal} = h) = (1 - p)(1 - z)(1 - x)$$

In the special case $p = 0$, which we considered earlier, these expressions collapse to probabilities x and $(1 - z)(1 - x)$. In the special case of no lending standards, $z = 0$, the expressions collapse to $(1 - p)x$ and $(1 - p)(1 - x)$ as the signal reveals no information.

Conditional on being active, banks make two choices now: first, they decide whether to screen at all; second, if they screen, they decide the screening intensity z, after which they lend if signal realization is "h". The bank profit maximization problem (4) then turns into

$$\Pi_t(r_t) \equiv \begin{cases} \kappa_{Ht} r_t + \kappa_{Lt} r_L & \text{if not screen} \\ \max_{z \in [0,\bar{z}]} (1 - p(1 - z)) \kappa_{Ht} r_t + (1 - p)(1 - z)\kappa_{Lt} r_L - (\kappa_{Ht} + \kappa_{Lt})\tilde{c}z & \text{if screen} \end{cases}.$$

It is straightforward to see that due to linearity, banks effectively choose either not to screen or to screen with intensity $z = \bar{z}$. The relevant condition for screening is then

$$(1 - p(1 - \bar{z})) \kappa_{Ht} r_t + (1 - p)(1 - \bar{z}) \kappa_{Lt} r_L - (\kappa_{Ht} + \kappa_{Lt})\tilde{c}\bar{z} > \kappa_{Ht} r_t + \kappa_{Lt} r_L \qquad (A.27)$$

Moreover, the equilibrium interest rate at the point of indifference is pinned down by the zero profit condition

$$\kappa_{Ht} r_t + \kappa_{Lt} r_L = 0$$

which, as before, simplifies to

$$r_t = (-r_L)x_t^{-1}\{1 - x_t\} \qquad (A.28)$$

Combining (A.27) and (A.28), we find that screening at $z = \bar{z}$ is still optimal when

$$x_t < \bar{x} = 1 - c \qquad (A.29)$$

which is the same threshold as before. In particular, it is independent of p (and $\bar{z}$) and hence independent of how much misidentification of type-H borrowers there is.

How is this possible? The intuition for this surprising result is that what matters for the informativeness of the signal is not how well type-H borrowers are identified in absolute terms; or

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[A-6]For any informative binary signal, we can label the signal realization with the greater conditional probability for type L as "l" and express the signal structure as in (A.25)–(A.26).

[A-7]In this section we assume for simplicity that $\varphi_t^H = \varphi_t^L = \theta_t = 1$.

how well type-L borrowers are identified in absolute terms. What matters instead is that the signal distributions conditional on type-H differs from that conditional on type-L by a certain amount. In equations (A.25) and (A.26), a natural measure of the distance between the two conditional signal distributions is the difference between the probabilities in (A.25) and (A.26). That distance is z, independent of p, which is why threshold (A.29) continues to apply here unchanged.

With an aggregate lending standard $z_t \in [0, \bar{z}]$, a fraction $z_t / \bar{z}$ of banks screen, while the rest do not. This means that a flow of

$$\frac{z_t}{\bar{z}} \left(1 - p \left(1 - \bar{z}\right)\right) \kappa_{Ht} + \left(1 - \frac{z_t}{\bar{z}}\right) \kappa_{Ht}$$

type-H borrowers and a flow of

$$\frac{z_t}{\bar{z}} \left(1 - p\right) \left(1 - \bar{z}\right) \kappa_{Lt} + \left(1 - \frac{z_t}{\bar{z}}\right) \kappa_{Lt}$$

type-L borrowers all get funded. The law of motion of pool quality x_t, (7), in this economy is then slightly different,

$$\dot{x}_t = \left[\frac{z_t}{\bar{z}} \left(1 - p\right) \left(1 - \bar{z}\right) + \left(1 - \frac{z_t}{\bar{z}}\right)\right] \kappa_{Lt} \lambda - \left[\frac{z_t}{\bar{z}} \left(1 - p \left(1 - \bar{z}\right)\right) + \left(1 - \frac{z_t}{\bar{z}}\right)\right] \kappa_{Ht} (1 - \lambda) + \delta(\lambda - x_t).$$

At a steady state, $\dot{x}_t = 0$, and we find the steady state pool quality as

$$x = \lambda - \frac{z(1 - \lambda)\lambda}{\delta \kappa^{-1} + \zeta(z) + z(1 - \lambda)}$$

where

$$\zeta(z) = \frac{z}{\bar{z}} \left(1 - p\right) \left(1 - \bar{z}\right) + \left(1 - \frac{z}{\bar{z}}\right).$$

In particular, the screening steady state has pool quality

$$x^s = \lambda - \lambda \frac{(1 - \lambda)\bar{z}}{\delta \kappa^{-1} + (1 - p) \left(1 - \bar{z}\right) + \bar{z}(1 - \lambda)}. \tag{A.30}$$

Greater p has two effects on pool quality. More type-H borrowers are misidentified, which increases pool quality when banks are screening, all else equal. On the other hand, more type-L borrowers are correctly identified (holding $\bar{z}$ constant). This lowers pool quality. If we want to separate out the first effect, we write

$$x^s = \lambda - \lambda \frac{(1 - \lambda) \left(q_L - q_H\right)}{\delta \kappa^{-1} + 1 - q_L + \left(q_L - q_H\right) (1 - \lambda)} \tag{A.31}$$

where we defined

$$q_j \equiv Prob(\text{signal} = l \,|\, \text{type-}j)$$

As (A.31) shows, x^s is decreasing in the ability to screen out type-L borrowers, q_L. x^s increases, however, in the misidentification q_H of type-H borrowers.

Our approach in this section generalizes to arbitrary signal structures. Whenever banks can pay for an informative signal about their potential borrowers, it leads them to select, on average, type-H borrowers. This, however, directly implies that the borrower pool is, on average, worsened by the signal acquisition of banks. The steady state with signal acquisition, which in our paper

corresponds to the screening steady state x^s, always has a worse pool quality than the steady state without signal acquisition.

E.2 Lending standards with convex costs

Our main results rely on intermediate costs of lending standards. Neither too low, so that the banks always screen, nor too high, so that banks never screen. While it is natural to assume that the costs of lending standards are linear the same way it is natural to assume constant returns to scale in production at an industry level, in this section we show that the main implications of our model continue to hold when lending standards have increasing marginal cost of an intermediate size. Instead of linear cost $\tilde{c}z$, we now assume that banks have to pay a strictly convex cost $\tilde{c}(z)$. As before, we write normalized cost as $c(z) \equiv \frac{\tilde{c}(z)}{-r_L}$. We continue to assume, for simplicity, that $\varphi_t^H = \varphi_t^L = \theta_t = 1$. A bank's optimization problem is now given by

$$\max_{z \in [0,1]} \kappa_{Ht} r_t + (1-z) \kappa_{Lt} r_L - (\kappa_{Ht} + \kappa_{Lt}) \tilde{c}(z). \tag{A.32}$$

The first order condition now pins down an interior value for the optimal screening intensity z_t,

$$c'(z_t) = 1 - x_t. \tag{A.33}$$

Given that $c(z)$ is strictly convex, the optimal screening intensity z_t falls in pool quality x_t. This corresponds to a downward sloping line in Figure 1, instead of a step function. Figure E.1 illustrates the new steady state diagram. Since both (A.33) and (9) slope downward, there is the potential for multiple intersections. Each intersection's lending standard z is a solution to the equation

$$c'(z) = (1 - \lambda) \frac{1 + \delta \kappa^{-1}}{(1 - \lambda z) + \delta \kappa^{-1}} \tag{A.34}$$

which follows directly from combining (A.33) with (9).

The law of motion of pool quality is now given by

$$\dot{x}_t = \kappa (1 - x_t) \left(1 - (c')^{-1} (1 - x_t) \right) \lambda - \kappa x_t (1 - \lambda) + \delta(\lambda - x_t).$$

This differential equation predicts that $\dot{x}_t > 0$, that is pool quality improves whenever the solid line lies below the dashed one in Figure E.1; and vice versa, it predicts pool quality to deteriorate, $\dot{x}_t < 0$, whenever the solid line lies above the dashed one in Figure E.1.

In the example shown in Figure E.1, we denote by $\bar{x}$ the last intersection of (A.33) and (9). To the right of $\bar{x}$, no screening is optimal, and pool quality improves, ultimately converging to x^p. To the left of $\bar{x}$, screening is optimal and pool quality converges to x^s.

E.3 Screening with collateral

Our modeling assumptions imply that banks cannot use contract terms like fees or covenants to screen out type-L borrowers. Consider instead a situation in which banks use collateral requirements to screen borrowers. Let borrowers be endowed with a collateral asset and suppose banks require borrowers to post their collateral to receive a loan. As long as the value of the collateral exceeds the

Figure E.1: Steady-state equilibria with convex screening cost.
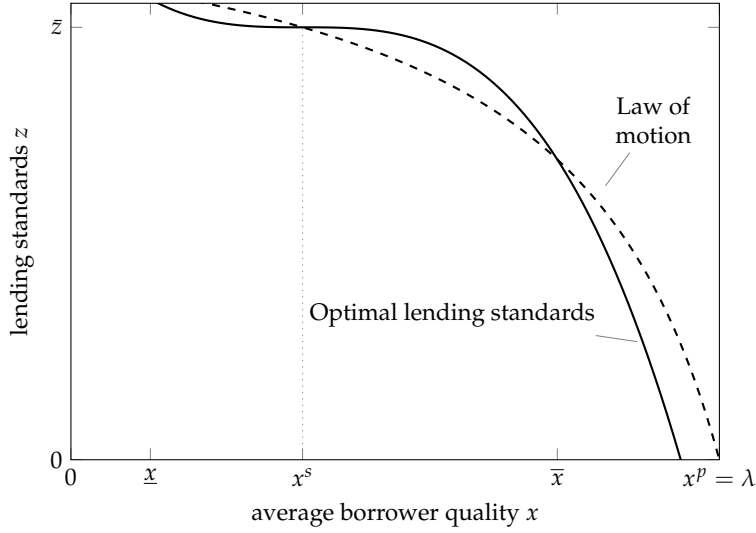
Note: This figure shows two curves whose intersections yield the steady-state pool quality x and the steady-state lending standard z. The solid line represents the optimal choice of the lending standard, (A.33). The dashed line represents the pool quality x that is caused by any given lending standard z through the law of motion. The cost chosen for this example is of the form $c(z) = c_0 z + c_1 (\bar{z} - z)^{c_2} + c_3$ with $c_1 > 1$, $c_2 > 1$, $c_0 > c_1 c_2 \bar{z}^{c_2 - 1}$, $c_3 = -c_1 \bar{z}^{c_2}$.

private benefits, type-L borrowers would not apply for loans if all banks require collateral. In this situation, the strategic complementarity remains, but the negative externality is eliminated.

However, in practice, several issues arise. First, when private benefits are unknown to banks, as long as the private value is greater than the collateral value for some type-L borrowers, they still apply for loans. In this case, collateral does not resolve the negative externality. Second, collateral values vary, and banks may be able to acquire costly private information about this value (e.g. appraise the collateral). In this case, the share of borrowers with good collateral would determine whether banks applied normal or tight lending standards to the collateral of borrowers, and these lending standards would act much like those in our model, exhibiting strategic complementarities and negative externalities among banks.

E.4   Positive bargaining weight of banks

In this section, we briefly describe what happens if we assume that there is a positive probability β with which banks are able to make a take-it-or-leave-it offer, rather than borrowers.

If banks make a take-it-or-leave-it offer, they pick the credit spread r_t in order to maximize $\Pi_t(r_t)$ subject to type-H's participation constraint,

$$r_H - r_t + u = J_t^H.$$

This means,

$$r_t = r_H + u - J_t^H.$$

Type-L borrowers always accept a loan, under any credit spread r_t. Thus, with probability $\beta > 0$,

banks obtain the entire extractable surplus,

$$\Pi_t(r_t) = S_t = \max_z \frac{\kappa_{Ht}}{\mathcal{K}_t} \left\{ r_H + u - J_t^H \right\} + \frac{\kappa_{Lt}}{\mathcal{K}_t} \left\{ (1 - z)r_L \right\} - \tilde{c}z$$

Thus, from an ex-ante perspective, a bank entering a meeting expects a profit of

$$\Pi_t^{ante}(r_t) = \beta S_t.$$

This means that banks strictly prefer to be active, $\mathcal{J}_t = \mathcal{J}$, whenever $S_t > 0$, and are indifferent about being active, $\mathcal{J}_t \in [0, \mathcal{J}]$, whenever $S_t = 0$.

Observe that the optimal lending standard is unaffected by the value of β. This leaves our results in Sections 4.3, 5, 6.1, and 6.2 unaffected by $\beta > 0$. The analyses in Sections 4.2 and 6.3 have to be adjusted somewhat whenever $\beta > 0$ (though by very little if β is small). We leave such an adjustment for future work.

E.5   Further non-essential assumptions

We simplified the analysis by assuming that the banking sector is competitive so that banks make zero profits. We conjecture that the qualitative features of the steady-states, dynamics and welfare results would remain if banks shared the surplus of a match with a given borrower.

Other changes to the screening technology are less consequential. There is also the possibility of more than two stable steady states, which would occur when the optimal lending standards line in Figure 1 decreases more smoothly and so has more intersections with the $\dot{x} = 0$ line. We assumed that screening produces a binary signal, and it would be inconsequential to instead assume a continuous signal as banks would simply choose a cutoff value for their binary lending decision. Lastly, if, when lending standards are tight, the probability that a given type-L borrower is funded by any bank were correlated across banks, then the dynamic strategic complementarity at the heart of our model would be stronger. This would occur because when one bank screens and rejects a borrower, such a correlation would make it easier for the next bank to detect that borrower as bad and so would raise the private value to screening.

Finally, our model has debt contracts. But because the model has only two borrower types, an equity contract can deliver the same payoffs to banks and borrowers of each type. With more types, our model could become significantly more complex. While the degree of complexity would depend on how well the screening technology detected different types, the extensions we have considered have all involved more state variables, which raises the possibility of non-linear dynamics that can occur in such systems.

F   Credit bureaus

One reason underlying the negative externality from tight lending standards is the assumption that information on previous rejections is unobservable and non-verifiable. Does this mean our model is inapplicable to credit markets in which credit bureaus track borrowers?

First, note that our model applies to information above and beyond publicly available information. As we described, it applies to borrowers within a given credit score bracket, which summarizes the past credit information contained in the credit bureau. Second, credit bureaus typically do *not* track much of the information that lenders might investigate prior to making a loan, and that might be uncovered by tight lending standards in our model. Appendix F contains extensive details about

the prevalence, coverage, and information provided by credit bureaus around the world. None of the countries that we investigated have credit bureaus that report whether credit was denied or instead turned down by the borrower. While many credit bureaus, like consumer bureaus in the U.S., delineate whether a credit check is *hard*, meaning associated with an application for credit, or *soft*, due to account review, marketing, or possibly hiring, only about half of the credit bureaus report the purpose of previous credit checks. Even with information on hard credit checks, lenders typically cannot tell whether a borrower who recently applied for a loan, applied for a mortgage, a car loan, or a credit card (again, see Appendix F for details).[A-8]

Finally, we note that our model may apply even in situations in which credit bureaus accurately track borrowers if lending standards are applied to collateral instead of borrowers. That is, if tight lending standards evaluate and reject on the basis of collateral (e.g. an appraisal of a house), then credit bureaus do not address the externality of tight lending standards because they track borrowers, not the assets they wish to fund.

Given the negative externality in our model, why don't credit bureaus track rejections? Our model suggests that bureaus do not track credit rejections because it is not incentive compatible for a bank-borrower pair to report a negative evaluation or to report a rejection.[A-9] Statements from credit bureaus are consistent with this reasoning and suggest that credit bureaus are unable to enforce the reporting of soft information that it is not privately optimal to report (see Appendix F).

While mature credit markets in legal environments with low-cost enforcement mechanisms may track information about borrowers that mitigates the negative externality associated with tight lending standards, we conclude that this tracking appears to be insufficient to eliminate the key externality in most countries' credit markets.

This appendix provides additional information on credit bureaus around the world. The on-line Appendix D provides the data that underlie this Appendix.

Credit bureaus, as opposed to credit registries, track borrowers and provide information about them to lenders.[A-10] When a borrower approaches a lender that is a member of a credit bureau, the lender can perform a *credit check* before making a loan, which involves getting a *credit report* from the bureau. Credit reports provide information on borrowers including existing credit and payment histories. In addition, many credit bureaus keep track of information about past credit checks and include this information on credit reports. Table F.1 describes credit reports for credit bureaus in different countries around the world (underlying data sources are provided in on-line Appendix D).

[A-8]Further, a past credit check without a subsequent loan does not indicate that a given borrower failed a past lending standard. The borrower may have applied for a job, or may have simply decided not to take the loan (decided not to buy that house or car, or decided to pick a different credit card). Importantly, in practice, lenders can evaluate borrowers before verifying their information via a credit report and leave no trail of credit checks for rejected borrowers. That is, as is common in mortgage markets for example, banks can fully apply their lending standards on the basis of information reported by the borrower and additional information gathered by the bank, and only verify information with a credit report for borrowers that pass the lending standard. Thus many applications are not recorded.

[A-9]Other models suggest different reasons. For example Axelson and Makarov (2020) show the striking result that introducing a credit registry that tracks borrowers' loan application histories but not the borrowing rates offered can lead to more adverse selection and quicker market breakdown. In that model, acquiring information on a borrower is costless and the result follows from the fact that a lender who knows that a borrower's offer was rejected does not know whether the borrower was bad or whether the borrower demanded a too-low interest rate.

[A-10]Credit registries are more widespread than credit bureaus, but registries only track the history of outstanding credit and/or loan payments and delinquencies. In our model, and probably in reality, outstanding loans do not assist banks in discriminating among borrowers who have recently been rejected. Credit registries seem to serve the purpose of providing information to assist a bank in setting loan terms, such as loan amount and interest rate based on payments-to-income ratio and/or pre-existing liens on collateral.

Table F.1: Data captured by credit bureaus

	Coverage? (consumers/firms)	Reporting required?	Credit checks				Rejections reported?
			On report for ≤ months	Hard check labeled?	Who requested?	Purpose?	
Advanced Economies							
Australia	Both		60	Yes	Yes	Yes	No
Canada	Both		72	Yes	Yes	No	No
European Union (AnaCredit)	Firms	By law	0	NA	No	No	No
France	Firms	By law	0	NA	No	No	No
Germany	Both	For access	12	Yes	No	No	No
Ireland	Both	By law	60	Yes	Yes	Yes	No
Italy	Both		6	Yes		Yes	No
Japan	Consumers	For access	6	Yes	Yes	Yes	No
Singapore	Both		24	Yes	Yes	Yes	No
South Korea	Both		0	NA	No	No	No
Taiwan	Both		3	Yes	Yes	Yes	No
United Kingdom	Both		24	Yes	Yes	Yes	No
United States	Both	Voluntary	24	Yes	Yes	Yes	No
Emerging Economies							
China	Both	By law	24	Yes	Yes	Yes	No
India	Both		24	Hard only	Yes	Yes	No
Malaysia	Both		12	Hard only	Yes	No	No

Blank cells are missing data.
Note: All information is from consumer credit reports and Bureau FAQs, except for EU and France, see Appendix for sources.

In most countries, a bank that conducts a credit check can generally observe past credit checks and whether the borrower subsequently did or did not receive a loan. The information in the bureaus tends to be available only to entities in the bureau's network, although some countries' bureaus sell the information to entities outside the credit market. In some countries like Japan and Germany, bureau members are required to report in exchange for access, but in other countries reporting is voluntary or only required by bureau members (second column of Table 1,). Most credit bureaus, like consumer bureaus in the US, delineate whether a credit check is *hard*, meaning associated with an application for credit, or *soft*, due to account review, marketing, or possibly hiring. Records of credit inquiries stay in credit report from 2 months in Taiwan to 24 months in the U.S. to 60 months in Ireland.

Importantly, however, none of the countries that we investigated have credit bureaus that report whether credit was denied or turned down by the borrower (final column of Table 1).[A-11] Further, credit bureaus generally contain only rudimentary information about the initiator of previous credit checks, such as whether they were banks, mortgage brokers, utilities, etc., and some in some countries, such as South Korea, France, and Germany, even this information is not recorded (fifth column). And only about half of the credit bureaus report the purpose of previous credit checks (sixth column), so that a credit card issuer for example does not know if a previous credit check was associated with an application for a credit card, mortgage, car loan or job.

A past credit check without a subsequent loan does not indicate that a given borrower failed a past lending standard. The borrower may have simply decided not to take the loan (decided not to buy that house or car, or decided to pick a different credit card). Importantly in practice, lenders can evaluate borrowers before verifying their information via a credit report and leave no trail of credit checks for rejected borrowers. That is, as is common in mortgage markets for example, banks can fully apply their lending standards on the basis of information reported by the borrower and additional information gathered by the bank, and only verify information with a credit report for

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[A-11]For example, Experian UK states "Here's what our role doesn't involve: - We aren't told which applications are successful or refused. - We don't know why you may have been refused credit."A possible exception is Experian Italy.

borrowers that pass the lending standard. Thus many applications are not recorded.

As noted, our theoretical model suggests that bureaus do not track credit rejections because it is not incentive compatible for banks to report rejections. Statements from credit bureaus are consistent with this reasoning and suggest that credit bureaus are not able to enforce the reporting of soft information that it is not privately optimal to report. First, bureaus state that they want to avoid arbitrating arguments over rejections. Rejection is easy to hide (e.g. just offer unfavorable loan terms) and hard to verify (consistent with our assumptions). Second, bureaus store only verifiable information due to privacy and legal concerns. Credit checks are hard information, rejections are not. Every credit bureau lists data verification and correction measures on their website.

Finally, we re-emphasize that our model may apply even in situations in which credit bureaus accurately track borrowers if lending standards are applied to collateral instead of borrowers. That is, if tight lending standard evaluate and reject on the basis of collateral (e.g. an appraisal of a house), then credit bureaus do not address the externality of tight lending standards because they track borrowers not the assets they wish to fund.

We conclude that mature credit markets in legal environments with low-cost enforcement mechanisms may exhibit various mechanisms for mitigating, but maybe not eliminating, the negative externality associated with tight lending standards.